

# The Complementarities of Accounting Information and Its Off-equilibrium Role\*

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## Abstract

This paper studies the role of financial reporting in enhancing the credibility of other sources of information. We find that an interim accounting report can discipline management's investment behavior and its effectiveness increases with accounting quality. As a consequence, when accounting quality increases, the investment itself becomes a sufficiently credible signal of management's more valuable private information, and therefore induces a dramatic market response. In this sense, the two pieces of information serve as compliments. However, since most information has already been preempted by the investment, the market's response to subsequent earnings announcement declines. Our results are consistent with the confirmatory role by [Gigler and Hemmer \(1998\)](#) in that the value of accounting is off equilibrium. Specifically, it is the threat that accounting will reveal an inefficient investment that prevents it from happening. Nevertheless, the mechanism in our paper is completely different, and the economic magnitude of the off-equilibrium role is first order, rather than a result of improved risk sharing. Therefore, to assess the value of accounting, we cannot simply rely on the equilibrium market responsiveness to earnings.

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# 1 Introduction

One of the most enduring questions in accounting research is studying whether mandatory financial reporting is an important source of information for investors. Starting with [Ball and Brown \(1968\)](#), numerous studies have found that accounting earnings are informative to the capital market, in that stock prices, on average, adjust to the public release of accounting earnings. However, several other studies have also noted that, for the most part, the price reaction *anticipates* the earnings disclosure, evidenced by the notorious low earnings response coefficient and  $R^2$  from earnings-returns regressions. For example, [Lev \(1989\)](#) finds that the low explanatory power ( $R^2$  of 2%-7%) is robust to different lengths of the return window, and suggests that financial reporting fails to fulfill its goal. [Ball and Shivakumar \(2008\)](#) also find low information content of quarterly earnings announcements. The weak market response to earnings could result from measurement errors in earnings ([Easton, Harris, and Ohlson, 1992](#)), investors' limited attention ([Hirshleifer, Lim, and Teoh, 2009](#)), the increasing complexity of financial reporting ([You and Zhang, 2009](#)), etc.

Meanwhile, investors are deluged with large amounts of information in reality, from sources as varied as a firm's detailed financial statements, to fast-changing bits of gossip in social media such as Twitter. These other sources of information are usually timely, readily comprehensible, and forward looking; whereas financial statements are periodic, more complex, and backward looking. As a consequence, other information has played an increasingly important role in facilitating investors' decision making compared to firms' financial reports, consistent with the survey evidence.<sup>1</sup> In other words, financial reports seem to play a trivial role in informing investors' about the firm's performance, as it can be easily *substituted* by other sources of information. Given these arguments, we face a conundrum of understanding the fundamental role of accounting and why it is important.

[Gigler and Hemmer \(1998\)](#) are among the first to tackle this question. They find that mandatory disclosures play a *confirmatory role* in creating an environment for management to credibly communicate their more value-relevant information. As a result, the market reaction to subsequent mandatory disclosures is the weakest when they are work-

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<sup>1</sup>For instance, a recent survey conducted by the National Financial Capability Study find that 68% of investors use information directly from the company, 62% use information from financial service companies such as analysts' reports, and 44% use media such as newspapers or online news, however, only 21% are aware of the SEC's EDGAR database. See [http://www.usfinancialcapability.org/downloads/NFCS\\_2015\\_Inv\\_Survey\\_Full\\_Report.pdf](http://www.usfinancialcapability.org/downloads/NFCS_2015_Inv_Survey_Full_Report.pdf).

ing the best to provide credibility for voluntary disclosures.<sup>2</sup> In other words, accounting *complements* rather than substitutes for other sources of information. Ball, Jayaraman, and Shivakumar (2012) provide supporting evidence for the confirmatory role: when firms commit more resources to financial statement verification as measured by audit fees, management forecasting activity becomes more frequent and accurate, and market reactions to those forecasts also become more significant.

In this paper, we broaden insights into why weak market reactions to mandatory disclosures could be a sign of accounting's fulfilling its role of ensuring better informed markets. The commonality between the explanation we propose and that of Gigler and Hemmer (1998) is that mandated accounting disclosures are disciplining some other visible activities, and thus compliment the value relevance of other information. In their paper, accounting disciplines voluntary disclosures; whereas in this paper, as in Kanodia and Lee (1998), accounting disciplines firms' investment decisions. Ball (2001) suggests that one of the most important economic roles of accounting is disciplining managers' voluntary disclosures and their investment behavior. In this sense, our paper compliments Gigler and Hemmer (1998). Another commonality is that in both cases, when accounting is working, its value is *off-equilibrium*. More specifically, it is the *threat* of accounting's revealing an inefficient activity that *prevents* the activity, and since the activity in question is prevented, it is neither revealed by accounting disclosures nor by any rational pricing. Consequently, one cannot measure the effectiveness of accounting by only looking to the equilibrium disclosures or to equilibrium market prices.

We model a simple overinvestment problem similar to that of Kanodia and Lee (1998) to feature the disciplinary role of periodic performance reporting, such as an earnings report. Our first result is qualitatively similar to theirs: increasing the information in the earnings report decreases the amount of overinvestment. However, because overinvestment manifests itself differently across the two models, the informational (and therefore market pricing) implications are dramatically different. In their model, every firm overinvests, and the amount of that overinvestment decreases with the precision of earnings reports. Even so, market participants are able to perfectly infer the private information on which the firm based its investment, so the price always perfectly impounds the information regardless of the precision of the earnings report. In our model, however, some firms

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<sup>2</sup>A similar model is used by Gigler and Hemmer (2001, 2002).

overinvest and others do not. Increasing the information in the earnings report decreases the likelihood that a firm will overinvest, but in general it is impossible to tell whether a particular firm is or is not overinvesting. This feature is crucial for studying the main point of our paper: how the degree to which an investment decision reveals a firm's private information *changes* with the precision of the earnings report—and how this in turn *affects* the earnings-return relationship studied in the aforementioned empirical literature.

Our main result is that the price reaction to the earnings report can actually decrease when its precision increases. To understand the intuition, since the overinvestment is curtailed as the precision of earnings reports increases, investors are more assured that the right investment decision was made in the first place. In other words, the decision of whether or not to invest becomes more informative about the manager's private information and about the efficiency of the investment. As a consequence, investors' uncertainty about the firm's fundamental decreases dramatically upon observing the investment, which means the market reaction to the investment is overwhelmed. By contrast, when the actual earnings report comes out, investors only obtain incremental information because most uncertainty has already been resolved when the investment decision was made. This could happen because the earnings report is verifiable yet a noisy and, possibly, less timely signal of management's private information (Dye, 1983; Gigler and Hemmer, 1998). Therefore, the price reaction to earnings is only marginal.

Our results also have important empirical implications. The prior literature has been relying on value relevance, as measured by the price reaction to earnings, to evaluate the effectiveness of the financial reporting (see, as reviewed by Barth, Beaver, and Landsman, 2001). Relevance, however, is introduced by the Financial Accounting Standard Board (FASB) as the ability to evaluate the potential effects on future cash flows (predictive value) *or* to confirm/correct their previous evaluations (confirmatory value).<sup>3</sup> Therefore, in our context, value relevance corresponds to the notion of *predictive value*, whereas the complementary role is consistent with the *confirmatory value*. Our results imply that focusing one role on a *stand-alone basis* while ignoring the other can cause tremendous loss of information, which is also inconsistent with the objective of financial reporting. Instead, the value of accounting should be measured based on its contribution to the entire information environment.

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<sup>3</sup>See the FASB Conceptual Framework: [http://www.fasb.org/pv\\_conceptual\\_framework.pdf](http://www.fasb.org/pv_conceptual_framework.pdf)

The remainder of this paper is organized as follows. Section 2 describes the model setup. Section 3 solves the equilibrium, characterizes the complementary role of accounting, and provides the intuition. Section 4 discusses robustness, and Section 5 concludes. All proofs are included in the appendix.

## 2 Model Setup

We model a situation in which a firm's manager makes an investment decision on behalf of current shareholders. Specifically, the firm has investment capital of  $K$  and, if the capital is invested, the investment can either succeed or fail. The return of the investment, or equivalently the terminal cash flow is

$$\tilde{C} = \begin{cases} R, & \text{if success} \\ 0, & \text{if failure} \end{cases}$$

While nobody can perfectly predict the outcome of the investment, we can think of the eventual return on the investment as the true unknown state of the world (because it is exogenous), and we denote the state as  $S$  and  $F$ . However, when the manager decides whether or not to make the investment, he has superior private information, which we denote as  $\theta \in \{G, B\}$ , about the expected return on the investment. In the following text, we call them firm  $G$  and firm  $B$  respectively. The manager's private information  $\theta$  is correlated with  $\tilde{C}$  as follows:

$$Prob(S|G) = P_g; \quad Prob(S|B) = P_b,$$

where  $0 \leq P_b < P_g \leq 1$ . The commonly held prior belief about the investment is represented by  $\rho = Prob(G)$ .

Furthermore, we assume the investment is ex ante profitable only for firm  $G$ , i.e.,  $P_g R > K > P_b R$ . After an investment decision is made, it becomes publicly known, that is, whether or not the firm invested is perfectly observable.<sup>4</sup> The manager, however, has no way of verifiably informing outsiders of what he learned about the investment's

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<sup>4</sup>Since the investment is a binary variable and requires fixed capital  $K$ , measurement error means the market may observe the investment even though the manager chose not to invest, or vice versa. Including this additional uncertainty does not change our results.

prospects,  $\theta$ , at the time when the investment was made.<sup>5</sup>

At date 1, an interim performance report is produced if the manager invested at date 0. If the manager chose not to invest, there is no performance report. We refer to this interim performance report as an earnings report  $\tilde{Y} \in \{y_h, y_l\}$ . The earnings report is informative about the return of the investment and hence is correlated with  $\theta$ , and the correlation is summarized as follows:

$$\begin{aligned} \text{Prob}(y_h|G) &= \tau \\ \text{Prob}(y_h|B) &= 1 - \tau \end{aligned}$$

It can be easily verified that both posterior probabilities  $\text{Prob}(G|y_h)$  and  $\text{Prob}(B|y_l)$  are strictly increasing in  $\tau$ , so we denote  $\tau$  as accounting quality or earnings quality. Without loss of generality, we assume  $\frac{1}{2} \leq \tau \leq 1$ .<sup>6</sup>

We also assume that the future cash flow  $\tilde{C}$  and the earnings report  $\tilde{Y}$  are independent conditional on  $\theta$ , i.e.,  $\theta$  is a sufficient statistic for  $Y$  with respect to  $\tilde{C}$ . In addition, the earnings report is mandatory and audited so the manager cannot manipulate. These assumptions altogether imply that mandatory disclosures are ex post verifiable, but also less informative and less timely compared to the manager's private information.

At date 2, the firm is sold to a second generation of owners for exogenous reasons such as liquidity needs or shorter life cycles. The new generation of shareholders will receive ownership to the return on the firm's investment,  $\tilde{C}$ , if the capital is invested, or to the capital,  $K$ , if it was not invested. At the time of sale, potential buyers (investors) know whether or not the manager chose to invest its capital, but they do not know what the manager privately knew about the investment's prospects,  $\theta$ . Instead, they must infer  $\theta$  from the firm's investment decision at date 1. Buyers are assumed risk neutral and perfectly competitive, so the price of the firm is equal to the expected return on the firm's investment given all information available to the market.

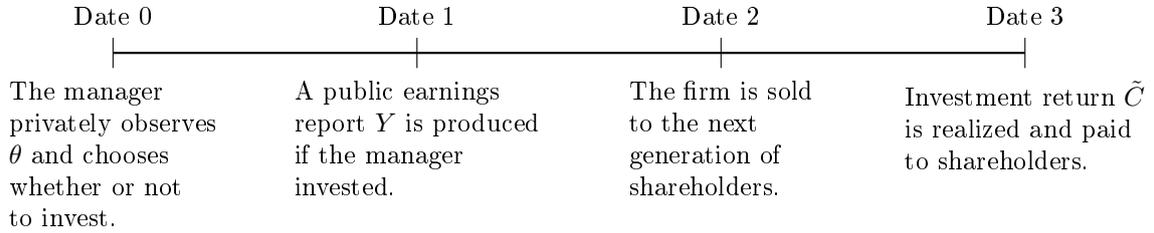
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<sup>5</sup>In classic disclosure theory, direct communication such as voluntary disclosures is truthful by assumption. Other papers find that voluntary disclosures are credible for some endogenous reasons. For example, [Gigler \(1994\)](#) finds that proprietary costs can make voluntary disclosures credible even without verification. [Gigler and Hemmer \(1998\)](#) find that mandatory disclosures can enhance the credibility of voluntary disclosure. Since we do not focus on optimal communication in this paper, the direct communication channel is excluded from the model.

<sup>6</sup>Throughout the paper, we use the terms accounting quality, earnings quality and information precision interchangeably. If  $\tau = \frac{1}{2}$ , the accounting signal is completely uninformative.

Finally, the investment return  $\tilde{C}$  is realized and distributed to the new generation of shareholders. If the manager did not invest the initial capital  $K$ , the capital is returned to the new owners.

The following timeline summarizes the model:



TimeLine

### 3 Analysis

#### 3.1 The Equilibrium

We first examine two benchmark cases.

**Lemma 1.** *1. In the first best, firm  $G$  always invests and firm  $B$  never invests, and the earnings report is irrelevant.*

*2. In the second best, suppose there is no earnings report; all firms invest when  $\rho \geq \rho^*$ , and no firm invests when  $\rho < \rho^*$ .*

In the first best where  $\theta$  is publicly observable, the earnings report  $Y$  is not incrementally informative about  $\tilde{C}$  because they are assumed independent conditional on  $\theta$ . As a result, the market price does not respond to the earnings report. On the contrary, since investors observe  $\theta$  and whether the investment was made, the market price of the firm will be  $P_g R$  or  $P_b R$  if the firm did invest; or will be  $K$  if the firm did not invest. Because  $P_g R > K > P_b R$ , firm  $G$  always invests and firm  $B$  never invests. In addition, the first best outcome can be attained if the manager is not myopic, i.e., the manager maximizes expected returns rather than market prices.

Suppose  $\theta$  is privately observed by the manager, we consider two extreme cases. In the first case when the earnings report perfectly reveals  $\theta$ , it is equivalent to  $\theta$  being publicly observable and the first best is obtained. In the other extreme, however, when the earnings

report is completely uninformative, it will be ignored and the market price is the same for all firms that chose to invest. Therefore, if the *a priori* expected payoff of the investment is greater than its cost, i.e.,  $\rho P_g R + (1 - \rho) P_b R \geq K$ , all firms will invest. In other words, the information asymmetry between the manager and investors leads to overinvestment compared to the first best. Similarly, if the *a priori* expected payoff is lower than its cost, no firm will invest, leading to underinvestment compared to the first best. Furthermore, since “investing” is off the equilibrium path, investors cannot use Bayes’ theorem if the investment is observed, but instead, they will hold certain beliefs. In this case, a natural off-equilibrium belief is that, any firm that chose to invest is  $B$ , and consequently no firm will deviate and the equilibrium is sustained.

We next consider a more interesting case in which the earnings report is informative, yet not perfectly so, i.e.,  $\frac{1}{2} < \tau < 1$ . We construct a perfect Bayesian equilibrium (*PBE*). At date 0, the manager privately observes  $\theta$  and chooses whether or not to invest initial capital  $K$  to maximize the expected price at date 2. At date 1, an earnings report  $\tilde{Y}$  becomes public, and investors update their beliefs about  $\theta$  based on the realized report as well as the conjectured investment strategy. At date 2, the stock price changes based on available public information, and current shareholders sell their shares to the next generation. Lastly, rational expectation requires that investors’ conjectured investment strategy be consistent with the actual strategy taken by the manager at date 0. Therefore, the equilibrium can be written as follows:

**Definition 1.** A *PBE* consists of a triplet  $\{I(\theta), P(\hat{I}(\theta), y), \phi(\theta|\hat{I}(\theta), y)\}$ :

1.  $I(\theta) \in \arg \max_I E_y[P(\hat{I}(\theta), y)|\theta]$  maximizes the expected market price conditional on observing the actual  $\theta$ .
2.  $P(\hat{I}, y) = E[\tilde{C}|\hat{I}(\theta), y]$  is the date 2 market price given the earnings report  $y$  and investors’ conjectured strategy  $\hat{I}(\theta)$ .
3.  $\phi(\theta|\hat{I}(\theta), y)$  is investors’ belief about  $\theta$ , which depends on the conjectured strategy, whether the manager invested, and the earnings report  $y$ .
4.  $I(\theta)$  and  $\hat{I}(\theta)$  coincide, which means that the conjecture obeys rational expectations.

We solve the equilibrium by backwards induction. At date 2, investors make inferences about  $\theta$  using two pieces of information: the earnings report and the investment. The

information content of earnings is fixed and exogenous, whereas the information content of the investment is driven by investors' conjecture  $\hat{I}$ . In equilibrium, even though  $\hat{I}$  coincides with the actual  $I$ , the manager is unable to change the conjecture. Therefore, we can derive the market price by expanding the following conditional expectation:

$$\begin{aligned} E[\tilde{C}|\phi(\hat{I}, y)] &= RPr(S|\hat{I}, y) = Pr(G|\hat{I}, y)Pr(S|G, \hat{I}, y)R + Pr(B|\hat{I}, y)Pr(S|B, \hat{I}, y)R \\ &= Pr(G|\hat{I}, y)P_gR + Prob(B|\hat{I}, y)P_bR \end{aligned}$$

The last equality follows the assumption that  $\tilde{C}$  and  $Y$  are independent conditional on  $\theta$ . Therefore, the stock price only depends on the posterior probabilities  $Pr(G|\hat{I}, y)$  given the firm invested, i.e.,

$$\begin{aligned} P(\hat{I}, y_h) &= Pr(G|\hat{I}, y_h)(P_gR - P_bR) + P_bR \\ P(\hat{I}, y_l) &= Pr(G|\hat{I}, y_l)(P_gR - P_bR) + P_bR \end{aligned}$$

For brevity, we denote  $P(y_h) = P(\hat{I}, y_h)$  and  $P(y_l) = P(\hat{I}, y_l)$ . Therefore, anticipating that investors will hold the right conjecture  $\hat{I} = I$ , the manager makes the optimal investment decisions at date 0.

**Proposition 1.** *There is a unique PBE in which firm  $G$  always invests and firm  $B$  invests with probability  $q \in (0, 1]$ .*

*Proof.* All proofs are in the appendix. □

Compared to the first best, firm  $B$  invests with probability  $0 < q \leq 1$ . If  $q = 1$ , both firm  $G$  and  $B$  invest and the equilibrium is pooling. If  $q < 1$ , firm  $G$  always invests but firm  $B$  takes a mixed strategy, leading to a hybrid equilibrium. Therefore, the equilibrium is unique—it is either pooling or hybrid, and investors' conjecture of  $\hat{I}$  is equivalent to a conjecture of  $\hat{q}$ . Furthermore, the parameter  $q$  captures the level of investment inefficiency resulting from the information asymmetry between the manager and investors.

Obviously, firm  $B$  is better off in the pooling equilibrium, i.e., firm  $B$  always has incentives to mimic firm  $G$ . However, this pooling equilibrium may not be attainable. The intuition is as follows. Suppose the equilibrium is pooling and investors also correctly believe that  $\hat{q} = 1$ ; as for firm  $B$ , if his expected payoff from investing is greater than the

cost, i.e.,  $E_y[P(y; \hat{q} = 1)|B; invest] \geq K$ , he will always invest and the pooling equilibrium is sustained. However, if the expected payoff from investing is less than the cost, i.e.,  $E_y[P(y; \hat{q} = 1)|B; invest] < K$ , firm  $B$  will deviate to never invest, suggesting that the conjecture  $\hat{q} = 1$  is incorrect. As a result, the pooling equilibrium is not attainable and investors will revise the belief to a point such that firm  $B$  becomes indifferent between investing and not investing.<sup>7</sup>

Next, we further characterize the equilibrium.

**Proposition 2.** 1. If  $\rho < \rho^*$ , firm  $G$  invests with probability 1 and firm  $B$  invests with probability  $q(\tau, \rho) < 1$ .

2. If  $\rho \geq \rho^*$ , there exists a unique threshold  $\frac{1}{2} \leq \hat{\tau}(\rho) < 1$  such that when  $\tau \leq \hat{\tau}(\rho)$ , firm  $G$  and firm  $B$  both invest with probability 1; otherwise, firm  $G$  invests with probability 1 and firm  $B$  invests with probability  $q(\tau, \rho) < 1$ . The threshold  $\hat{\tau}(\rho)$  is strictly increasing in  $\rho$ .

3. Furthermore,  $q(\tau, \rho)$  is decreasing in  $\tau$  and increasing in  $\rho$ , and  $\frac{\partial q^2}{\partial \tau \partial \rho} \leq 0$ .

In Proposition 2, we find that high quality accounting works more effectively in mitigating the investment inefficiency resulting from asymmetric information and management myopia, consistent with the prior literature. For example, [Biddle and Hilary \(2006\)](#); [Biddle, Hilary, and Verdi \(2009\)](#) find that higher quality accounting reduces information asymmetry between managers and outside suppliers of capital, thereby enhancing firms' investment efficiency. The intuition of our result is very simple: When accounting quality increases, investors can differentiate firm  $G$  and  $B$  with a more informative earnings report  $y$ . As a consequence, the investment becomes less profitable for firm  $B$  and thus he invests with lower probabilities.

To outline the proof of Proposition 2, we first consider the possibility of pooling equilibrium. Recall that if the *a priori* value of the investment is negative and the earnings report is uninformative, neither firm  $G$  or  $B$  would invest in equilibrium. After introducing an informative earnings report, however, the market price will depend on  $y$ . As a result, firm  $G$  is able to separate himself from  $B$  and thus chooses to invest. On the other hand, because accounting is not perfectly informative, firm  $B$  still has incentives to mimic firm

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<sup>7</sup>Following the literature, we assume that when an "agent" is indifferent, he follows what the "principal" expects him to do.

$G$  so the equilibrium  $q > 0$ . Therefore, mandatory disclosures eliminate the inefficiency of firm  $G$ 's underinvestment at the expense of firm  $B$ 's overinvestment. Nonetheless, the pooling equilibrium is never attainable. To see this point, suppose the equilibrium is pooling and investors also believe so; the law of iterated expectation implies

$$\mathbb{E}_\theta\{\mathbb{E}_y[P(y)|\theta]\} = \mathbb{E}_\theta\{\mathbb{E}_y[\mathbb{E}(\tilde{C}|y)|\theta]\} = \mathbb{E}[\tilde{C}] < K$$

where  $\mathbb{E}$  denotes investors' expectation based on conjecture  $\hat{q} = 1$ . In other words, the weighted average of  $\mathbb{E}_y[P(y)|B]$  and  $\mathbb{E}_y[P(y)|G]$  is less than  $K$ , suggesting that  $\mathbb{E}_y[P(y)|B]$  must be lower than  $K$ .

In contrast, when the *a priori* value of the investment is positive, we find that the equilibrium can be pooling if and only if accounting quality is adequately low. To understand the intuition, if the earnings report is completely uninformative, firm  $B$ 's default strategy is to pool with firm  $G$  by proposition 1. When the earnings report becomes informative, firm  $B$  will realize signal  $y_l$  with a high probability and consequently receive a low market price. Therefore, the earnings report helps investors to distinguish  $\theta$  and therefore reduces firm  $B$ 's benefit of mimicking firm  $G$ . However, if accounting quality is not sufficiently high, the threat of being revealed is not strong enough such that firm  $B$  still takes his default strategy. As a consequence, the equilibrium is pooling and accounting fails to discipline firm  $B$ 's overinvestment.

To see why the equilibrium threshold is decreasing in  $\tau$ , note that firm  $B$  must be indifferent between investing and not investing in the hybrid equilibrium, i.e.,

$$\underbrace{(1 - \tau)P(y_h)}_{\text{decrease or increase with } \tau (?)} + \underbrace{\tau P(y_l)}_{\text{decrease with } \tau (-)} = K \quad (1)$$

Suppose investors' conjectured  $\hat{q}$  does not change with  $\tau$ ; intuitively,  $P(y_l)$  will go down and  $P(y_h)$  will go up as the information content of  $y_h$  and  $y_l$  increases. Firm  $B$ , on the one hand, is more likely to receive  $y_l$ , suggesting that high quality accounting will reduce the second term  $\tau P(y_l)$ . On the other hand, firm  $B$  is less likely to receive  $y_h$ , which means the first term  $(1 - \tau)P(y_h)$  can be decreasing or increasing in  $\tau$ , depending on the magnitude of  $P(y_h)$ . Taking them together, we conjecture that the negative effect on the second term will always dominate the ambiguous effect on the first term. As a

result, to bring the LHS back to  $K$ , investors will revise their conjecture  $\hat{q}$  down, which consequently decreases the actual investment strategy  $q$  as well. The result confirms our conjecture. Therefore, high quality accounting information must reduce firm  $B$ 's benefit of mimicking firm  $G$  and ultimately his incentive.

More importantly, Proposition 2 suggests an *off-equilibrium role* of accounting: It is the threat that interim reports will reveal the manager's inefficient investment that prevents the manager from taking the investment in the first place. However, the inefficient action becomes less likely to be observed or revealed by accounting, precisely because it is not on the equilibrium path. In other words, when we observe an inefficient investment being revealed by accounting, it probably suggests that accounting was not fulfilling its objectives in the first place. As a consequence, we cannot evaluate the effectiveness of accounting by looking to the equilibrium accounting disclosures or to equilibrium prices.

Our results are also consistent with the *disciplinary effect* of periodic performance report raised by [Kanodia and Lee \(1998\)](#), but there are several differences. First, in their model, the manager chooses the level of investment and the precision of disclosures simultaneously, and they find that more precise disclosure can more effectively discipline the manager's investment. The bundling of the two choices is essential and has a value: "...This bundling creates a need for regulating the precision of performance reports; without regulatory intervention the economy would face a difficult implementation problem." However, in our model, the information structure is exogenous. In reality, managers usually have limited discretion choosing the precision of accounting rules, instead, regulators design and enforce a more efficient accounting system.

In addition, we assume that the action space is discrete for two reasons. First, for modeling purposes, we want to avoid the fully revealing equilibrium of [Kanodia and Lee \(1998\)](#) in which the manager's action perfectly reveals his private information. Second, to ensure the information content conveyed by the manager's action can be sufficiently high relative to his private information, we assume that the dimension of the action space is the same as that of the state space. However, the intuition behind our model is generalizable beyond the binary setting.

Lastly, the real effects literature mainly focuses on how different accounting measurement rules affect resource allocations in various settings. For example, [Kanodia, Sapra, and Venugopalan \(2004\)](#) find that measuring intangibles may be undesirable when the

noise of the measurement is sufficiently severe, because the firm inefficiently changes its allocation of tangible and intangible assets. Our definition of accounting, however, is more generic—it maps from the underlying state to some interim performance reports such as earnings. As a result, one limitation is that we are unable to provide policy implications for regulators and accounting standard settings.

**Corollary 1.** *Investment inefficiency  $q$  increases when*

1. *The project yields higher cash flow on success.*
2. *The project is more likely to succeed.*
3. *The investment cost is lower.*

The comparative statics are also intuitive. When the investment is more profitable as represented by a larger  $P_g, P_b$  or  $R$ , or when the cost of investing becomes lower, investors are willing to pay higher prices to purchase the stocks, which in turn provides more incentives for firm  $B$  to mimic firm  $G$  in equilibrium.

### 3.2 The Complementary Role of Accounting

So far, we find that accounting can discipline the manager's inefficient investment in addition to information provision for valuation purposes. In this section, we will disentangle these two effects by examining how accounting quality affects equilibrium market prices.

**Proposition 3.** 1. *When  $q = 1$ ,  $P(y_h)$  is strictly increasing in  $\tau$ , and  $P(y_l)$  is strictly decreasing in  $\tau$ .*

2. *When  $q < 1$ ,*

(a)  *$P(y_h)$  is strictly increasing in  $\tau$ .*

(b) *There exists a threshold  $\tau_0^*$  s.t.  $P(y_l)$  is strictly increasing in  $\tau$  if and only if  $\tau > \tau_0^*$ .*

We discuss the effect of accounting quality on  $P(y_h)$  and  $P(y_l)$  separately. To see why  $P(y_h)$  is unambiguously increasing in accounting quality, we rewrite  $P(y_h)$  as a function

of the conjectured  $\hat{q}$  and  $\tau$ , and the marginal effect becomes

$$\frac{dP(y_h, \hat{q}, \tau)}{d\tau} = \underbrace{\frac{\partial P(y_h, \hat{q}, \tau)}{\partial \tau}}_{\text{informational effect (+)}} + \underbrace{\frac{\partial P(y_h, \hat{q}, \tau)}{\partial \hat{q}} \times \frac{d\hat{q}}{d\tau}}_{\text{disciplinary effect (+)}} \quad (2)$$

First, investors update their beliefs on  $\theta$  upon observing the earnings report. When accounting is more informative, investors believe that firms that receive  $y_h$  are more likely of  $G$  type, so the market price  $P(y_h)$  increases, i.e.,  $\frac{\partial P(y_h, \hat{q}, \tau)}{\partial \tau} > 0$ . We define this direct effect as the *informational effect*. Second, when accounting becomes sufficiently precise, it starts to discipline firm  $B$ 's overinvestment—as evidenced by a lower  $q$ —and investors' conjectured  $\hat{q}$  also changes correspondingly. To be more precise, since investors anticipate that firm  $B$  invests less frequently, upon observing an investment, they rationally believe that the firm is more likely to be  $G$ , i.e.,  $\frac{\partial P(y_h, \hat{q}, \tau)}{\partial \hat{q}} < 0$ . Meanwhile, by Proposition 2, high quality accounting is more effective in preventing firm  $B$  from investing, i.e.,  $\frac{d\hat{q}}{d\tau} < 0$ . Taken together, accounting quality can also indirectly affect  $P(y_h)$  by changing the equilibrium  $\hat{q}$ , which we define as the *disciplinary effect*. Since both effects imply that  $P(y_h)$  increases with  $\tau$ , the net effect must be positive. Finally,  $q = 1$  is a special case in which the disciplinary effect does not exist, i.e.,  $\frac{\partial P(y_h, \hat{q}, \tau)}{\partial \hat{q}} = 0$ .

The case of  $P(y_l)$  is relatively straightforward. Next, we focus on a more interesting case on  $P(y_l)$ . In a similar fashion, we separate the two effects as follows:

$$\frac{dP(y_l, \hat{q}, \tau)}{d\tau} = \underbrace{\frac{\partial P(y_l, \hat{q}, \tau)}{\partial \tau}}_{\text{informational effect (-)}} + \underbrace{\frac{\partial P(y_l, \hat{q}, \tau)}{\partial \hat{q}} \times \frac{d\hat{q}}{d\tau}}_{\text{disciplinary effect (+)}} \quad (3)$$

First, different from  $P(y_h)$ , the informational effect on  $P(y_l)$  becomes negative, that is,  $\frac{\partial P(y_l, \hat{q}, \tau)}{\partial \tau} < 0$ . This happens because upon observing  $y_l$ , investors rationally believe that the firm is more likely of  $B$  type when the earnings report is more informative. On the other hand, as  $\tau$  increases, firm  $B$  is more disciplined and therefore investors' conjectured  $q$  decreases. In other words, investors still believe that firms that chose to invest are more likely to be  $G$ , even if the subsequently earnings reports are  $y_l$ . As a result, the disciplinary effect remains positive. Therefore,  $\tau$  moves  $P(y_l)$  in opposite directions, and the net effect is not obvious.

Proposition 3 suggests that  $P(y_l)$  increases in  $\tau$  if and only if  $\tau > \tau_0^*$ .<sup>8</sup> We first consider the extreme case in which  $\tau$  approaches 1. From the perspective of statistics,  $y_l$  reveals firm  $B$  almost for sure so  $P(y_l)$  should reach its minimum. However, the indifference condition  $(1-\tau)P(y_h) + \tau P(y_l) = K$  implies that  $\lim_{\tau \rightarrow 1} P(y_l) = K$ , which is the maximum of  $P(y_l)$ . The intuition for  $P(y_l)$  being increasing in  $\tau$  is that when accounting becomes sufficiently precise, most uncertainty about  $\theta$  has already been resolved upon which the investment decisions were made. By contrast, investors only receive incremental information from the earnings report itself. In other words, the disciplinary effect dominates the informational effect. To see this point, we define  $P_0(\text{invest})$  as the market price after the investment being observed but before the earnings reports, i.e.,

$$P_0(\text{invest}) = Pr(G|I)(P_g - P_b)R + P_bR \quad (4)$$

Because there is no earnings report, the price only depends on the market's conjecture  $\hat{q}$ , and the posterior probability is  $Pr(G|I) = \frac{\rho}{\rho + (1-\rho)\hat{q}}$ . As shown in Figure 1, when  $\tau$  increases,  $P_0(\text{invest})$  converges to  $P(y_h)$  and  $P_0(\text{no invest})$  converges to  $P(y_l)$ , consistent with our intuition.

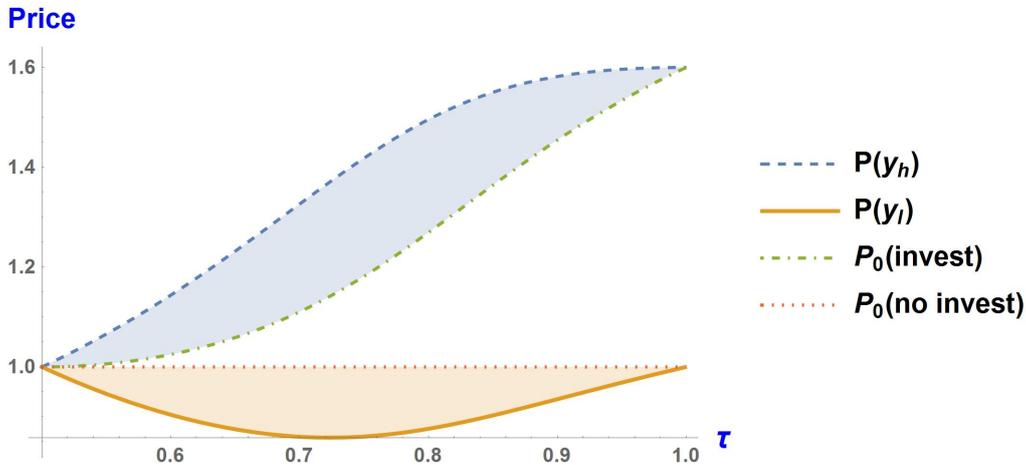


Figure 1: Stock Price:  $P_0(\text{invest})$  and  $P(y)$   
A numerical example:  $P_g = 0.8, P_b = 0.2, K = 1, R = 2$  and  $\rho = 0.5$

To formally show how investors' uncertainty varies, we adopt a commonly used measure of uncertainty: conditional variance. Specifically, investors' information set at date

<sup>8</sup>If we compare  $\tau_0^*$  the threshold in Proposition 2,  $\hat{\tau}(\rho)$ , we find a sufficient and necessary condition for  $\hat{\tau}(\rho) < \tau_0^*$  is that  $M < \rho < \frac{1}{2}\sqrt{8M+1} - \frac{1}{2}$ , and the parameter  $M$  is defined in the proof.

0 is  $\Omega_1 = \{I, NI\}$ , and the level of uncertainty is  $Var(\theta|\omega \in \Omega_1)$ .<sup>9</sup> Similarly, investors' information set at date 1 is  $\Omega_2 = \{(I, y_h), (I, y_l), NI\}$ , so the level of uncertainty is  $Var(\theta|\omega \in \Omega_2)$ . Finally, to evaluate the overall effect of an information system, we need to calculate the ex ante conditional variance for all possible realization.<sup>10</sup>

**Definition 2.** 1. Investors' prior uncertainty is  $Var[\theta]$ .

2. Investors' uncertainty after observing the investment decision is  $E_I[Var(\theta|\Omega_1)]$ .

3. Investors' uncertainty after observing the investment decision and the earnings report is  $E_{I,y}[Var(\theta|\Omega_2)]$ .

Now, we can quantify the disciplinary effect and informational effect as follows:

**Definition 3.** 1. The disciplinary effect is equivalent to  $W_D = Var[\theta] - E_I[Var(\theta|\Omega_1)]$ .

2. The informational effect is equivalent to  $W_I = E_I[Var(\theta|\Omega_1)] - E_{I,y}[Var(\theta|\Omega_2)]$ .

Specifically, the disciplinary effect is equal to the amount of uncertainty resolved by firms' investment decisions; the informational effect is equal to the amount of uncertainty resolved by the earnings report itself.

**Proposition 4.** 1. If  $q = 1$ ,  $W_D = 0$ ,  $W_I > 0$ . When  $\tau \rightarrow 1$ ,  $W_I \rightarrow Var[\theta]$ .

2. If  $q < 1$

(a)  $W_D > 0$ ,  $W_I > 0$  and when  $\tau \rightarrow 1$ ,  $W_D \rightarrow Var[\theta]$  and  $W_I \rightarrow 0$ ;

(b)  $W_D$  is strictly increasing in  $\tau$ ;

(c) there exists a threshold  $\tau_1^*$  such that  $W_I$  is decreasing in  $\tau$  if and only if  $\tau > \tau_1^*$ .

Proposition 4 confirms our intuition. In the pooling equilibrium, the investment decision has no information content, which means no uncertainty was resolved by observing the investment, i.e.,  $W_D = 0$ . However, the performance report is still useful for valuation purposes and therefore the informational effect is positive and strictly increasing in  $\tau$ . In

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<sup>9</sup>We use the conditional variance of  $\theta$  rather than the conditional variance of  $\tilde{C}$ , because the market price only depends on the posterior belief of  $\theta$ . Whether more uncertainty of  $\theta$  is equivalent to more uncertainty of  $\tilde{C}$  is unclear—it also depends on  $P_g$  and  $P_b$ .

<sup>10</sup>In general, we cannot apply Bayes' theorem when the action is an endogenous choice variable. In our setting, as firm  $B$  uses a mixed strategy and investors hold the correct conjecture  $\hat{q}$ , the investment decision is equivalent to a random variable.

## Uncertainty

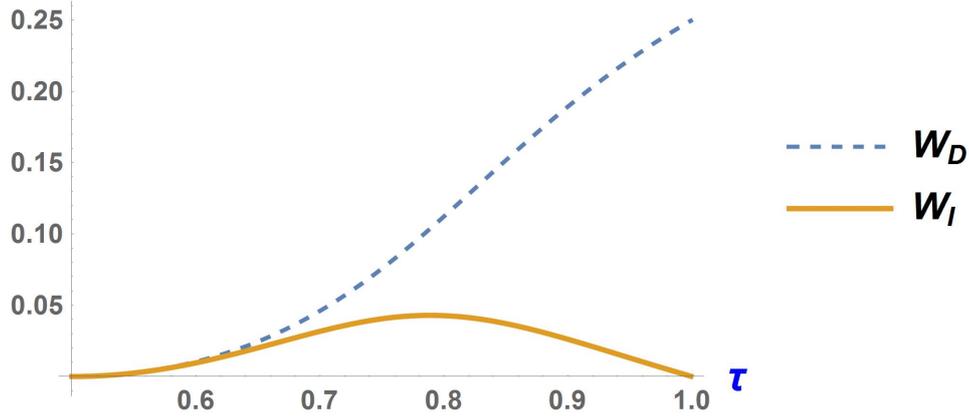


Figure 2: Accounting Quality and Investors' Uncertainty  
A numerical example:  $P_g = 0.8, P_b = 0.2, K = 1, R = 2$  and  $\rho = 0.5$

the more interesting case where  $q < 1$ , the investment decision starts to be informative about  $\theta$ , i.e.,  $W_D > 0$ . As accounting becomes more informative,  $W_D$  strictly increases and finally converges to the maximum  $Var[\theta]$ . By contrast,  $W_I$  starts to decline in  $\tau$  when  $\tau$  is sufficiently high, implying that the informational effect becomes weaker. Ultimately,  $W_I$  converges to 0. This happens because when  $\tau$  is sufficiently high, the firm's investment decision almost perfectly reveals  $\theta$ , and the residual uncertainty that could potentially be resolved by earnings reports is only marginal.<sup>11</sup> A numerical example is plotted in Figure 2. In summary, we find that when the earnings report is sufficiently informative, accounting serves its primary role in complementing other sources of information; whereas the role of information provision for valuation purposes is at best of second order.

The disciplinary effect in our paper corresponds to the confirmatory role of financial reporting raised by [Gigler and Hemmer \(1998\)](#), that is, mandatory disclosures serve their primary role of enhancing the credibility of managers' voluntary disclosures. We find a similar result that under certain conditions, financial reports can only provide a modest, but not overwhelming amount of information, because most information has already been preempted from other sources. However, the mechanism is totally different in our paper.

<sup>11</sup>We implicitly assume that the investment must be publicly observable before the earning report. To see why, suppose the two pieces of information are revealed at the same time, then we cannot rule out an alternative explanation—most uncertainty about  $\theta$  was resolved by the interim earnings report and the investment is only marginally informative. As a result, we need to be very cautious about the differential value relevance of “balance sheet” versus “income statement” that will be discussed in the text later, because financial statements are simultaneously disseminated to the public. However, firms' investments can be continuously disclosed or revealed via many other channels in reality, which are usually more timely than financial statements such as the earnings report. Therefore, it is still reasonable to assume the balance sheet information comes out prior to the earnings report.

First, in their model, voluntary disclosures are motivated by the principal’s goal of efficient contracting, because the manager’s private information is more value relevant compared to the interim performance report. We made the same assumption about management’s informational advantage, but also assume that any direct communication, such as voluntary disclosures, is unverifiable. Second, the economic magnitude of the disciplinary effect is probably much significant in our model than the confirmatory role in [Gigler and Hemmer \(1998\)](#), because the confirmatory role only enhances effective risk-sharing between the risk averse manager and risk neutral shareholders. However, the disciplinary effect in our paper changes firms’ real decisions such as investments, and therefore increases shareholders’ value and ultimately improves resource allocation from the social planner’s perspective. Finally, [Gigler and Hemmer \(1998\)](#) do not explicitly model the “quality” of earnings reports nor the capital market, but rather use the frequency of mandatory disclosures to represent the informativeness of an accounting system. In that regard, our paper more directly tackles the off-equilibrium role of accounting by examining the association between accounting quality and equilibrium prices.

Our results also relate to the literature on differential value relevance of balance sheets and income statements. For example, [Collins, Maydew, and Weiss \(1997\)](#) and [Francis and Schipper \(1999\)](#) find that the value relevance of earnings (book value) has decreased (increased), even though the combined value relevance has not changed. The decline of usefulness in earnings was attributed to more transitory items or potential managerial manipulation; in other words, earnings quality declines. We provide a completely different explanation, that is, the low (high) value relevance of income statements (balance sheets) possibly suggests that accounting is working more effectively in preventing inefficient investment and improving the overall information environment.

### **3.3 Accounting Quality, Price Sensitivity and ERC**

Prior literature defines “high quality accounting” as earnings providing more information about a firm’s financial performance and thus improving decision making ([Dechow, Ge, and Schrand, 2010](#)). Commonly used proxies for earnings quality can be classified into three categories: properties of earnings, investor responsiveness to earnings, and external indicators of earnings misstatements. The first category includes earnings persistence ([Nissim and Penman, 2001](#)), abnormal accrual ([Dechow and Dichev, 2002](#)), timely loss

recognition (Ball and Shivakumar, 2005), etc. In the second category, earnings response coefficient (ERC) or the  $R^2$  from the earnings-returns model are often interpreted as earnings quality (Holthausen and Verrecchia, 1988; Liu and Thomas, 2000). Finally, the last category includes Accounting and Auditing Enforcement Releases (AAERs), restatements, and internal control deficiencies reported are often viewed as indicators of errors or earnings management (Hennes, Leone, and Miller, 2008; Marinovic, 2013). In our model, accounting quality is defined as the extent to which earnings reports reflect future cash flow, consistent with the previous definition. Although this concept is theoretically intuitive, it is difficult to match with a good measure using archival data. Therefore, in this section, we examine the association between our definition of accounting quality with two proxies in the second category, price sensitivity and ERC.

**Proposition 5.** 1. When  $q = 1$ ,  $\frac{\partial[P(y_h) - P(y_l)]}{\partial\tau} > 0$ .

2. When  $q < 1$ , there exists a unique  $\tau_2^*$ , such that  $\frac{\partial[P(y_h) - P(y_l)]}{\partial\tau} \leq 0$  if and only if  $\tau \geq \tau_2^*$

First, the price sensitivity to earnings can be written as  $\frac{P(y_h) - P(y_l)}{y_h - y_l}$  in our model.<sup>12</sup> Proposition 5 suggests that that the price difference could be decreasing in accounting quality if  $\tau$  is sufficiently high. The intuition is the same as above: When the performance report is working the best in disciplining managers' inefficient investment, the investment can already convey a large amount of information and thus induce a strong market response. However, since most information has already been preempted, the market responsiveness to the subsequent earnings announcement, as measured by price sensitivity to earnings, becomes marginal.

Second, ERC, defined as unexpected returns ( $UR$ ) divided by unexpected earnings ( $UE$ ), measures the average change in prices associated with a dollar change in earnings, and therefore captures capital market responsiveness to earnings. Before the earnings announcement, investors only observe whether the firm made an investment. Therefore, the expected earnings are

$$E(y|I) = Pr(y_h|I) \times y_h + Pr(y_l|I) \times y_l$$

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<sup>12</sup>To see why this term represents price sensitivity, consider a regression model  $Price = \alpha + \beta Investment + \gamma Earnings + \varepsilon$ . Put in the data and we have  $P(y_h) = \alpha + \beta K + \gamma y_h$  and  $P(y_l) = \alpha + \beta K + \gamma y_l$ , and therefore  $\gamma = \frac{P(y_h) - P(y_l)}{y_h - y_l}$ .

In addition,  $Pr(y_h|I)$  further depends on investors' conjecture  $\hat{q}$ , which means

$$Pr(y_h|I) = \tau Pr(G|I) + (1 - \tau)Pr(B|I) = \frac{\tau\rho + (1 - \rho)(1 - \tau)\hat{q}}{\rho + (1 - \rho)\hat{q}}$$

Suppose the realized earnings at date 2 are  $y_h$ ; then  $UE$  is the difference between realized and expected earnings

$$UE = y_h - E(y|I) = Prob(y_l|I)(y_h - y_l)$$

Now we turn to unexpected returns. After the firm makes the investment, the price becomes  $P_0(invest)$  as in equation (4). Meanwhile, the law of iterated expectation implies that

$$E(P(y)|I) = E(E(\tilde{C}|y, I)|I) = P_0(invest)$$

As a result, the expected return is  $\frac{E(P(y)|I) - P_0(invest)}{P_0(invest)} = 0$ . In other words, because stock price instantaneously reflects all available information, the expected return at any time is 0. Similarly, given  $y_h$ , the realized return is  $\frac{P(y_h) - P_0(invest)}{P_0(invest)}$  and

$$\begin{aligned} UR &= \frac{P(y_h) - P_0(invest)}{P_0(invest)} - 0 \\ &= \frac{Prob(y_l|I)[P(y_h) - P(y_l)]}{P_0(invest)} \end{aligned}$$

Therefore,

$$ERC = \frac{UR}{UE} = \frac{P(y_h) - P(y_l)}{(y_h - y_l)P_0(invest)} \quad (5)$$

**Corollary 2.** *ERC is strictly decreasing in accounting quality when  $\tau$  is sufficiently high.*

Corollary 2 is a direct result of Proposition 5. ERC may be decreasing in  $\tau$  for two reasons: On the one hand, as Proposition 5 shows, the price sensitivity to earnings goes down when  $\tau$  is sufficiently large, which means the numerator of equation (5) decreases. On the other hand, since firm  $B$  is better disciplined when  $\tau$  increases, investors in turn attach a higher price to firms that made the investment, i.e.,  $P_0(invest)$  increases. As a result, the denominator of equation (5) also goes up, implying that it becomes more expensive to acquire the stock holding everything else constant. Overall, taking the two effects together, ERC is decreasing in accounting quality when  $\tau$  is adequately high.

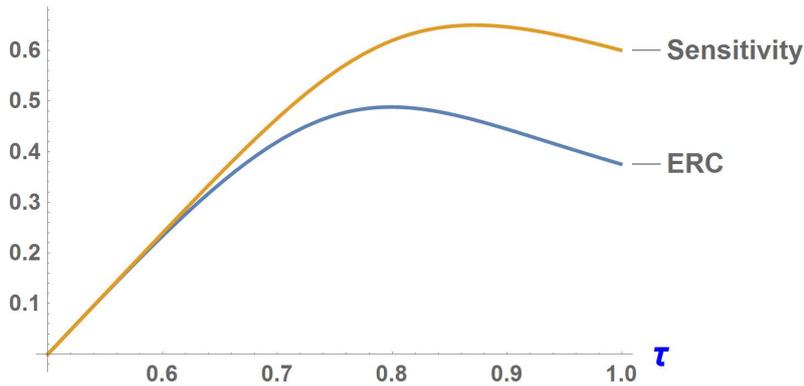


Figure 3: *ERC* and Price Sensitivity

In summary, the rationale of using market responsiveness to earnings to measure accounting quality is that, given an efficient market, it is an estimate of the amount of new information conveyed by earnings announcements. However, we find a counter-intuitive result that less intensive market responsiveness to earnings may imply higher quality accounting, because the earnings report may have enhanced the credibility of other sources of information. In other words, more intensive market responsiveness to earnings is not equivalent to an *overall* more informative accounting system. Therefore, to evaluate the usefulness of accounting, one cannot only look at the equilibrium actions or to the equilibrium market responsiveness.

## 4 Discussions

### 4.1 Alternative Representation

We extend the model in this section. Specifically, we assume

1. Productivity  $\theta \in [0, 1]$  and  $Pr(S|\theta) = \theta$ . The prior distribution is  $\theta \sim f(\theta)$  and  $f(\theta)$  is the density function.
2. The interim performance report is a mapping from  $\tilde{C}$  to  $Z$ <sup>13</sup>:  $Prob(z_h|S) = \lambda$ ,  $Prob(z_h|F) = 1 - \lambda$ , and  $\frac{1}{2} < \lambda < 1$ .
3.  $\theta$  and  $Z$  are independent conditional on  $\tilde{C}$ , i.e., earnings are correlated with the manager's private information only because they are both correlated with the terminal

<sup>13</sup>To differentiate from the main model, here we denote accounting signals as  $Z$ , and  $\lambda$  is the precision of the earnings report.

cash flow.

We study this alternative representation for two reasons: First, given a binary  $\theta$  in our main model, the equilibrium is inevitably discrete and the mixed strategy is difficult to interpret. Second, to capture the idea that the manager has superior private information, we assume the earnings report  $Y$  is independent of  $\tilde{C}$  conditional on  $\theta$ . However, in reality, accounting is a set of summarized statistics from past transactions, and thus may not be completely subsumed by the manager's information. In this alternative specification, we assume that accounting is associated with the manager's private information only because they are both informative about the expected return of the investment. Consequently, accounting quality means the extent to which accruals map into cash flow realizations (Dechow and Dichev, 2002), and a better match signifies high accounting quality (Gigler, Kanodia, Sapra, and Venugopalan, 2014).

**Proposition 6.** *There exists a unique threshold  $\theta^*$  such that the manager invests if and only if  $\theta \geq \theta^*$ . Specifically,*

1. *In the first best,  $\theta_{FB} = \frac{K}{R}$ .*
2. *In the second best  $\theta_{SB} \leq \theta_{FB}$ , and  $\theta_{SB} = 0$  if and only if  $\lambda$  is sufficiently small and  $E(\theta)$  is sufficiently high. Furthermore,  $\theta_{SB}$  is increasing in  $\lambda$ .*

Similar to Lemma 1, we find that in the first best, the manager invests if and only if *a priori* the investment has positive NPV. However, although there is no information asymmetry between the manager and outside investors, the interim report is incrementally useful in predicting future cash flow. In other words, accounting always serves its informational role even in a frictionless world.

In the second best, however, the manager always overinvests, as evidenced by  $\theta_{SB} \leq \theta_{FB}$ . Therefore, when the manager observes  $\theta \in [\theta_{SB}, \theta_{FB})$ , he still chooses to invest in the negative NPV project. This happens because investors infer  $\theta$  from the firm's investment decisions rather than by directly observing  $\theta$ . As a result, some low types are able to pool with high types by investing and thereby obtain higher stock prices. At the extreme, where  $\theta_{SB} = 0$ , all types will invest, leading to the worst pooling equilibrium. Similar to the main model, we find that the pooling equilibrium is sustained only when the *a priori* mean of  $\theta$  is sufficiently high and accounting quality is sufficiently low.

Finally, we find that high quality accounting works more effectively in alleviating overinvestment, consistent with Proposition 3. We only discuss the intuition and leave the formal proof to the appendix. Suppose accounting quality increases from  $\lambda_L$  to  $\lambda_H$ , but the equilibrium threshold remains  $\theta_L$ ; the law of iterated expectation implies that  $E(P(z)|\theta > \theta_{SB}) = E[E(\tilde{C}|\theta > \theta_{SB}, z)|\theta > \theta_{SB}] = E(\tilde{C}|\theta > \theta_{SB}) = E(\theta|\theta > \theta_{SB})R$ . In other words, if more information makes certain types better off, some other types must become worse off. What's more, a high type would naturally prefer more precise information to separate himself from other low types, whereas a low type prefers less information to pool with other high types. Therefore, as accounting quality increases to  $\lambda_H$ , the expected payoff for a low (high) type decreases (increases). When accounting quality is  $\lambda_L$ , type  $\theta_{SB}$  is indifferent between investing or not investing. Nevertheless, when accounting quality increases to  $\lambda_H$ , type  $\theta_{SB}$  must become worse off and thus choose not to invest.

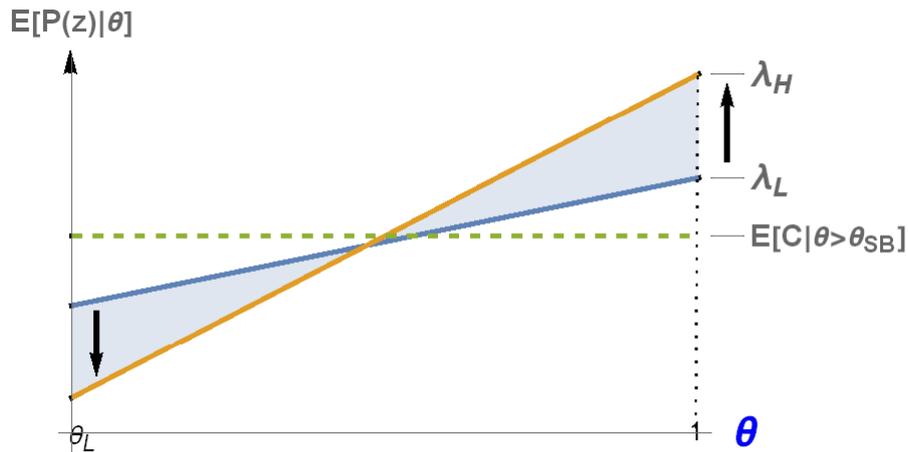


Figure 4: The Expected Payoff for Type  $\theta$  Holding Investors' Belief Fixed

Similar to the main model, accounting serves two roles: an information role and a disciplinary role.

$$\frac{dP(z, \lambda)}{d\lambda} = \underbrace{\frac{\partial P(z, \lambda)}{\partial \lambda}}_{\text{informational effect}} + \underbrace{\frac{\partial P(z, \tau)}{\partial \theta_{SB}} \times \frac{d\theta_{SB}}{d\lambda}}_{\text{disciplinary effect (+)}} \quad (6)$$

Since both  $\frac{\partial P(z, \tau)}{\partial \theta_{SB}}$  and  $\frac{d\theta_{SB}}{d\lambda}$  are positive, the disciplinary effect drives the price up for both  $z_h$  and  $z_l$ . By contrast, the informational effect will increase the price  $P(z_h)$  and decrease the price  $P(z_l)$ . Different from Proposition 3, however, the comparison of these two effects becomes extremely sensitive to the distribution of  $\theta$ , and we cannot make

any comparison unless the function  $f(\theta)$  is specified. In Appendix B, we find that two simple distributions can lead to completely different results. The key driving forces in our main model are that the manager’s private information is more value relevant compared to accounting information; and the manager’s action is able to convey a large amount of his private information. We conjecture that with these two forces, our result on market responsiveness will continue to hold qualitatively in other model specifications.

## 4.2 Entropy and Informational Gain

Another commonly used measure to quantify the amount of information is Shannon’s entropy (Shannon, 2001). Formally, for a discrete random variable  $X$  with possible values  $\{x_1, \dots, x_n\}$  and probability mass function  $P(X)$ , Shannon’s entropy, or information entropy,  $H$  is defined as

$$H(X) = E[-\ln(P(X))] = - \sum_{i=1}^n P(X_i) \log_b P(X_i) \quad (7)$$

Entropy refers to disorder or uncertainty of the state, and a larger value means the level of uncertainty is higher. Furthermore, information gain, a synonym for Kullback–Leibler divergence, measures how much information was obtained from observing an outcome of a random variable that is correlated with an unknown state. Formally, it is defined as the change in Shannon’s entropy from a prior state  $P$  to a state that takes some information as given:  $IG(X, a) = H(X) - H(X|a)$ . The expected value of the information gain, i.e., the weighted average of  $IG(X, a)$  for all possible  $a$ , captures the reduction in entropy of  $X$  by learning the state of the random variable  $A$ . This definition has been used in the literature, for example, Jiang and Yang (2017) use entropy to capture the informational constraint faced by the manager.

With the above notation, we have the following definition:

- Definition 4.**
1. *The prior information is  $H_0(\theta) = -(\rho \log_b \rho + (1 - \rho) \log_b (1 - \rho))$ .*
  2. *The information after observing the investment decision is  $H_1(\theta) = H_1(\theta|I)$ .*
  3. *The information after observing the investment decision and accounting information is  $H_2(\theta) = H(\theta|I, Y)$ .*

Obviously, more information can always reduce investors' uncertainty about  $\theta$ , i.e.,  $H_0(\theta) \geq H_1(\theta) \geq H_2(\theta)$ . However, the marginal effect is different. The informational gain resulting from observing firms' investment decisions is  $V_D = H_0(\theta) - H_1(\theta)$ , which is also equivalent to the magnitude of the disciplinary effect. In a similar way, the informational gain resulting from observing firms' earnings reports is  $V_I = H_1(\theta) - H_2(\theta)$ , which is equivalent to the magnitude of the information effect.

**Proposition 7.** 1. When  $q = 1$ ,  $V_D = 0$ ,  $V_I > 0$  and strictly increases in  $\tau$ .

2. When  $q < 1$ :

(a)  $V_D > 0, V_I > 0$ .

(b)  $V_D$  is strictly increasing in  $\tau$ .

(c) When  $\tau$  is sufficiently small, the informational effect dominates; when  $\tau$  is sufficiently large, the disciplinary effect dominates.

Proposition 7 is consistent with Proposition 4 which measures uncertainty with conditional variance. Specifically, when  $q < 1$ , both the disciplinary and informational effect are positive, and the disciplinary effect is strictly increasing in accounting quality. The last result in Proposition 7 is weaker as the monotonicity conditions for  $V_I$  cannot be solved analytically. Nevertheless, we still find a consistent result that the informational effect is overwhelmed by the disciplinary effect when accounting quality is sufficiently high, because most information has already been preempted before the actual earnings announcement.

## 5 Concluding Remarks

Value relevance has been one of the most prevalent criteria for assessing the usefulness of financial reporting by accounting academics and standard setters. Nevertheless, several studies (Wallman, 1995; Lev, 1989) have noted that earnings can explain very little of the contemporaneous price movement, and conclude that financial reporting fails to fulfill its mission. Gigler and Hemmer (1998) propose a confirmatory role to explain the empirical regularity and get the opposite conclusion. In this paper, we broaden the insights of Gigler and Hemmer (1998) and study the complementary role of financial reporting in enhancing the credibility of other sources of information.

We adopt the setting from the real effects literature: A benevolent manager with some private information chooses whether to invest in a risky project in order to maximize interim stock prices. The earnings report is produced and disseminated after the manager makes the investment, but is always less valuable and timely compared to the manager's private information, similar to [Gigler and Hemmer \(1998\)](#). We find that information asymmetry between the management and investors leads to investment inefficiency. When the quality of the earnings report is higher, the mitigating effect becomes stronger, suggesting that financial reporting plays a disciplinary role for the manager's misbehavior.

The main result of this paper is that, the disciplinary role will dominate the information provision role when earnings quality becomes sufficiently high. This could happen because investors only receive an incremental but not overwhelming amount of information from the earnings announcement, as more valuable information has already been incorporated when the investment decision was made. Using two commonly used measures of uncertainty, conditional variance and entropy, we confirm our intuition. As a result, when the market responsiveness to earnings is weak, accounting is possibly working the best in fulfilling its off-equilibrium role, which is in stark contrast with the aforementioned empirical literature. We argue that to evaluate the effectiveness of accounting, one cannot look to the equilibrium prices on a stand-alone basis; instead, we should measure its contribution to the overall information environment.

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# Appendices

## A Proofs

### Proof of Proposition 1

*Proof.* We prove a more general result: the equilibrium holds for any information structure  $Y$  which satisfies (strict) monotone likelihood ratio property (MLRP). Suppose  $\tilde{Y} \in \Omega$  and  $\Omega$  is bounded,  $\tilde{Y}$  is correlated with  $\theta$  in the sense of MLRP:  $\frac{f(y|G)}{g(y|B)}$  is strictly increasing in  $y$ . Therefore, high earnings are more favorable than low earnings, because the posterior belief  $Pr(G|y)$  is strictly increasing in  $y$ .

Suppose in equilibrium firm  $G$  invests with probability  $h \in [0, 1]$ , and firm  $B$  invests with probability  $q \in [0, 1]$ . The following results together prove the proposition.

**Lemma 2.**  $q = 0, h > 0$  can never be sustained in equilibrium.

We prove by contradiction. Suppose  $q = 0, h > 0$  is an equilibrium; in this case, investment perfectly reveals firm  $G$ . By rational expectation,  $P(\text{invest}) = P_g R$  and  $P(\text{no invest}) = K$ . Anticipating this price rule, firm  $G$  will take  $h = 1$ , and firm  $B$  will also deviate to  $q = 1$  as investing yields a higher price. Therefore, in equilibrium firm  $B$  will always invest with some positive probability. Furthermore, fully separation is not attainable.

**Lemma 3.** If firm  $B$  chooses  $q < 1 \Rightarrow E(P(y)|B) = K$ .

Suppose in equilibrium  $q < 1$ , but  $E(P(y)|B) > K$ . It implies that firm  $B$  can obtain a higher payoff by always investing, i.e.,  $B$  deviates to  $q = 1$ . Investors in turn anticipate that  $B$  has incentives to deviate, and therefore revise their belief. Similarly, if  $E(P(y)|B) < K$ , firm  $B$  will deviate to never invest and investors' belief also changes accordingly. Therefore, when firm  $B$  chooses  $q < 1$ , he must be indifferent between investing and not investing. Following the literature, we assume that when the manager is indifferent, he will take the conjectured  $\hat{q}$ .

**Lemma 4.** For any strategy of  $B$ , firm  $G$  will always invest, i.e.,  $h = 1$ .

First, MLRP suggests that  $Pr(G|y)$  is strictly increasing in  $y$

$$P(y) = Pr(G|\hat{I}, y)(P_g - P_b)R + P_bR \quad (8)$$

so price  $P(y)$  is strictly increasing in  $y$ . Suppose firm  $B$  chooses  $q = 1$  in equilibrium, the expected return from investing is greater or equal to  $K$ , i.e.,

$$\begin{aligned} E(P(y)|B) &= \int_{\Omega} Pr(y|B)P(y) > K \\ E(P(y)|G) &= \int_{\Omega} Pr(y|G)P(y) \end{aligned}$$

Because  $Y|\theta$  satisfies first order stochastic dominance and  $P(y)$  is strictly increasing in  $y$ , we have  $E(P(y)|G) > E(P(y)|B) = K$ , suggesting  $h = 1$ . The off-equilibrium belief is that  $Prob(B|no\ invest) = 1$ , i.e., if the firm did not invest, investors believe it is  $B$ .

Suppose firm  $B$  chooses  $q < 1$  in equilibrium, from the previous result,  $E(P(y)|B) = K$ . Similarly, we have  $E(P(y)|G) > E(P(y)|B) = K$ , suggesting  $h = 1$ .

**Lemma 5.** *Firm  $B$  chooses  $q = 1$  if and only if  $E(P(y)|B) \geq K$ , and investors believe  $\hat{h} = \hat{q} = 1$ .*

The pooling and hybrid equilibria are mutually exclusive. Given investors believe that  $\hat{h} = \hat{q} = 1$ , if the corresponding prices are such that  $E(P(y)|B) \geq K$ , firm  $B$  finds it optimal to always invest, and the conjecture is sustained. Otherwise, Given the belief  $\hat{h} = \hat{q} = 1$ , if  $E(P(y)|B) < K$ , firm  $B$  will never invest, suggesting that the belief  $\hat{h} = \hat{q} = 1$  is incorrect. As a result, investors must revise their belief to the point at which firm  $B$  is indifferent between investing or not.

Before closing the proof, we check a special case in which no firm invests, i.e.,  $h = 0, q = 0$ . If this is an equilibrium, investors must believe that  $\hat{h} = \hat{q} = 0$ , and prices are constant  $K$ . The action “investing” is off equilibrium, so Bayes’ theorem does not apply when “investing” is observed. To support this equilibrium, we assume that the off-equilibrium belief is  $Pr(B|I) = 1$ . Anticipating this belief, both  $G$  and  $B$  have no incentive to deviate and invest. However, this equilibrium is not interesting or sustainable. In fact, certain equilibrium refinement rules can eliminate this case. Since it is not our focus to eliminate multiple equilibria, we ignore this special case in the paper.  $\square$

## Proof of Proposition 2

*Proof.* As Proposition 1 suggests, firm  $G$  invests with probability 1 and firm  $B$  invests with probability  $q \leq 1$ , so we can derive the posterior probabilities:

$$\begin{aligned} Pr(G|I, y_h) &= \frac{Pr(G)Pr(I|G)Pr(y_h|I, G)}{Pr(G)Pr(I|G)Pr(y_h|I, G) + Pr(B)Pr(I|B)Pr(y_h|I, B)} \\ &= \frac{Pr(G)Pr(I|G)Pr(y_h|G)}{Pr(G)Pr(I|G)Pr(y_h|G) + Pr(B)Pr(I|B)Pr(y_h|B)} \\ &= \frac{\rho\tau}{\rho\tau + (1-\rho)\hat{q}(1-\tau)} \\ Pr(G|I, y_l) &= \frac{\rho(1-\tau)}{\rho(1-\tau) + (1-\rho)\hat{q}\tau} \end{aligned}$$

The second equality is because  $\theta$  is a sufficient statistic for *invest* w.r.t.  $Y$ . At date 0, the firm maximizes its expected price, i.e.,  $\text{Max}_I E[P(\hat{I}, y)|\theta]$ .

Firm  $B$  is indifferent between investing and not investing if

$$\begin{aligned} E(P(y)|B) &= Pr(y_h|B) \times P(y_h) + Pr(y_l|B) \times P(y_l) = K \\ \Rightarrow (1-\tau) \times \frac{\rho\tau}{\rho\tau + (1-\rho)\hat{q}(1-\tau)} + \tau \times \frac{\rho(1-\tau)}{\rho(1-\tau) + (1-\rho)\hat{q}\tau} &= \frac{K - P_b R}{(P_g - P_b)R} \end{aligned}$$

Define  $M = \frac{K - P_b R}{(P_g - P_b)R}$ , and  $0 < M < 1$ . Rational expectation requires that  $\hat{q} = q$ , so we can replace  $\hat{q}$  with  $q$ .

However, by the proof of Proposition 1 suggests, the equilibrium may be pooling. If so, investors believe  $\hat{h} = \hat{q} = 1$ , and under this belief, firm  $B$  still finds it optimal to always invest, i.e.,  $E(P(y)|B) \geq K$ . Expand this equation and we have

$$(1-\tau) \times \frac{\rho\tau}{\rho\tau + (1-\rho)(1-\tau)} + \tau \times \frac{\rho(1-\tau)}{\rho(1-\tau) + (1-\rho)\tau} \geq M \quad (9)$$

$$\Leftrightarrow (\rho - M + 4M\rho - 4M\rho^2)(\tau^2 - \tau) + M\rho - M\rho^2 \leq 0 \quad (10)$$

To examine when equation (10) is satisfied, we discuss two cases as follows:

Case 1:  $M > \rho$ . Define  $\Gamma \triangleq \rho - M + 4M\rho - 4M\rho^2$ , and the sign of  $\Gamma$  is ambiguous. If  $\Gamma < 0$ , because  $\tau - \tau^2$  is negative and  $M\rho - M\rho^2$  is positive, (10) cannot be satisfied. If  $\Gamma > 0$ , the function  $\Gamma(\tau^2 - \tau) + M\rho - M\rho^2$  is strictly increasing in  $\tau \in (\frac{1}{2}, 1)$ , and the minimum value is  $0.25M - 0.25\rho > 0$ , so (14) is also not satisfied. Therefore, when

$M > \rho$ , equation (10) can never be satisfied.

Case 2:  $M \leq \rho$ , and so  $\Gamma > 0$ . In this case,  $\Gamma(\tau^2 - \tau) + M\rho - M\rho^2$  is increasing in  $\tau \in (\frac{1}{2}, 1)$ . When  $\tau = \frac{1}{2}$ , the minimum is  $0.25M - 0.25\rho < 0$ , and when  $\tau = 1$ , the maximum is  $M\rho - M\rho^2 > 0$ . So there exists a unique threshold  $\hat{\tau}(\rho)$  such that (10) is satisfied if and only if  $\tau < \hat{\tau}(\rho)$ , where  $\hat{\tau}(\rho) = \frac{1}{2} + \frac{1}{2}\sqrt{1 - \frac{4(M\rho - M\rho^2)}{\Gamma}}$ . Since  $\Gamma > 4(M\rho - M\rho^2) \Rightarrow 0 < \hat{\tau}(\rho) < 1$ , it is easy to verify that  $\frac{4(M\rho - M\rho^2)}{\Gamma}$  is decreasing in  $\rho$ , so  $\hat{\tau}(\rho)$  is strictly increasing in  $\rho$ .

Therefore, we find that firm  $B$  chooses  $q = 1$  if and only if  $M \leq \rho$  and  $\tau \leq \hat{\tau}(\rho)$ .

Lastly, we show how  $q$  varies with  $\tau$  and  $\rho$ . When  $q < 1$ , rewrite the indifference condition as

$$\frac{\rho\tau(1-\tau)}{\rho\tau + (1-\rho)q(1-\tau)} + \frac{\rho(1-\tau)\tau}{\rho(1-\tau) + (1-\rho)q\tau} - M = 0 \quad (11)$$

Define LHS as a function  $L(\tau, q, M) \Rightarrow$

$$\frac{\partial L(\tau, q, M)}{\partial \tau} = \frac{-q(1-\rho)(\rho + (1-\rho)q)(2\tau - 1)\rho^2}{(\rho\tau + (1-\rho)q(1-\tau))^2(\rho(1-\tau) + (1-\rho)q\tau)^2} < 0$$

$$\frac{\partial L(\tau, q, M)}{\partial q} < 0$$

Based on implicit function theorem,

$$\frac{dq}{d\tau} = -\frac{\frac{\partial L(\tau, q, M)}{\partial \tau}}{\frac{\partial L(\tau, q, M)}{\partial q}} < 0 \quad (12)$$

To see why  $\frac{dq}{d\rho} > 0$ , define  $s = \frac{(1-\rho)}{\rho}q$  and rewrite equation (11) as

$$\frac{\tau(1-\tau)}{\tau + s(1-\tau)} + \frac{(1-\tau)\tau}{(1-\tau) + s\tau} - M = 0 \quad (13)$$

Obviously, the solution of equation (17) does not depend on  $\rho$ ; in other words,  $\frac{ds}{d\rho} = 0$ .

In addition, by definition, we have

$$\frac{ds}{d\rho} = \frac{(1-\rho)}{\rho} \frac{dq}{d\rho} - \frac{q}{\rho^2} = 0 \Rightarrow \frac{dq}{d\rho} = \frac{q}{\rho(1-\rho)} > 0$$

Lastly, the cross partial is  $\frac{\partial q^2}{\partial \tau \partial \rho} = \frac{1}{\rho(1-\rho)} \frac{\mathbf{d}q}{\mathbf{d}\tau} < 0$   $\square$

### Proof of Corollary 1

*Proof.* Following Proposition 2, we have  $\frac{\partial L(\tau, q, M)}{\partial M} = -1$ . By the implicit function theorem

$$\frac{\partial q}{\partial M} = -\frac{\frac{\partial L(\tau, q, M)}{\partial M}}{\frac{\partial L(\tau, q, M)}{\partial \tau}} < 0$$

Meanwhile  $M = \frac{K - P_b R}{(P_g - P_b)R} \Rightarrow \frac{\partial M}{\partial R} = \frac{K}{-(P_g - P_b)K^2} < 0$ ,  $\frac{\partial M}{\partial P_g} = -\frac{K - P_b R}{(P_g - P_b)^2 R} < 0$ ,  $\frac{\partial M}{\partial P_b} = \frac{K - P_g R}{(P_g - P_b)^2 R} < 0$  and  $\frac{\partial M}{\partial K} = \frac{1}{(P_g - P_b)R} > 0$ .

In summary, by the chain rule, we have (1)  $\frac{\partial q}{\partial R} > 0$ , (2)  $\frac{\partial q}{\partial P_g} > 0$ ,  $\frac{\partial q}{\partial P_b} > 0$ , and (3)  $\frac{\partial q}{\partial K} < 0$ .  $\square$

### Proof of Proposition 3

*Proof. Case 1:  $q = 1$ .* In this case,

$$\begin{aligned} \frac{\partial P(y_h)}{\partial \tau} &= \frac{(1-\rho)\rho}{(\rho(2\tau-1) - \tau + 1)^2} (P_g - P_b)R > 0 \\ \frac{\partial P(y_l)}{\partial \tau} &= \frac{-(1-\rho)\rho}{(\rho + \tau - 2\rho\tau)^2} (P_g - P_b)R < 0 \end{aligned}$$

*Case 2:  $q < 1$ .* From Proposition 2, we rewrite  $P(y_h)$  and  $P(y_l)$  in terms of  $s$

$$\begin{aligned} \Rightarrow P(y_h) &= \frac{\tau}{\tau + s(1-\tau)} (P_g - P_b)R + P_b R \\ P(y_l) &= \frac{(1-\tau)}{(1-\tau) + s\tau} (P_g - P_b)R + P_b R \end{aligned}$$

(a)

$$P(y_h) = \frac{1}{1 + s \frac{(1-\tau)}{\tau}} (P_g - P_b)R + P_b R \quad (14)$$

Since  $\frac{\partial q}{\partial \tau} < 0$ ; and  $\frac{1-\tau}{\tau}$  strictly decreases in  $\tau$ , we must have  $\frac{\partial P(y_h)}{\partial \tau} > 0$ .

(b)

$$P(y_l) = \frac{1}{1 + \frac{s}{1-\tau}} (P_g - P_b)R + P_b R \quad (15)$$

To see the monotonicity, we define  $t \triangleq \frac{\tau}{1-\tau}s$  and  $\gamma \triangleq \frac{\tau}{1-\tau} \Rightarrow \gamma \in (1, \infty)$  and  $\frac{\partial \gamma}{\partial \tau} = \frac{1}{(1-\tau)^2} > 0$ . We are ultimately interested in  $\frac{\partial t}{\partial \tau}$ , but based on the chain rule  $\frac{\partial t}{\partial \tau} = \frac{\partial t}{\partial \gamma} \frac{\partial \gamma}{\partial \tau} = \frac{1}{(1-\tau)^2} \times \frac{\partial t}{\partial \gamma}$ , therefore  $\frac{\partial t}{\partial \tau}$  has the same sign of  $\frac{\partial t}{\partial \gamma} > 0$ .

Now we replace  $q$  and  $\tau$  in Equation (10)

$$\Leftrightarrow Mt^2 + (M\gamma^2 - \gamma + M)t + (M-1)\gamma^2 = 0 \quad (16)$$

Equation (16) has a unique positive solution

$$t = \frac{-M + \gamma - M\gamma^2 + \sqrt{(M\gamma^2 - \gamma + M)^2 + 4M(1-M)\gamma^2}}{2M} \quad (17)$$

Define  $V(\gamma) \triangleq (M\gamma^2 - \gamma + M)^2 + 4M(1-M)\gamma^2 \Rightarrow$

$$\begin{aligned} \frac{\partial t}{\partial \gamma} > 0 &\Leftrightarrow \frac{1}{2M} \left[ 1 - 2M\gamma + \frac{\frac{\partial V(\gamma)}{\partial \gamma}}{2\sqrt{V(\gamma)}} \right] > 0 \Leftrightarrow \frac{\partial V(\gamma)}{\partial \gamma} > 2(2M\gamma - 1)\sqrt{V(\gamma)} \\ &\Leftrightarrow 1 > (2M\gamma - 1) \times \frac{\gamma}{\sqrt{V(\gamma)} + (M\gamma^2 - \gamma + M)} \end{aligned}$$

Since  $V(\gamma) > (M\gamma^2 - \gamma + M)^2 \Rightarrow \sqrt{V(\gamma)} + (M\gamma^2 - \gamma + M) > 0$ , it is equivalent to

$$\begin{aligned} &\Leftrightarrow \sqrt{V(\gamma)} > M(\gamma^2 - 1) \\ &\Leftrightarrow -\gamma \left( \gamma - \frac{1 + 4M - \sqrt{1 + 8M}}{4M} \right) \left( \gamma - \frac{1 + 4M + \sqrt{1 + 8M}}{4M} \right) > 0 \end{aligned}$$

Because  $\gamma > 1 > \frac{1 + 4M - \sqrt{1 + 8M}}{4M}$ , we have  $\frac{\partial t}{\partial \gamma} > 0 \Leftrightarrow \tau < \frac{1 + 4M + \sqrt{1 + 8M}}{1 + 8M + \sqrt{1 + 8M}} \triangleq \tau_0^*$ .

In a nutshell,  $P(y_t)$  is decreasing in  $\tau$  if and only if  $\tau < \tau_0^*$ .  $\square$

#### Proof of Proposition 4

*Proof.* First, we have  $Var[\theta] = \rho(1-\rho)(G-B)^2$  by definition.

Second, since not investing perfectly reveals  $B$ ,  $Var(\theta|NI) = 0$ . However, when the firm invests,

$$Var(\theta|I) = Pr(G|I)(1 - Pr(G|I))(G-B)^2 = \frac{\rho(1-\rho)\hat{q}}{(\rho + (1-\rho)\hat{q})^2}(G-B)^2$$

Finally, for realized signal  $y$ , we have

$$\begin{aligned} Var(\theta|I, y_h) &= Pr(G|I, Y_h)(1 - Pr(G|I, y_h))(G - B)^2 = \frac{\rho\tau(1 - \rho)\hat{q}(1 - \tau)(G - B)^2}{[\rho\tau + (1 - \rho)\hat{q}(1 - \tau)]^2} \\ Var(\theta|I, y_l) &= \frac{\rho(1 - \tau)(1 - \rho)\hat{q}\tau}{[\rho(1 - \tau) + (1 - \rho)\hat{q}\tau]^2}(G - B)^2 \end{aligned}$$

To evaluate the expected conditional variance, we replace  $\hat{q}$  with  $q$  and

$$\begin{aligned} E_I[Var(\theta|\Omega_1)] &= Pr(I)Var(\theta|I) = (\rho + (1 - \rho)q)Var(\theta|I) \\ &= \frac{\rho(1 - \rho)q}{\rho + (1 - \rho)q}(G - B)^2 \\ E_{I,y}[Var(\theta|\Omega_2)] &= Pr(I, y_h)Var(\theta|I, y_h) + Pr(I, y_l)Var(\theta|I, y_l) \\ &= \left[ \frac{\rho(1 - \rho)\tau(1 - \tau)q}{\rho\tau + (1 - \rho)q(1 - \tau)} + \frac{\rho(1 - \rho)\tau(1 - \tau)q}{\rho(1 - \tau) + (1 - \rho)q\tau} \right] (G - B)^2 \\ &= \frac{\rho(1 - \rho)\tau(1 - \tau)q[\rho + (1 - \rho)q]}{[\rho\tau + (1 - \rho)q(1 - \tau)][\rho(1 - \tau) + (1 - \rho)q\tau]}(G - B)^2 \end{aligned}$$

If  $q = 1$ ,  $E_I[Var(\theta|\Omega_1)] = Var[\theta]$ , suggesting  $W_D = 0$ . In addition,

$$W_I = \left[ \rho(1 - \rho) - \frac{\rho(1 - \rho)\tau(1 - \tau)}{[\rho\tau + (1 - \rho)(1 - \tau)][\rho(1 - \tau) + (1 - \rho)\tau]} \right] (G - B)^2 > 0$$

Furthermore, when  $\tau \rightarrow 1$ ,  $W_I \rightarrow Var[\theta]$ , and  $W_I$  is strictly increasing in  $\tau$ .

If  $q < 1$ , obviously,  $E_I[Var(\theta|\Omega_1)] < Var[\theta]$ . In addition,

$$\begin{aligned} E_I[Var(\theta|\Omega_1)] &< E_{I,y}[Var(\theta|\Omega_2)] \\ \Leftrightarrow \tau(1 - \tau)[\rho + (1 - \rho)q]^2 &< [\rho\tau + (1 - \rho)q(1 - \tau)][\rho(1 - \tau) + (1 - \rho)q\tau] \\ \Leftrightarrow \rho(1 - \rho)q(2\tau - 1)^2 &> 0 \end{aligned}$$

Therefore  $E_{I,y}[Var(\theta|\Omega_2)] < E_I[Var(\theta|\Omega_1)] < Var[\theta]$ , i.e.,  $W_D > 0, W_I > 0$ . Now we examine the effect of  $\tau$  on  $W_D$  and  $W_I$ . First,

$$\frac{dE_I[Var(\theta|\Omega_1)]}{d\tau} = \frac{E_I[Var(\theta|\Omega_1)]}{\partial q} \frac{\partial \hat{q}}{\partial \tau} = \frac{(1 - \rho)\rho^2}{(\rho + (1 - \rho)q)^2}(G - B)^2 \frac{\partial \hat{q}}{\partial \tau} < 0$$

it suggests that high quality accounting can more effectively resolve investors' uncertainty

upon observing investment. As a result, by our definition,

$$\frac{dW_D}{d\tau} = \frac{d[Var(\theta) - E_I[Var(\theta|\Omega_1)]]}{d\tau} > 0$$

implying that the disciplinary effect is strictly increasing in  $\tau$ . Similarly,

$$\begin{aligned} \frac{dE_{I,y}[Var(\theta|\Omega_2)]}{d\tau} &= \frac{E_{I,y}[Var(\theta|\Omega_2)]}{\partial q} \frac{\partial \hat{q}}{\partial \tau} + \frac{E_{I,y}[Var(\theta|\Omega_2)]}{\partial \tau} \\ \frac{E_{I,y}[Var(\theta|\Omega_2)]}{\partial \tau} &= \frac{q^2(1-\rho)^2\rho^2(2\tau-1)(q(\rho-1)-\rho)}{[\rho\tau+(1-\rho)q(1-\tau)]^2[\rho(1-\tau)+(1-\rho)q\tau]^2} < 0 \\ \frac{E_{I,y}[Var(\theta|\Omega_2)]}{\partial q} &= \frac{q^2(\rho-1)^2(3\tau^2-3\tau+1)-\rho^2(\tau-1)\tau+2q\rho(\rho-1)(\tau-1)\tau}{[\rho\tau+(1-\rho)q(1-\tau)]^2[\rho(1-\tau)+(1-\rho)q\tau]^2} \\ &\times (1-\rho)\rho^2(1-\tau)\tau > 0 \\ &\Rightarrow \frac{dE_{I,y}[Var(\theta|\Omega_2)]}{d\tau} < 0 \end{aligned}$$

So investors' uncertainty at date 3 is also decreasing in  $\tau$ . But the incremental effect  $W_I$ ,

$$W_I = \frac{q^2(\rho-1)^2\rho^2(1-2\tau)^2(G-B)^2}{[\rho+(1-\rho)q][\rho\tau+(1-\rho)q(1-\tau)][\rho(1-\tau)+(1-\rho)q\tau]}$$

Plug in equilibrium  $q$ , and after some tedious algebra we find that  $\frac{dW_I}{d\tau} < 0$  if and only if  $\tau > \tau_1^*$ , where  $\tau_1^* \triangleq \frac{1}{2}\sqrt{\frac{1}{4M+1}} + \frac{1}{2}$ .  $\square$

### Proof of Proposition 5

*Proof.* When  $q = 1$ , it is a direct result from Proposition 3.

When  $q < 1$ , by the indifference condition

$$\begin{aligned} (1-\tau)P(y_h) + \tau P(y_l) &= K \\ \Rightarrow P(y_h) - P(y_l) &= \frac{P(y_h) - K}{\tau} = R(P_g - P_b) \times \frac{Pr(G|I, y_h) - M}{\tau} \end{aligned}$$

Define  $u = \frac{Pr(G|I, y_h) - M}{\tau}$  and we want to examine when  $\frac{\partial u}{\partial \tau} < 0$ . By the definition of  $s$ , we have  $s = \frac{\tau}{1-\tau} \times \frac{1-\tau u - M}{\tau u + M}$ . Replace  $s$  with  $u$  in equation (16) and we have

$$\Rightarrow u^2\tau(\tau-1)(2\tau-1) + u[(1-\tau)\tau + M(2\tau-1)^2] + (M^2 - M)(2\tau-1) = 0 \quad (18)$$

Since  $\tau(\tau - 1)(2\tau - 1) < 0$ ,  $(1 - \tau)\tau + M(2\tau - 1)^2 > 0$  and  $(M^2 - M)(2\tau - 1) < 0$ , the above equation has two positive solutions. Nonetheless, as  $u < \frac{1 - M}{\tau}$ , one solution can be eliminated. Therefore, the unique solution is

$$u^* = \frac{(\tau - 1)\tau - M(2\tau - 1)^2 + \sqrt{W(\tau)}}{2(\tau - 3\tau^2 + 2\tau^3)} \quad (19)$$

where  $W(\tau) = [(1 - \tau)\tau + M(2\tau - 1)^2]^2 - 4M(1 - M)\tau(1 - \tau)(2\tau - 1)^2$ .

$$\begin{aligned} \Rightarrow \frac{\partial u}{\partial \tau} &= \frac{1}{2\tau^2(1 - 3\tau + 2\tau^2)^2} \times \left\{ [(2\tau - 1) - 4M(2\tau - 1) + \frac{W'(\tau)}{2\sqrt{W(\tau)}}](\tau - 3\tau^2 + 2\tau^3) \right. \\ &\quad \left. - (1 - 6\tau + 6\tau^2)[(\tau - 1)\tau - M(2\tau - 1)^2 + \sqrt{W(\tau)}] \right\} \\ \Rightarrow \frac{\partial u}{\partial \tau} < 0 &\Leftrightarrow [(2\tau - 1) - 4M(2\tau - 1) + \frac{W'(\tau)}{2\sqrt{W(\tau)}}]\tau(2\tau - 1)(\tau - 1) < \\ &\quad (1 - 6\tau + 6\tau^2) \frac{4M(1 - M)\tau(1 - \tau)(2\tau - 1)^2}{(\tau - 1)\tau - M(2\tau - 1)^2 - \sqrt{W(\tau)}} \end{aligned}$$

Now we plug in  $W'(\tau)$  and rearrange terms. After some algebra we get

$$\begin{aligned} 2(1 - M)M[M(1 - 2\tau)^2 + \tau(\tau - 1)] &> 2(1 - M)M(1 - 2\tau)^2\sqrt{W(\tau)} \\ \Leftrightarrow \begin{cases} M(1 - 2\tau)^2 + \tau(\tau - 1) > 0 & \text{(a) and} \\ [M(1 - 2\tau)^2 + \tau(\tau - 1)]^2 > (1 - 2\tau)^4W(\tau) & \text{(b)} \end{cases} \end{aligned}$$

i.e.,  $\frac{\partial u}{\partial \tau} < 0$  requires conditions (a) and (b) be satisfied at the same time.

Condition (a)  $\Leftrightarrow (4M + 1)\tau^2 - (4M + 1)\tau + M > 0$ , and because  $0 < M < 1$  and  $0.5 < \tau < 1$ , it is equivalent to  $\tau > \frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4M + 1}}$ .

Condition (b)  $\Leftrightarrow 4\tau(1 - \tau)\{(32M + 4)\tau^6 - (96M + 12)\tau^5 + (14 + 120M + 16M^2)\tau^4 - (8 + 80M + 32M^2)\tau^3 + (2 + 28M + 24M^2)\tau^2 - (4M + 8M^2)\tau + M^2\} > 0$ . Define the function in the curly bracket as  $G(\tau)$ , which is a polynomial function of  $\tau$  to the power of 6. First, when  $0 < M < 1$ ,  $G(\tau) = 0$  has four real solutions in the interval  $(0, 1)$ ; second, function  $G(\tau)$  is symmetric as  $G(\tau) = G(1 - \tau)$ <sup>14</sup>. Therefore,  $G(\tau) = 0$  has two solutions  $\tau_1, \tau_2$  in  $(0, \frac{1}{2})$ , and two solutions  $\tau_3, \tau_4$  in  $(\frac{1}{2}, 1)$ . Condition (b) is then equivalent to  $0.5 < \tau < \tau_3$

<sup>14</sup>Mathematica code is available upon request.

or  $\tau > \tau_4$ <sup>15</sup>. As a result,

$$\frac{\partial u}{\partial \tau} < 0 \Leftrightarrow \begin{cases} \tau > \frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4M+1}} & \text{(a) and} \\ 0.5 < \tau < \tau_3 \text{ or } \tau > \tau_4 & \text{(b)} \end{cases} \quad (20)$$

Meanwhile, since  $G(\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4M+1}}) = -\frac{M^2}{(1+4M)^3} < 0$ , we have  $\tau_3 < \frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4M+1}} < \tau_4$ . Therefore, we can further simplify (20)  $\Leftrightarrow \tau > \tau_4$ .

In summary,  $P(y_h) - P(y_l)$  is decreasing in  $\tau$  if and only if  $\tau > \tau_4$ .  $\square$

### Proof of Proposition 6

*Proof.* In the first best, investors observe  $\theta$  and  $Z$  and the stock price will be  $P(z, \theta) = E(\tilde{C}|z, \theta)$ . A manager who observes  $\theta$  maximizes the expected price at date 0, because of the law of iterated expectations.

$$\begin{aligned} \max_I E[E(\tilde{C}|z, \theta)|\theta] &\Leftrightarrow \max_I E(\tilde{C}|\theta) \\ &\Rightarrow \max_I Pr(S|\theta)R \Rightarrow \theta_{FB} = \frac{K}{R} \end{aligned}$$

In the second best, we first show that the equilibrium must be a threshold. Suppose the market believes that managers who observe  $\theta \in \Omega$  will invest, then the corresponding prices are  $P(z_h) = Pr(S|I, z_h)R$  and  $P(z_l) = Pr(S|I, z_l)R \Rightarrow$

$$\begin{aligned} P(z_h) &= \frac{Pr(S|I)Pr(z_h|I, S)}{Pr(S|I)Pr(z_h|I, S) + Pr(F|I)Pr(z_h|I, F)}R \\ &= \frac{Pr(S|I)Pr(z_h|S)}{Pr(S|I)Pr(z_h|S) + Pr(F|I)Pr(z_h|F)}R \\ &= \frac{E(\theta|\theta \in \Omega)\lambda}{E(\theta|\theta \in \Omega)\lambda + (1 - E(\theta|\theta \in \Omega))(1 - \lambda)}R \\ P(z_l) &= \frac{E(\theta|\theta \in \Omega)(1 - \lambda)}{E(\theta|\theta \in \Omega)(1 - \lambda) + (1 - E(\theta|\theta \in \Omega))\lambda}R \end{aligned}$$

The second equality uses the assumption that  $\theta$  and  $Z$  are independent conditional on  $\tilde{C}$ . Therefore,  $P(z_h) > P(z_l)$  as long as  $0 < E(\theta|I) < 1$ . For the manager who observes  $\theta$ ,

<sup>15</sup>It is a general property for any polynomial equation  $T(x)$ : If  $T(x) = 0$  has  $N$  real solutions  $x_1 < x_2 < \dots < x_N$ . Suppose  $T(x) \rightarrow \infty$  when  $x \rightarrow \infty$ , then  $T(x) > 0$  is equivalent to  $(x_N, +\infty) \cup (x_{N-2}, x_{N-1}) \cup (x_{N-4}, x_{N-3}) \dots$

because of conditional independence

$$\begin{aligned} Pr(z|\theta) &= Pr(S|\theta)Pr(z|S, \theta) + Pr(F|\theta)Pr(z|F, \theta) \\ &= \theta Pr(z|S) + (1 - \theta)Pr(z|F) \end{aligned}$$

So for type  $\theta$ , the expected payoff for investing is

$$\begin{aligned} E(P(z, I)|\theta) &= Pr(z_h|\theta)P(z_h) + Pr(z_l|\theta)P(z_l) \\ &= [\theta\lambda + (1 - \theta)(1 - \lambda)](P(z_h) - P(z_l)) + P(z_l) \\ \Rightarrow \frac{\partial E(P(z, I)|\theta)}{\partial \theta} &= (2\lambda - 1)(P(z_h) - P(z_l)) > 0 \end{aligned}$$

Therefore, the expected payoff is strictly increasing in  $\theta$ , suggesting that the equilibrium must be a threshold  $\theta_{SB}$ .

Now we examine when  $\theta_{SB} = 0$ . For the lowest type  $\theta = 0$ , its expected payoff is

$$E[P(z, I)|\theta = 0] = (1 - \lambda)P(z_h) + \lambda P(z_l) \quad (21)$$

If prices are such that equation (21) is greater than  $K$ , the lowest type will invest, leading to a pooling equilibrium. If so, investment is completely uninformative and  $Pr(S|I) = E(\theta) = \bar{\theta}$ . Define  $N = \frac{K}{R}$ , we have

$$E(P(z, I)|\theta = 0) \geq K \Leftrightarrow \begin{cases} 0 < N < \bar{\theta} & \text{(c) and} \\ \lambda < \frac{1}{2} \left( 1 + \sqrt{\frac{\bar{\theta} - N}{\bar{\theta} - N + 4N\bar{\theta} - 4N\bar{\theta}^2}} \right) & \text{(d)} \end{cases} \quad (22)$$

So  $\theta_{SB} = 0$  if and only if conditions (c) and (d) are satisfied at the same time.

We next prove  $\theta_{SB} < \theta_{FB}$ . Suppose instead  $\theta_{SB} \geq \theta_{FB}$ ; type  $\theta_{FB}$  will not invest in the second best. However, given that investors believes the equilibrium is  $\theta_{SB}$ , the price for any realization  $z$  is  $P(z) = E(\tilde{C}|\theta \geq \theta_{SB}, z) > E(\tilde{C}|\theta = \theta_{FB}, z)$ . Therefore, the expected payoff for  $\theta_{FB}$  is

$$\begin{aligned} E(P(z)|\theta_{FB}) &= Pr(z_h|\theta_{FB})P(z_h) + Pr(z_l|\theta_{FB})P(z_l) \\ &> Pr(z_h|\theta_{FB})E(\tilde{C}|\theta_{FB}, z_h) + Pr(z_l|\theta_{FB})E(\tilde{C}|\theta_{FB}, z_l) = K \end{aligned}$$

Contradiction. Therefore,  $\theta_{SB} < \theta_{FB}$ .

Lastly, we show that  $\theta_{SB}$  increases in  $\lambda$ . Define the conditional expectation  $E(\theta|\theta \geq t) = H(t)$ , obviously  $t < H(t) < 1$  and  $H(t)$  is increasing in  $t$ . In equilibrium, type  $\theta_{SB}$  must be indifferent between investing and not investing:

$$[\theta_{SB}\lambda + (1 - \theta_{SB})(1 - \lambda)]P(z_h) + [\theta_{SB} + \lambda - 2\lambda\theta_{SB}]P(z_l) = K \quad (23)$$

$$\Leftrightarrow \frac{[\theta_{SB}\lambda + (1 - \theta_{SB})(1 - \lambda)]H(\theta_{SB})\lambda}{[H(\theta_{SB})\lambda + (1 - H(\theta_{SB}))(1 - \lambda)]} + \frac{[\theta_{SB} + \lambda - 2\lambda\theta_{SB}]H(\theta_{SB})(1 - \lambda)}{[H(\theta_{SB})(1 - \lambda) + (1 - H(\theta_{SB}))\lambda]} = N \quad (24)$$

Define equation (24) as  $L_1(\theta_{SB}, \lambda)$  and we have the the partial derivative:

$$\frac{\partial L_1(\theta_{SB}, \lambda)}{\partial \lambda} = \frac{(2\lambda - 1)H(\theta_{SB})(1 - H(\theta_{SB}))(\theta_{SB} - H(\theta_{SB}))}{[H(\theta_{SB})\lambda + (1 - H(\theta_{SB}))(1 - \lambda)]^2 [H(\theta_{SB})(1 - \lambda) + (1 - H(\theta_{SB}))\lambda]^2}$$

It is negative because  $\theta_{SB} < H(\theta_{SB})$ . Meanwhile,

$$\begin{aligned} \frac{\partial L_1(\theta_{SB}, \lambda)}{\partial \theta_{SB}} &= (2\lambda - 1)(P(z_h) - P(z_l)) + [\theta_{SB}\lambda + (1 - \theta_{SB})(1 - \lambda)] \frac{\partial P(z_h)}{\partial \theta_{SB}} \\ &\quad + [\theta_{SB} + \lambda - 2\lambda\theta_{SB}] \frac{\partial P(z_l)}{\partial \theta_{SB}} \end{aligned}$$

For any signal realization,  $\frac{\partial P(z)}{\partial \theta_{SB}} = \frac{\partial P(z)}{\partial H(\theta_{SB})} \frac{\partial H(\theta_{SB})}{\partial \theta_{SB}} > 0$ , therefore  $\frac{\partial L_1(\theta_{SB}, \lambda)}{\partial \theta_{SB}}$  must be positive. By the implicit function theorem

$$\frac{d\theta_{SB}}{d\lambda} = -\frac{\frac{\partial L_1(\theta_{SB}, \lambda)}{\partial \lambda}}{\frac{\partial L_1(\theta_{SB}, \lambda)}{\partial \theta_{SB}}} > 0. \quad (25)$$

□

## Proof of Proposition 7

*Proof.* First,  $H_0(\theta) = -(\rho \log_b \rho + (1 - \rho) \log_b (1 - \rho))$  and  $H_1(\theta) = Pr(\text{invest}) \times H(\theta|I) + Pr(\text{no invest}) \times H(\theta|\text{no invest})$ . Since investors know for sure  $\theta = B$  upon observing no investment, there is no residual uncertainty and  $H_1(\theta) = Pr(\text{invest}) \times H(\theta|I)$ . Similarly,

$H_2(\theta) = Pr(invest, y_h) \times H(\theta|I, y_h) + Pr(invest, y_l) \times H(\theta|I, y_l)$ . Therefore, we have

$$\begin{aligned} H_1(\theta) &= -\rho \log_b \frac{\rho}{\rho + (1-\rho)q} - (1-\rho)q \log_b \frac{(1-\rho)q}{\rho + (1-\rho)q} \\ H_2(\theta) &= -\rho\tau \log_b \frac{\rho\tau}{\rho\tau + (1-\rho)(1-\tau)q} - (1-\rho)(1-\tau)q \log_b \frac{(1-\rho)(1-\tau)q}{\rho\tau + (1-\rho)(1-\tau)q} \\ &\quad -\rho(1-\tau) \log_b \frac{\rho(1-\tau)}{\rho(1-\tau) + (1-\rho)\tau q} - (1-\rho)\tau q \log_b \frac{(1-\rho)\tau q}{\rho(1-\tau) + (1-\rho)\tau q} \end{aligned}$$

By definition,  $V_D = H_0(\theta) - H_1(\theta)$ , and  $V_I = H_1(\theta) - H_2(\theta)$ . Without loss of generality, we assume that  $b$  is equal to the natural logarithm  $e$ .

When  $q = 1$ , investment does not provide any information, so  $H_0(\theta) = H_1(\theta) \Rightarrow V_D = 0$ . In addition,  $V_I = H_2(\theta) - H_0(\theta) > 0$ , and it is easy to verify that  $V_I$  is strictly increasing in  $\tau$ .

However, when  $q < 1$ ,  $V_D > 0$  and

$$\begin{aligned} \frac{\partial V_D}{\partial \tau} &= -\frac{\partial H_1(\theta|I)}{\partial \tau} = -\frac{\partial H_1(\theta|I)}{\partial q} \times \frac{\partial q}{\partial \tau} \\ &= \frac{(1-q)(1-\rho)\rho}{q(q(\rho-1) - \rho)} \times \frac{\partial q}{\partial \tau} > 0 \end{aligned}$$

Next, we prove that  $V_D < V_I$  when  $\tau$  is sufficiently small, and  $V_D > V_I$  when  $\tau$  is sufficiently large.

When  $\tau \rightarrow \hat{\tau}(\rho)$ ,  $V_D \rightarrow 0$  and  $V_I > 0$ . Because of continuity, when  $\tau$  is sufficiently small,  $V_D < V_I$ , i.e., the informational effect dominates. Similarly, when  $\tau$  goes from  $\hat{\tau}(\rho)$  to 1,  $V_D$  increases from 0 to  $H(\Theta)$ , where  $H(\Theta) = H_0(\theta)$  is the maximal entropy. Meanwhile,  $V_D + V_I = H_0(\theta) - H_2(\theta) \leq H(\Theta)$ , so if  $\tau$  is large enough, we must have  $V_D > V_I$ , suggesting the disciplinary effect dominates.  $\square$

## B Two Ad Hoc Distributions in Section 4.1

As we discussed in Section 4, how market prices move with accounting quality is very sensitive to the distribution of  $\theta$ . To highlight the distinction, we provide two ad hoc examples in this section and make a qualitative statement at the end.

*Example 1: Uniform Distribution:  $\theta \sim U[0, 1]$ .*

A uniform prior allows us to calculate the conditional expectation as  $E[\theta|\theta > t] = \frac{1+t}{2}$

for  $0 \leq t \leq 1$ . It turns out that  $\theta_{SB}$  is a solution for a cubic function. Therefore, we can compare the date 1 and date 2 stock price as illustrated in the following graph.

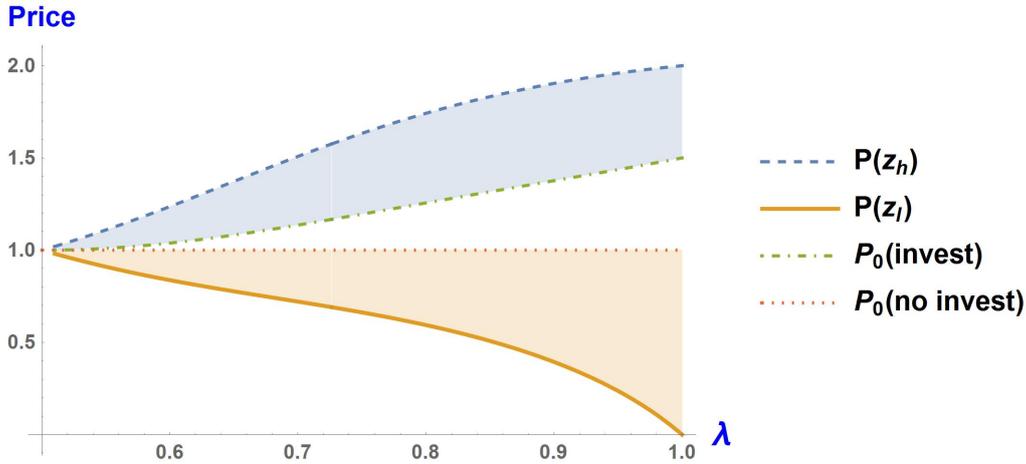


Figure 5: Uniform Distribution  $K = 1, R = 2$

*Example 2: Bernoulli Distribution:* suppose  $\theta$  is degenerated to a Bernoulli distribution:

$$\theta = \begin{cases} G = 0.95 & \text{with prob } 0.4 \\ B = 0.05 & \text{with prob } 0.6 \end{cases}$$

The threshold equilibrium is then degenerated to the mixed strategy, similar to the main model. However, a fully revealing equilibrium is plausible in this case if the accounting information is adequately precise. The intuition is that, if the earnings report almost perfectly reveals future cash flow, the manager’s private information becomes marginally useful. As a result, even if firm  $B$  pretends to be firm  $G$  by investing and investors also believe so, he will ultimately be revealed with high chances and thus receive the low price. In other words, even under the most optimistic belief of  $Pr(G|\text{invest}) = 1$ , firm still  $B$  finds it not optimal to invest. The fully equilibrium is not interesting as the first best is attained, so we only focus on the case in which firm  $B$  takes a mixed strategy. For example, suppose  $K = 1, R = 2$ , we find that firm  $B$  takes a mixed strategy if and only if  $0 < \lambda < 0.939$ . Results are plotted in Figure 5 and 6..

With the two examples above, we can observe the difference qualitatively. Under the uniform distribution, the manager’s private information is by nature of “low quality” in the sense that the variance is extremely high. Since firms’ investment decisions can only reflect the manager’s private information, the level of uncertainty resolved by investment is at most moderate. For example, in the first best, investors only know that  $\theta > \theta_{FB}$  upon

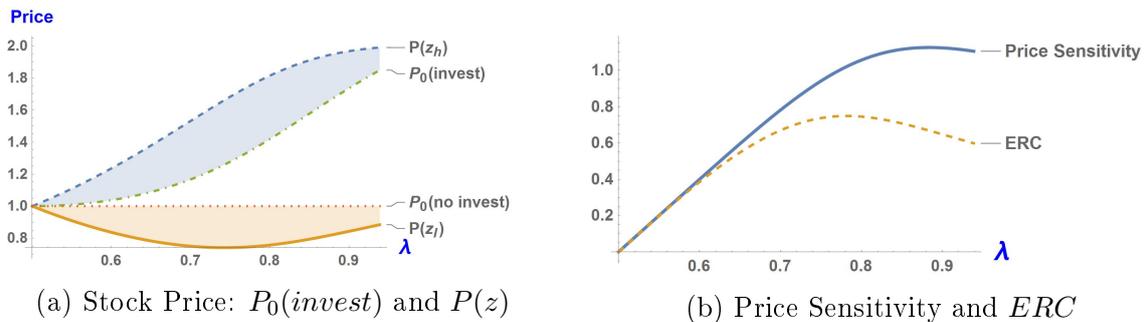


Figure 6: Bernoulli Distribution

observing the investment. However, unless  $\theta_{FB} \rightarrow 1$ , investors still face a large amount of uncertainty about  $\tilde{C}$ . On the contrary, the exogenous earnings report is of “high quality” and consequently resolves a large amount of uncertainty. Figure 4 confirms our intuition:  $P_0(invest)$  does not converge to  $P(z_h)$ , and  $P(z_l)$  does not converge to  $P(no invest)$ .

However, under the degenerated Bernoulli distribution, the manager’s private information is of “high quality” as evidenced by its low variance a prior. The earnings report, however, is only incrementally more informative about the fundamental  $\tilde{C}$ , similar to the assumption in our main model and in [Gigler and Hemmer \(1998\)](#). Furthermore, since both the manager’s private information and the investment decision are of the same dimension, investment can potentially deliver most information content from management’s private knowledge. Taken together, when accounting is fulfilling its objective, firms’ investment decisions becomes sufficiently useful in resolving investors’ uncertainty, whereas the actual earnings report is only marginal. Figure 6 shows that  $P_0(invest)$  converges to  $P(z_h)$  and  $P(z_l)$  converges to  $P(no invest)$ , consistent with Figure 1. In addition, both the price sensitivity and ERC are decreasing in  $\lambda$  when accounting is sufficiently precise, suggesting that high quality accounting may decrease capital market responsiveness to earnings reports.

In summary, we conjecture that our results will hold as long as *the manager has more valuable private information compared to the earnings report, and the manager’s actions can effectively convey his private information.*