

The Choice of Commercial Breaks in Television Programs: The Number, Length, and Timing

Wen Zhou*

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Abstract

This paper examines the choice of commercial breaks by a television network in both monopoly and duopoly settings. Viewers dislike commercials in the model, while the networks seek to maximize the total audience for these commercials through their choice of the number, length, and timing of commercial breaks. The model's prediction of a network's optimal choice seems to match what we observe in reality; in particular, commercial breaks appear more frequent toward the end of the program, and the optimal length of breaks is single-peaked. In duopoly equilibrium, commercial breaks on two networks must have the same length and appear at the same time. Compared to monopoly, the breaks in a duopoly equilibrium are longer and/or appear later.

Keywords: Commercial breaks, timing games, synchronization, zapping

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*Department of Economics, Hong Kong University of Science and Technology. wenzhou@ust.hk. I would like to thank Lin Zhou, Hervé Moulin, Daniel Graham, Nikolaos Vettas, and James Anton for advice and comments. The paper has also benefited from seminar participants at Duke University, University of North Carolina at Chapel Hill, University of Kentucky, Hong Kong University of Science and Technology, and the Southeast Economic and International Conference.

1 Introduction

Television networks in the United States have complete freedom of arranging commercial breaks in a TV program: they can freely choose any number of breaks and any length and timing of each break.¹ Given this fact, it is interesting to observe that networks tend to make very similar choices. The following picture shows a typical pattern of commercial breaks when the three major networks simultaneously broadcast the half-hour evening news programs:



Figure 1: Real-life arrangement of commercial breaks on the three major networks, recorded on Thursday, February 12, 1998.

The three networks all choose eight minutes of commercials with five breaks. The lengths of each break are exactly the same between NBC and CBS, while ABC differs only slightly. The synchronization of commercial breaks across channels is also obvious. The average overlap between any two channels is 70%,² while random timing yields an expected overlap of only 30%. A natural question is why networks want to make the same choice, in particular, why do they want simultaneous timing of commercial breaks? Is it a coincidence or an intentional act?

Television is not a business of producing programs; television produces audience, the access to whom is sold to advertisers. The fact is, viewers do not like commercials.

¹The FCC does not regulate the amount or frequency of television commercials, although regulators in many other countries do. The National Association of Broadcasters, through its industry code, once limited the amount of TV commercials to be no more than 6 minutes per hour in prime time. The practice was declared to violate the antitrust law in 1981 and was abandoned thereafter. The Congress has passed a law in 1992 requiring that the commercials in children's program can not exceed 10.5 minutes per hour on weekdays and 12 minutes per hour on weekends.

²Because commercial breaks are inserted between different news stories, it is impossible to perfectly synchronize breaks even if networks intend to do so.

They try to escape whenever possible.³ The typical behavior during a commercial break is to switch to another channel, go and get a drink, go to the restroom, talk to another person, or simply leave the room without coming back.⁴ Today with the help of remote control and more than 30 channels available to American households at any time, switching channels during commercial breaks, commonly referred to as “zapping,” has become an all-American fashion. Pahwa(1990) estimates that “72% of US households now have at least one remote control” and that “the average household zaps once every 3 minutes and 42 seconds.” On the other hand, a television network’s revenue comes solely from selling commercial times. It has every incentive to keep viewers during its commercial breaks. The advertising industry has long realized the threat posed by viewers’ zapping behavior (Kaplan, 1985; Heeter and Greenberg, 1985; York and Kitchen, 1985; Knealy, 1988; Gross, 1988; Pahwa, 1990; Nakra, 1991; Ward, 1991; *American Demographics*, 1995; Danaher, 1995; Siddarth and Chattopadhyay, 1998). One solution they came up with is the “roadblock” in which the same commercial is broadcast at exactly the same time on several channels. Gross(1988) claimed that “roadblocks can be a somewhat effective deterrent to flipping. The idea behind this time-honored media tactic is to frustrate flippers as they find commercial materials at every turn of the dial.”⁵

The incentive to synchronize commercial breaks is now easy to understand. Television networks coordinate the choice of commercial breaks to force viewers to watch commercials.⁶ In fact viewers’ tendency to avoid commercials is ubiquitous even when

³In this aspect television is different from other media such as newspaper or magazine, where readers might actively look for certain advertisements.

⁴See, for example, Thorson and Zhao(1997), Danaher(1995), and Steiner(1966). In 1952 the water commissioner in Toledo, Ohio, noticed that whenever “I Love Lucy” was shown on the city’s only television station, each advertising break was marked by a huge drop in water pressure as thousands of toilets flushed at once (Knealy, 1988).

⁵This practice was more effective in the past when there were only three or four VHF stations in any local market. It has become less popular today because viewers have so many channels from which to choose. Also, it is not easy to exactly coordinate the timing unless it is some standard time slot such as the commercial time before the 11 o’clock local nightly news.

⁶This coordination might take the form of explicit agreement or tacit collusion. As shown in the paper, in both cases the outcome is simultaneous timing. In reality the decision of when to insert

a network does not have any competitors. The situation, which we call a monopoly, is simpler but more fundamental than duopoly in our attempt to understand the choice of commercial breaks. A monopoly could be a market where there is only one television channel, or a special program in a multi-channel environment in which there are no comparable programs elsewhere at the same time. The Super Bowl and the Academy Awards ceremony are two examples. To some extent a movie on TV can also be regarded as a monopoly. In contrast, duopoly or oligopoly is a situation where several channels broadcast the same type of programs at the same time. Examples are also abundant: NBC, ABC, and CBS all broadcast half-hour evening news programs simultaneously; NBC and CBS both have late night show at 11:30; CNN and Fox News have around the clock news, etc.

The goal of this paper is to study a network's choice of commercial breaks in both monopoly and duopoly settings. Since viewers do not like commercials, it is assumed that every commercial break drives away some viewers; the drop in audience size depends on the length and frequency of breaks. Given viewers' behavior, a network chooses the number of breaks and the length and timing of each break in order to maximize its commercial audience. With this simple setup, the paper is able to reproduce the real-life arrangement of commercial breaks as a network's optimal choice. Figure 2 shows the model's prediction of commercial breaks in monopoly. The pattern resembles what we observe in reality. Considering the model's prediction that the duopoly timing of breaks is later than a monopoly's choice, the picture will be even more similar to the pattern shown in Figure 1. Other interesting features also emerge in monopoly, including the conclusions that commercials become more frequent with the procession of a program, and that the optimal length of breaks is "single-peaked"—meaning that a network should never reduce the length of breaks

a commercial break into the program is made by the program executive manager. It is possible for the manager to monitor the timing of breaks on other channels and insert its own breaks at any appropriate time. Since the same types of programs are repeated every day and each network has a regular pattern, the coordination is not very difficult, provided that simultaneous timing is a Nash equilibrium, which is true in the model.

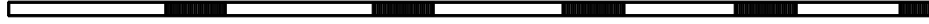


Figure 2: Optimal choice of commercial breaks predicted by the model

first and then increase it later.

The biggest finding in duopoly is that any pure strategy equilibrium must be symmetric; the two networks choose exactly the same number, length, and timing of commercial breaks. Symmetry also characterizes the collusive behavior when the two networks try to maximize the joint commercial audience. The second general result is that the breaks in a duopoly equilibrium are longer and/or appear later, compared to a monopolist's choice. Thirdly, we are able to establish the existence of a continuum of pure strategy equilibria, the range of which continuously shrinks to the unique monopoly choice when the interaction between channels (in the form of the flow of audience) decreases to zero.

Many conclusions that are reached by the study are supported by empirical data. For example, Epstein(1998) provided evidence of the following statements: (1) In equilibrium all networks broadcast commercials at the same time. (2) As the program progresses, commercial breaks become longer and/or news intervals become shorter. (3) The total commercial time in oligopoly might be longer than in monopoly.

Given the importance of television commercials in business and everyday life, surprisingly little research has been done on the choice of commercial breaks. The only paper that we found is Epstein(1998), which provides empirical evidence of many findings that are consistent with our model. Cancian, Bills, and Bergstrom(1995) studied the optimal timing of television news programs, and they found that no pure strategy equilibrium exists in the duopoly game where timing is modeled as a location with directional constraint. The study was followed by several other papers trying to recover the equilibrium: Cancian, Bills, and Bergstrom(1993), Barros(1998), Nilssen and Sorgard(1996, 1998). Our model is inherently much more complicated in two

senses: the duration of commercial breaks is a choice variable; and it involves the optimal number of breaks and the timing of every break. In contrast, these papers study the timing of a television program as one single point in time without any length.

Lal(1990) studies the alternating offer of price discounts by competing firms. Villas-Boas(1993) investigates the “pulse” behavior of firms’ advertising expenditure. Both papers look at the synchronization of certain economic activities, but there are no dynamics in the games. The focus is on whether two firms should have the same kind of economic activities at any point of time. In contrast, the flow of audience as a function of time plays a central role in our model.

Viewers’ behavior during commercial breaks has been studied by Steiner(1966), Ray and Webb(1978), and Thorson and Zhao(1997). Barwise and Ehrenberg(1988) described television program scheduling games.⁷ Owen and Wildman(1992) analyzed a monopolist’s scheduling activity called “windowing.” Advertising by itself has been a topic of economic studies(see, for example, Shy, 1995). Recent research models advertising in timing games(Erickson, 1995). What we study in this paper is essentially not advertising, but rather the timing and duration of different economic activities.

2 The Model

A single network (henceforth the monopolist) broadcasts a television program (we call it news) from time 0 to time 1. There are several commercial breaks in the program.

We use the following notation and assumptions:

n : the number of breaks;

c_i : the length of the i th break, $i = 1, 2, \dots, n$.

⁷One interesting phenomenon is that TBS always starts its programs five minutes later than other channels. This is obviously an intentional action to deal with viewers’ “surfing” at exact hours and half-hours when most channels change programs. Another observation is that a special event like the Super Bowl can produce some interesting economic activities: a competing channel might choose to broadcast programs that cater to women; some grocery stores offer special sales only during Super Bowl time.

x_i : the length of the news interval between the $(i - 1)$ th break and the i th break, $i = 1, 2, \dots, (n + 1)$.

r_i : the audience size (sometimes also called the rating) of the i th break, $i = 1, 2, \dots, n$. r_i is fixed for the duration of that break. When the program begins the audience size is $r_0 = 1$.

$$r_i = r_{i-1} - r_0 f(x_i, c_i), \quad i = 1, 2, \dots, n.$$

where $f(x_i, c_i) \geq 0$, $\frac{\partial f}{\partial x_i} < 0$, $\frac{\partial f}{\partial c_i} > 0$, $\frac{\partial^2 f}{\partial x_i^2} > 0$, $\frac{\partial^2 f}{\partial c_i^2} \geq 0$.

The monopolist's objective function is the cumulative total commercial audience:

$$\pi = \sum_{i=1}^n c_i r_i$$

The monopolist chooses n , $\{c_i\}_{i=1}^n$, and $\{x_i\}_{i=1}^{n+1}$ to maximize π , subject to the constraints: $c_i \geq 0$, $x_i \geq 0$, $\sum_{i=1}^n c_i + \sum_{i=1}^{n+1} x_i = 1$.

In this model, viewers do not like commercials. They stay with the channel so long as it is showing news. At the beginning of a commercial break,⁸ some viewers leave the program.⁹ Consequently the audience size is a downward step function of time as shown in Figure 3.

⁸This assumption is not essential and is only for the convenience of computation. An alternative is to model the audience size as a continuously decreasing function during a commercial break. By appropriately defining the immediate drop of audience size at the beginning of commercial breaks, the two approaches are mathematically equivalent.

⁹The flow of audience is one-directional in monopoly, but when there are more than one channel, viewers can jump back and forth between channels. In reality many viewers leave the program temporarily during a commercial break and come back when the program resumes. But if all of the viewers do this, the length or the timing of breaks would not matter at all for a network. It always loses that part of its audience whenever a commercial appears. There must be some viewers who do not come back. Take the 6:30 evening news as an example. At the beginning of the program there are a certain number of viewers who are willing to watch at least part of the news. They enjoy the program until being provoked by a commercial break, at which moment many decide whether it is worthwhile to tolerate one more break and watch the news thereafter. Some will stay, but some will quit and take up other activities (i.e. go to a movie) without watching the remaining news. Another interpretation is that there exists a network which does not have any commercial breaks in its programs, i.e. the PBS. Once a viewer is driven to PBS by commercials, he is trapped there and never comes back.

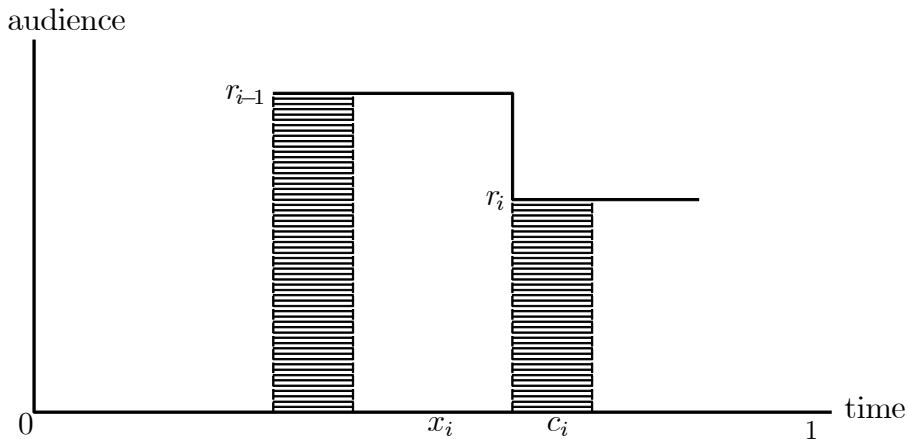


Figure 3: Flow of audience in monopoly

The drop (as a ratio of original audience size r_0) at the beginning of each break is influenced by two factors: the frequency of commercials and the length of the current break, where the frequency is represented by the news interval immediately preceding the break. This specification assumes very simple behavior by viewers. At the sight of a commercial, they decide whether or not to stay, and they only care about the program interval before this point and the length of commercials after the point. Viewers are partially rational in the sense that they can correctly anticipate the length of the current break and remember the length of the most recent news interval, but nothing else. Because viewers do not like commercials, the drop is decreasing in x_i and increasing in c_i . In other words, the drop in audience size is greater when the current break is longer and smaller when the program interval immediately preceding the break is longer.¹⁰ The signs of the second derivatives on f are assumed to guarantee an interior solution. Note that a zero second derivative on c_i is allowed.

¹⁰There is evidence to support the effect of the length of commercial breaks. Siddarth and Chattopadhyay(1998) found that “longer advertisements have a higher probability of being switched off.” Danaher(1995) made the same discovery. The second factor—the frequency of breaks—is both intuitive and necessary to the model, although it has not been identified by the literature. When the audience is annoyed by too frequent commercial breaks, more of them will quit. If this effect were not present, the timing of commercial breaks would not matter at all, and networks would squeeze in as many breaks as possible, but this is obviously not true.

The monopolist's payoff is simply the cumulative total commercial audience.¹¹ Current techniques can only report the rating of the whole program, i.e. the commercial audience cannot be distinguished from the news audience.¹² Therefore another possible (and seemingly more natural) specification of the objective function would be the rating times the total length of commercials, where the rating is represented by the total audience of the whole program. (Note that the total audience itself cannot be the objective function because that would lead the monopolist to broadcast no commercials at all.) Then a network has a strong incentive to broadcast commercials later rather than earlier in the program in order to retain a bigger audience for the early part of the program. All of the commercials might appear at the end of the program. Many viewers will leave once commercials begin. Consequently advertisers would raise the concern and require the commercials to be aired earlier. This implies that a network must give greater weight to the commercial audience than to the news audience. Simulations on the ratings as weighted averages of commercial audience plus news audience provide no meaningful new result while complicating computations.

¹¹ Advertisers buy commercial times up front for a whole season. The price they pay networks depends on the ratings of programs in which their commercials are broadcast. Consequently networks seek to maximize the ratings, which are simply the audience sizes. In principle an advertiser should care about the exact position of his commercials either in the program or in a series of commercials in a break, and the concern should be reflected in the pricing of different positions. While this might be true for a special event like the Super Bowl, normally a contract between a network and an advertiser goes no further than specifying the frequency of any particular commercial in a certain program, i.e. four times per week in the 6:30 evening news program.

¹² What happens is that a meter is attached to the TV set in a sample family and automatically records the duration of tuning to any channel for at least 5 minutes. Since normally a commercial break does not last for more than 3 minutes, it is simply not possible for this monitoring system to record the audience of any particular commercial break. Nielson Research Institute has told me that they are developing a technique to monitor audience minute by minute, or even second by second. This development justifies our specification of an advertiser's (thus a network's) concern about the commercial audience rather than the program audience.

3 A Monopolist's Optimal Choice

Before we proceed to the general results, it is helpful to look at some examples. Let $f(x_i, c_i) = de^{-kx_i} + e^{ac_i} - 1$ with $d = 1/7$, $k = 20$, $a = 2.2$. In the unique optimal solution the monopolist chooses five breaks with a total length of 0.32. The arrangement of breaks in the optimal solution is shown in Figure 2. The lengths of the breaks are 0.064, 0.065, 0.066, 0.066, and 0.061 respectively, while the lengths of news intervals are 0.17, 0.16, 0.14, 0.12, 0.09. The average drop of audience size at the beginning of each break is 16 percent of the original audience.¹³ Many other functional forms and parameters have also been tried. The results vary, but there emerges a robust pattern: the last commercial break always appears at the end of the program; news intervals become shorter and shorter when the program proceeds; and the length of breaks has a single peak. This is not a coincidence.

Lemma 1 In any optimal solution $x_{n+1} = 0$. In other words, the last commercial break always appears at the end of the program.

Proof: Suppose in some optimal solution $x_{n+1} \neq 0$. Now increase x_n while keeping c_n unchanged. Remember $\pi = \sum_{i=1}^n c_i r_i$. r_i is not affected by this action for $i = 1, 2, \dots, n-1$, but r_n has increased because $r_n = r_{n-1} - r_0 f(x_n, c_n)$ and f is strictly decreasing in x_n . The original arrangement could not have maximized π , thus the desired contradiction. **Q.E.D.**

The reason is easy to understand. Since viewers do not look into the far future, the audience size up to a break does not depend on any arrangement after this break. The monopolist's only concern on the last commercial break is to increase the audience of that break, so he pushes the last break to the very end of the program. In contrast, there are trade-off's in arranging the timing of all the previous breaks: increasing one

¹³Kneale(1988) reported a study by R.D. Percy and Co. in New York city where the rating is measured on a second-by-second basis. They found that ratings during commercial breaks dropped by 10 percent on average.

news interval will necessarily reduce another, leading to a bigger audience on the first break and a smaller audience on the second. In reality the commercials between two TV programs always appear right before exact hours or half-hours, which means we should regard them as happening at the end of the previous program rather than at the beginning of the next program.

Now the constraint faced by the monopolist becomes $\sum_{i=1}^n (x_i + c_i) = 1$. Rewrite the audience size of the i th break as $r_i = 1 - \sum_{t=1}^i f(x_t, c_t)$. Let $f_i = f(x_i, c_i)$. Then

$$\pi = \sum_{i=1}^n c_i r_i = \sum_{i=1}^n c_i - \sum_{j=1}^n \left(\sum_{i=j}^n c_i \right) f_j$$

The monopolist chooses $x_i \geq 0$ and $c_i \geq 0$, $i = 1, 2, \dots, n$ to maximize π , under the constraint $\sum_{i=1}^n (x_i + c_i) = 1$. Let λ be the Lagrangian coefficient. Assuming an interior solution, the first order conditions are:

$$\frac{\partial \pi}{\partial c_j} = 1 - \sum_{i=1}^j f_j - \left(\sum_{i=j}^n c_i \right) \frac{\partial f_j}{\partial c_j} = \lambda \quad (1)$$

$$\frac{\partial \pi}{\partial x_j} = - \left(\sum_{i=j}^n c_i \right) \frac{\partial f_j}{\partial x_j} = \lambda \quad (2)$$

$j = 1, 2, \dots, n$

The second order conditions are always satisfied.

Proposition 1 If $f(x_i, c_i)$ is separable and additive in x_i and c_i , or if the length of each break is the same, then x_i is decreasing over i : $x_i > x_j$ whenever $i < j$.

Proof: From (2),

$$\left(\sum_{t=i}^n c_t \right) \frac{\partial f_i}{\partial x_i} = \left(\sum_{t=j}^n c_t \right) \frac{\partial f_j}{\partial x_j}$$

because $i < j$, we have $\sum_{t=i}^n c_t > \sum_{t=j}^n c_t$. Since $\partial f_t / \partial x_t < 0$ for $t = 1, 2, \dots, n$,

$$\frac{\partial f(x_i, c_i)}{\partial x_i} > \frac{\partial f(x_j, c_j)}{\partial x_j}$$

If $f(x_i, c_i) = g(x_i) + h(c_i)$, the above inequality becomes $g'(x_i) > g'(x_j)$. Since $g'' > 0$, $x_i > x_j$. Alternatively, if $c_i = c_j$, the same conclusion is also reached. **Q.E.D.**

This result says that commercial breaks become more frequent toward the end of the program. It is consistent with what we observe in reality. Since the audience size at any moment is a subset of the previous audience, what happens to the audience at any particular time has a cumulative impact on the remaining part of the program. In other words, the audience size of an early break has a bigger weight in the monopolist's objective function than the audience of a later break. As a longer news interval tends to increase audience, the optimal timing should command a shorter and shorter news interval—therefore more frequent commercial breaks—toward the end of the program.¹⁴ The network does not want to drive viewers away very early, although it might have to do so later since it has a certain number of commercials to air.

There is an immediate corollary to Proposition 1:

Corollary. $x_1 > 0$ unless $\sum_{i=1}^n c_i = 1$.

The first commercial break will never appear at the very beginning of the program. There must be at least some regular program before the first commercial break begins. Again, this is consistent with what happens in reality.¹⁵

In line with the argument of a shorter news interval toward the end of the program, one might expect the length of breaks to be increasing over time. One might reason that a shorter break in the early part of the program would keep a bigger portion of the audience, and thus increase the total rating. This is not the case. Unlike the news interval, the length of breaks enters the objective function in two ways. First, it is the weight of the audience size of each break: $\pi = \sum_{i=1}^n c_i r_i$. Second, it influences the

¹⁴Another interpretation is that a viewer of a television movie might be “hooked” in the sense that the more he watches, the higher is the cost of switching to another program. The idea of getting hooked is consistent with our assumption that the longer a program interval is, the smaller is the number of viewers who quit.

¹⁵There are many commercials before the broadcast of a sporting event like the Super Bowl, but we do not regard the program as having started until the game actually begins.

payoff indirectly by influencing the audience of each break. The two forces operate in opposite directions. A short break in the early part of a program will increase the rating, but it will also reduce the weight on this higher rating. The net effect is ambiguous. Nonetheless, we are able to find one sure thing.

Proposition 2 Suppose $f(x, c) = g(x) + h(c)$. Then the length of breaks is single-peaked, i.e. it is never optimal to decrease the length of breaks first and then increase the length later.

Proof: Suppose in some optimal solution $c_i < c_{i-1}$ and $c_i < c_{i+1}$ for some i . From equation (1) we have:

$$(c_i + c_{i+1} + \dots + c_n) \frac{\partial f_i}{\partial c_i} = f_{i+1} + (c_{i+1} + c_{i+2} + \dots + c_n) \frac{\partial f_{i+1}}{\partial c_{i+1}}$$

Rearranging terms yields:

$$c_i h'(c_i) + \left(\sum_{t=i+1}^n c_t \right) [h'(c_i) - h'(c_{i+1})] = f_{i+1}$$

For the same reason we have:

$$c_{i-1} h'(c_{i-1}) + \left(\sum_{t=i}^n c_t \right) [h'(c_{i-1}) - h'(c_i)] = f_i$$

Since $c_i < c_{i-1}$, $c_i < c_{i+1}$, $h' > 0$, $h'' > 0$, we get

$$c_i h'(c_i) > f_{i+1} \quad \text{and} \quad c_{i-1} h'(c_{i-1}) < f_i$$

We already know that $c_i < c_{i-1}$ and $0 < h'(c_i) < h'(c_{i-1})$. Therefore $f_i > f_{i+1}$. On the other hand, from Proposition 1, $x_i > x_{i+1}$. By assumption $c_i < c_{i+1}$, $g' < 0$, $h' > 0$, then $f_i < f_{i+1}$, thus the contradiction. **Q.E.D.**

At the beginning of the program a network does not want the break to be very long for fear of driving many viewers away too early. At the end it does not want long breaks either, because the audience size is already small. In real life we see the

last break tends to be relatively short. Sometimes this time slot is filled with other “non-program elements” such as station declarations or “coming up next”

Thus far we have discussed the timing and the length of breaks. A network’s choice variables include the number of breaks. The following is true (because of limited space, the proof is omitted):

Lemma 2 When the length of each commercial break is the same, the payoff function is concave in the number of breaks n .

This property ensures us that the monopolist will choose a certain number of breaks within a program, neither too many nor too few. For a fixed total length of commercials, splitting into more breaks will decrease the length of every break, which tends to increase the rating, while at the same time the commercials become more frequent, which decreases the rating.

4 Two Networks

We maintain every assumption that was made for the monopoly case. Two networks (henceforth channel 1 and channel 2) each broadcast a television news program from time 0 to time 1. The following notation is used:

n_j : the number of breaks on channel j , $j = 1, 2$.

z_i^j : the starting time of the i th break on channel j .

w_i^j : the ending time of the i th break on channel j . $w_0^j = 0$ for $j = 1, 2$.

$c_i^j = w_i^j - z_i^j$: the length of the i th break on channel j .

$x_i^j = z_i^j - w_{i-1}^j$: the length of the news interval between the $(i - 1)$ th break and the i th break on channel j .

r_t^j : the audience size at time t on channel j . $r_0^1 = r_0^2 = 1/2$.

At $t = z_i^j$, r_t^j instantly drops by $r_0^j f(x_i^j, c_i^j)$; at $t = z_i^k$, r_t^j instantly increases by $pr_0^k f(x_i^k, c_i^k)$. $j, k \in \{1, 2\}$. $f(x, c) > 0$, $\frac{\partial f}{\partial x} < 0$, $\frac{\partial f}{\partial c} > 0$, $\frac{\partial^2 f}{\partial x^2} > 0$, $\frac{\partial^2 f}{\partial c^2} \geq 0$.

$p \in [0, 1]$: the switching coefficient.

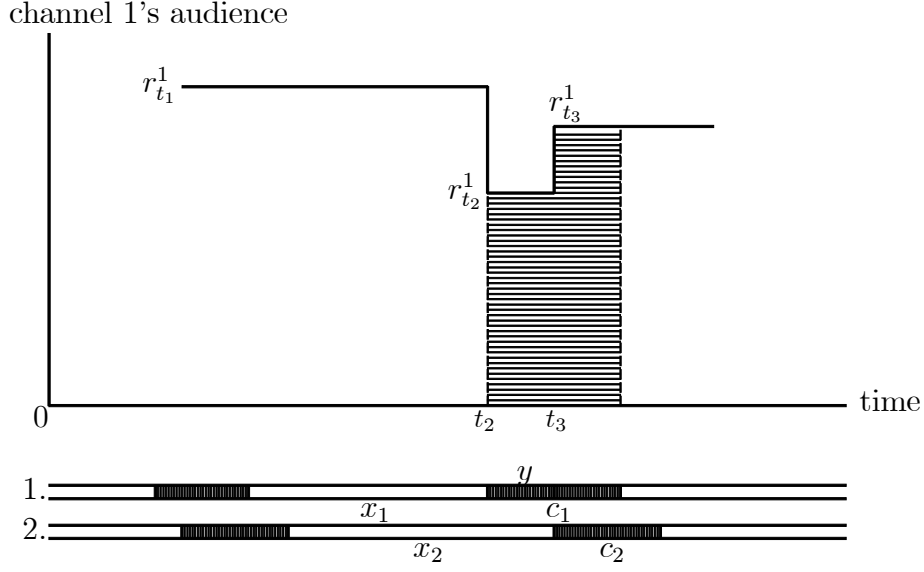


Figure 4: Flow of audience in duopoly

Each channel's payoff is its own cumulative total commercial audience:

$$\pi^j = \int_{t \in \cup_{i=1}^{n_j} [z_i^j, w_i^j]} r_t^j dt$$

The choice variables available to channel j are: $n_j, \{z_i^j\}_{i=1}^{n_j}, \{w_i^j\}_{i=1}^{n_j}$, namely the number of breaks, and the length and timing of each break. The constraints are: $0 \leq \dots \leq z_i^j \leq w_i^j \leq z_{i+1}^j \leq w_{i+1}^j \leq \dots \leq 1, i = 1, 2, \dots, n_j$.

Figure 4 shows the change of a channel's audience when viewers jump between the two channels. In this picture, one of channel 1's breaks starts at time t_2 , with x_1 and c_1 being the length of the preceding news interval and the length of the break, respectively. Before the end of this break, channel 2 starts its break at time t_3 with length c_2 and interval x_2 . Then

$$r_{t_2}^1 = r_{t_1}^1 - r_0^1 f(x_1, c_1), \quad r_{t_3}^1 = r_{t_2}^1 + pr_0^2 f(x_2, c_2)$$

and channel 1's audience of that break is $yr_{t_2}^1 + (c_1 - y)r_{t_3}^1$.

Viewers do not have any preference between the two channels' programs. Their only concern is to avoid commercials. When the program starts, the two channels split the market equally. The objective function of a channel is still the channel's total commercial audience. Whenever a commercial appears on screen, part of the channel's audience will leave. The drop function $f(x_i, c_i)$ is exactly the same as in monopoly: it decreases with the news interval immediately preceding the break and increases with the length of this break. However, unlike in monopoly, the departing audience has another choice. Not only can viewers avoid commercials by turning off the television, they can also choose to change the channel. The switching coefficient is $p \in [0, 1]$: out of the people who escape, a fixed proportion p changes channels, while the remaining $(1 - p)$ turns off the television.¹⁶ When $p = 0$, each channel becomes a *de facto* monopolist and there is no interaction at all between the two channels, which make their choices independently. At the other extreme, when $p = 1$, the total population is fixed at 1. Viewers must watch one of the programs; the total demand is perfectly inelastic. This is the situation where the interaction is the strongest.

Switching viewers will stay with the second channel even if they also find commercials there. Normally viewers perceive commercial breaks to have the same length. A break has just started on the first channel, while the second channel might be finishing its commercials at any time. The expected waiting time of staying with the second channel is shorter than that of the first. On the other hand, the switching viewers will not drop out of the market entirely after finding commercials on the second channel, because they are the people who decide to stay in the market in the first place. They are willing to tolerate at least one more commercial break.¹⁷

¹⁶Alternatively, we could assume that the number of viewers exiting the duopoly market is the same as that in the monopoly case. Furthermore, we could assume that there is an additional exodus to the other channel, i.e. $r_i = r_{i-1} - (1+p)r_0f(x_i, c_i)$. However, our objective is to compare how the flow of audience between channels influences the channels' choice of commercial breaks for monopoly and duopoly. The loss in audience when a commercial appears should be the same in both cases. The only difference is that a channel can now gain viewers from the other channel when that channel broadcasts commercials.

¹⁷Alternatively, when a commercial break appears on screen, the viewers who leave can look at the other channel first. If it is also showing commercials, they will exit the market and never come back;

As in monopoly, the drop function at the beginning of a break depends only on the news interval preceding the break x_i , and the length of the break c_i . There might be some viewers who have just switched from the other channel and do not know x_i , i.e. they should instead care about y_i , which is the news interval between the time they switch and the time the current break begins. For simplicity we assume switching viewers also know x_i .

Since the ratings and drops are all defined in the unit of the original audience size r_0^j , $j = 1, 2$, and since audience is originally distributed evenly between the two channels, the optimal choice of any channel does not depend on the value of r_0^j . For simplicity, we assume that $r_0^j = 1$ for both channels $j = 1, 2$. This won't affect any choice made by a channel. The only difference it makes is that the payoff we now talk about is twice that of the actual payoff when $r_0^j = 1/2$. When there is no ambiguity, subscriptions are also used to indicate channels: c_i and x_i represent the length of a break and the length of a news interval on channel i , $i = 1, 2$.

5 Collusive Behavior

We first study the collusive behavior in duopoly. Two channels coordinate choices of commercial breaks in order to maximize the joint total commercial audience.

First, it can be shown that in the optimal solution the last break on each channel always appears at the end of the program. Since there are too many variables, we need to simplify. We start by assuming that each channel has only one commercial break. There are two choice variables: the length of the break on each channel c_1 and c_2 . It turns out that the two channels must choose the same length. (Henceforth, all of the proofs are collected in the Appendix.)

Lemma 3 If each channel has one commercial break, the maximum joint payoff is achieved when $c_1 = c_2 = c$. c is increasing in p . If $p = 0$, then $c = c_0$, where c_0 is a

otherwise they will stay (with the second channel). In this case networks have a strong incentive to have non-simultaneous timing in order to preserve the total audience.

monopolist's choice of the length of a single commercial break.

A non-symmetric arrangement could not be optimal. Since a shorter break implies a later appearance (recall that the two breaks are both at the end of each program), it has more audience than the one on the other channel because of switching viewers. Consequently it should have longer length in the optimal solution than the other break, thus the contradiction. When $p = 0$, the two channels become two monopolists that each independently choose c_0 . When $p > 0$, a commercial break is less costly than in monopoly because each channel gains some viewers from the other channel, therefore both choose a longer break.

Now suppose each channel has two breaks and every break has the same fixed length c . Again there are two variables: the news interval before the first break on either channel, x_1 and x_2 .

Lemma 4 If each channel has two breaks and each break has the same length, then the joint payoff achieves the maximum when $x_1 = x_2 = x_0$, where x_0 is a monopolist's choice of the timing.

Suppose the two channels do not have simultaneous timing of their first breaks, i.e. channel 2's break appears later. Then channel 2's first break has a bigger audience than that of channel 1 because of switching viewers. On each channel, a later first break results in an increase in the audience of the first break and a decrease in the audience of the second break. The timing is optimally chosen when the audiences of the two breaks are balanced. Since the first break on channel 1 has a smaller audience than that on channel 2, it should be broadcast later, i.e. the two channels actually should have the same timing for their breaks.

In the above two cases (each channel has one break, and each channel has two breaks with fixed equal length), collusion always results in a symmetric arrangement of the commercial breaks—equal length and simultaneous timing. A channel is going to lose part of its audience anyway whenever commercials appear on its screen, but

the other channel will gain some escaping viewers. This is a positive externality. A symmetric arrangement of commercial breaks across the two channels can maximally internalize the externality. This should still be true in the general case when the two channels can choose any number of breaks with any length and timing.

6 Duopoly Equilibrium

Two channels compete with each other for commercial audience. We want to study equilibrium choice of commercial breaks in a duopoly game and how the choice compares to that of monopoly. Since there are too many choice variables, we need to concentrate on one type of variable at a time. We shall first consider the number of breaks.

First, it can be shown that in equilibrium the last break always appears at the end of the program on either channel. Now suppose two channels have the same total length of commercial breaks and they decide independently and simultaneously how many breaks they want. Up to two breaks are allowed. We have the following result:

Lemma 5 The two channels must choose the same number of breaks in any pure strategy Nash equilibrium: they either both choose one break or both choose two breaks.

This lemma says that it is never an equilibrium for the two channels to choose a different number of breaks. The trick is to use a monopolist's choice as a benchmark. Between the two channels' choices of the number of breaks, a monopolist must (weakly) prefer one to another. Then the channel whose choice is not preferred by a monopolist should not be maximizing its commercial audience, because by changing to its rival's number of breaks, it can benefit in two ways. First, it acts as if it is a monopolist when the original number of breaks is not optimal; second, its commercial audience becomes bigger because its commercials overlap more with the other channel. Note that we still do not know whether it is an equilibrium for the two channels

to choose the same number of breaks while a monopolist chooses a different number of breaks.

Now let's turn to the timing of breaks. Suppose each channel has the same number of breaks with the same fixed length. If there is only one break on each channel, the timing problem becomes trivial as both breaks appear at the end of the program. When there are two breaks with fixed length, each channel has only one choice variable: the timing of the first break. Not surprisingly, the two channels again must make the same choice.¹⁸

Lemma 6 If each channel has two breaks and every break has a fixed length c , then:

- (i) Non-simultaneous timing can never be a pure strategy equilibrium;
- (ii) There is a continuum of equilibria in simultaneous timing in the form of $x_0 \leq x \leq y$, where x is the equilibrium simultaneous timing by both channels, x_0 is the unique optimal timing by a monopolist, and $y \geq x_0$ is characterized by $pf(y) - 2cf'(y) + cf'(1 - 2c - y) = 0$.

The best response of channel 1 is shown in Figure 5. x_0 is a monopolist's optimal choice of the timing. If channel 2 airs the first break earlier than x_0 , channel 1 will stick to the timing of x_0 . So long as channel 1's first break is after channel 2's, whatever channel 1 gains from the switching viewers is independent of channel 1's timing; the optimization problem channel 1 faces is exactly the same as when it is the only channel to operate.

When channel 2's first break is very late, channel 1 would again stick to the monopolist's ideal timing x_0 . So long as channel 1's first break is before channel 2's without any overlap, channel 1 will not benefit from switching viewers on its first

¹⁸For simplicity we have assumed that the length of all four breaks is equal, but this is not necessary. The two breaks within one channel can have different lengths so long as the two channels are symmetric, i.e. the two channels' first breaks have the same length, and the two channels' second breaks have the same length. When the two channels are asymmetric, we might have non-simultaneous timing in equilibrium.

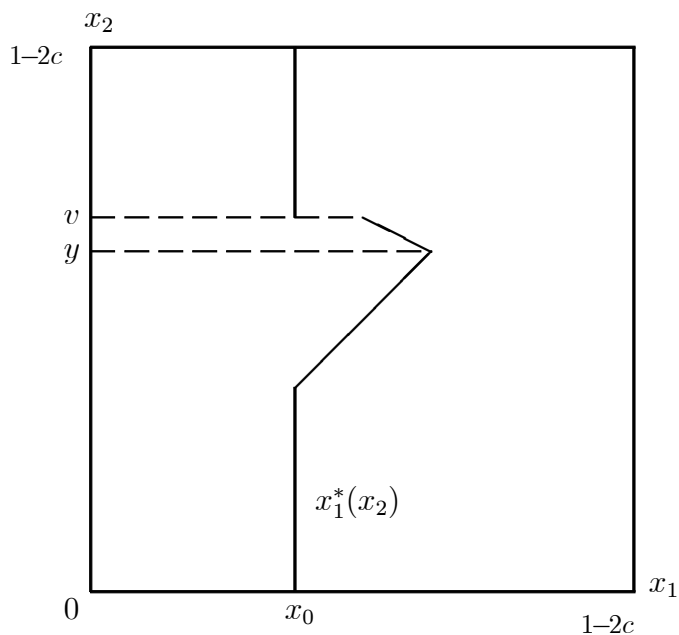


Figure 5: Best response in the timing of two breaks

break. (Its second break will always benefit, but that gain does not depend on the timing of the first break.) Therefore again channel 1 is solving an optimization problem faced by a monopolist: the trade-off of increasing x_1 is to increase the audience of the first break and decrease the audience of the second break.

Now suppose channel 2 starts its first break slightly after x_0 . If channel 1 sticks to x_0 , the trade-off on its own audience between the two breaks is optimally solved. However, channel 1 has an extra incentive to defer the first break so that it can maximally benefit from channel 2's switching audience. Over a certain range, channel 1's best response is to always start its first break when channel 2 does. This simultaneous timing can be weakly later than the monopoly timing x_0 , but never earlier.

When channel 2 starts its first break even later, the force to keep channel 1's two breaks apart is so strong that it will give up simultaneous timing. Instead it will adopt partial overlap by starting the first break slightly earlier than channel 2's. x_1 and x_2 move in opposite directions: the later channel 2's first break, the earlier is

channel 1's. Beyond some point, channel 1 will totally abandon partial overlap and jump back to x_0 .

The range of simultaneous timing that can be supported as an equilibrium is between x_0 and y , where y is determined by $pf(y) - 2cf'(y) + cf'(1 - 2c - y) = 0$. When $p = 0$, this equation becomes the first order condition that a monopolist needs to solve. Consequently $y = x_0$. The situation $p = 0$ means that the two channels are in fact two independent monopolists. Naturally they will both choose x_0 . The upper bound of all equilibria, y , is increasing in p —when the flow of viewers between the two channels becomes stronger, an even later simultaneous break could still be an equilibrium.

The two channels will have the same payoff in any equilibrium. For each channel, the equilibrium payoff is decreasing with the timing x . It achieves the maximum when $x = x_0$, which is also the optimal choice in both monopoly and collusion.

So far we have discussed the equilibrium number and timing of breaks. Now we turn to the length of breaks. To begin with, suppose each channel has only one break. Then the break will appear at the end of each program, and the only choice variable for each channel is the length of the break.

Lemma 7 When each channel has only one break:

- (i) in equilibrium the breaks always have the same length; and
- (ii) the equilibrium length is weakly longer than a monopolist's choice.

The best response is shown in Figure 6. c_0 is the optimal choice of the length by a monopolist. When channel 2's break is very short, channel 1's gain from switching audience is independent of the choice of c_1 , so long as channel 1 starts its break earlier than channel 2. Consequently channel 1 acts as a monopolist and choose the monopoly length c_0 .

When channel 2's break is slightly longer than c_0 , channel 1 would like to match channel 2's choice. Channel 1 does not want a longer break because, according to the

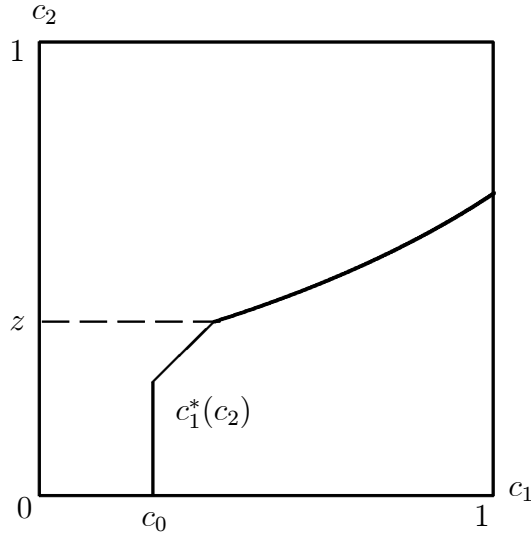


Figure 6: Best response in the length of one break

previous argument, if $c_1 > c_2$, channel 1 wants $c_1 = c_0$. However, c_2 is already greater than c_0 . On the other hand, channel 1 does not want a shorter break either, because now a commercial break is less costly than in monopoly and therefore channel 1 has an incentive to inflate the length of the break. When c_2 is not too long, channel 1 will choose $c_1^* = c_2$ to take full advantage of channel 2's switching audience.

When channel 2's break is even longer, channel 1 stops matching channel 2's choice and instead chooses a shorter break. c_1^* keeps increasing with c_2 , though at a slower speed.

Since the two channels are symmetric, the above best response will only produce symmetric equilibria: the two channels must choose the same length of breaks. The equilibrium symmetric length is bounded by c_0 and z . z is a function of p and is increasing in p . In the extreme case when $p = 1$, we have $z = 1$; that is, any symmetric adoption of commercial length that is greater than c_0 can be supported as an equilibrium. In particular, $c_1 = c_2 = 1$ is an equilibrium. This is simply because viewers are confined to the programs and the two channels do not have to worry about

departing audience.

Now suppose that each channel has two breaks. Within each channel, we further restrict the length of the two breaks to be equal in order to reduce the number of choice variables. Across channels, the choice of break length is independent. Each channel now has two choice variables: the interval before the first break x_i , and the length of each of the two breaks c_i , $i = 1, 2$. The constraint is $x_i + 2c_i = 1$ because the last break is always at the end. We first want to find if non-symmetric arrangement can be supported as an equilibrium. That is, could $c_1 \neq c_2$, $x_1 \neq x_2$ happen in any equilibrium?

Lemma 8 If each channel has two breaks, the breaks on one channel have the same length, and $f(x_i, c_i) = g(x_i) + h(c_i)$, then any equilibrium in (x_i, c_i) is symmetric: $x_1 = x_2$ and $c_1 = c_2$.

The argument is a combination of the case when timing is the only choice variable and the case when length is the only choice variable. Basically, if a break is shorter than its counterpart on the other channel, the shorter break enjoys more audience and hence should be longer. If a break appears earlier than its rival, the earlier break has fewer viewers and thus should be pushed further toward the end. This process won't stop until the two channels make exactly the same choice. The small area in Figure 7 represents all of the combination of (symmetric) x and c that can be supported as an equilibrium. We can see that the equilibrium length is always longer than the monopoly choice, because a commercial break is now less costly due to switching audience. On the other hand, the simultaneous timing could be either earlier or later than the timing in monopoly. This is because longer breaks push the first break toward the beginning of the program, while the interaction between the two channels pulls the first break to the end the program, and the net effect is ambiguous.

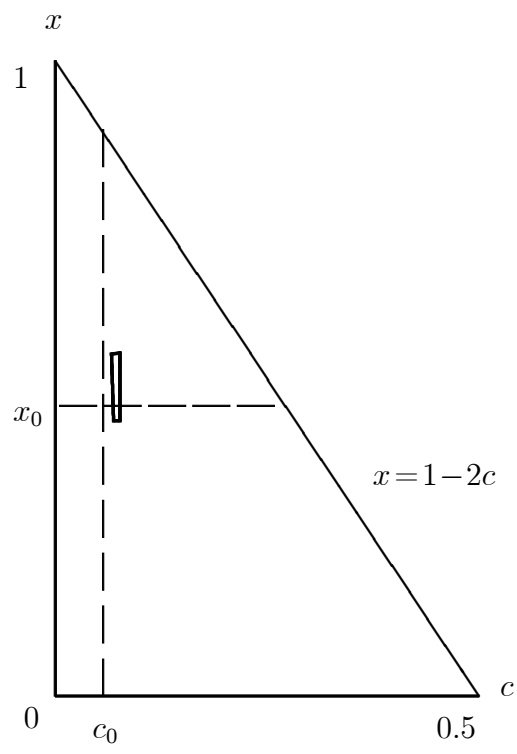


Figure 7: Equilibrium length and timing of two breaks.

Finally, we summarize the results of lemmas 5 through 8 in the following proposition:

Proposition 3

(i) Any duopoly equilibrium must be symmetric—the two channels choose exactly the same number, length, and timing of breaks. In particular, breaks on two channels always appear at the same time.

(ii) Compared to the monopoly choice, a duopoly equilibrium has longer and/or later breaks.

(iii) There exists a continuum of duopoly equilibria in the space of choice variables, and the range of these equilibria continuously shrinks to the monopoly choice as the interaction between the two channels continuously drops to zero.

7 Conclusion

This paper attempts to understand a television network’s choice of commercial breaks. With simple and natural specifications of both viewers’ behavior and a network’s objective function, the model seems to be able to explain the pattern that we observe in reality by monopoly optimal choice and duopoly equilibria. Most of the conclusions are supported by empirical evidence and have practical implications.

In this paper we have used television as an example. Naturally the radio industry is almost identical. To some extent any media advertising bears the same characteristics and the entire analysis applies. For example, we never see any advertisement on the front page of a magazine or newspaper, while the back page always bears commercials.¹⁹

While the results in monopoly are both general and satisfying, the conclusions in duopoly are limited to the case with no more than two breaks because of the presence of too many choice variables. One defense of this weakness is that we can

¹⁹I owe this observation to James Anton.

use “backward induction.” What happens in the later stage of the program does not influence the audience of the early part. We have already proved the conclusions when there are one or two breaks. When the networks decide to have three breaks, given the choice on the first break, the last two breaks must be symmetric. Then we can concentrate on the first break and eventually prove that they should also be symmetric. However, this is not a strict proof. In order to fully overcome the weakness, we can try a totally different approach. Instead of defining the number of breaks as a discrete variable, we can model a network’s choice to be a continuous binary decision (to show commercial or to show news) at any moment of time. This approach represents the first possible direction of the next research. Mathematically, it could be more difficult or easier; we still do not yet know.

The context of the current research could be very rich. Basically, it deals with the situation where a firm must alternate between different economic activities: regular program versus commercials, normal price versus discount price, news versus other programs, or the initiation of an advertising campaign. It has been observed that both Wendy’s and Burger King offer price discounts on Wednesdays. Consequently the second possible extension of the current research is to study other activities such as the offering of price discounts (Zhou, 2000).

A third extension is to study the situation with three or more commercial breaks, or with three or more channels. There are many interesting questions to ask: How does the pattern of duopoly competition in local news programs compare with that of national evening news? Is it easier or more difficult to coordinate the choice of breaks when there are three or more channels rather than two?²⁰ Is there any explicit coordination when two channels belong to the same owner, such as the case of Discovery and TLC, or CNN and CNN Headline News? What happens if two channels have different types of programs? What if the two channels are not symmetric? Is

²⁰The ratings of ABC’s “World News Tonight with Peter Jennings” have been consistently lower than those of the other two networks. (See the picture on the first page.) One might wonder if this has anything to do with ABC’s different arrangement of commercial breaks.

there any leader-follower relationship? Is it beneficial to preempt or to wait?

A fourth direction is to further test all of the conclusions against empirical data. Although some of the conclusions have already found support from previous studies, mainly from Epstein(1998), more work needs to be done, as the questions are both theoretically interesting and practically important.

A Proof of Lemma 3

Not losing generality, assume $c_1 \leq c_2$. Then the joint payoff is:

$$\pi = c_1 [1 - f(1 - c_1, c_1)] + pc_1 [f(1 - c_1, c_1) + f(1 - c_2, c_2)] + c_2 [1 - f(1 - c_2, c_2)]$$

Assuming an interior solution, we must have the following first order conditions:

$$\begin{aligned} \frac{\partial \pi}{\partial c_1} &= (1 - f_1) + c_1(1 - p) \left[\frac{\partial f_1}{\partial x} - \frac{\partial f_1}{\partial c} \right] + p(f_1 + f_2) = 0 \\ \frac{\partial \pi}{\partial c_2} &= (1 - f_2) + (c_2 - pc_1) \left[\frac{\partial f_2}{\partial x} - \frac{\partial f_2}{\partial c} \right] = 0 \end{aligned}$$

where $f_1 = f(1 - c_1, c_1)$, $f_2 = f(1 - c_2, c_2)$. Let $G(c_1) = \partial \pi / \partial c_1 = 0$. Now $G(c_2) = 2pf_2 + p(c_1 - c_2)(\partial f_2 / \partial x - \partial f_2 / \partial c) > 0 = G(c_1)$. Because $G' < 0$ by the second order condition, $c_1 > c_2$, contradicting the assumption that $c_1 \leq c_2$. That is, in the optimal solution, $c_1 = c_2 = c$.

Now each channel has a break at the end of the program with length c .

$$\pi = 2c [1 - (1 - p)f(1 - c, c)]$$

It is obvious that when $p = 0$, the payoff is exactly that of a monopolist, so $c = c_0$. For a general p , c^* is solved by the first order condition:

$$\frac{d\pi}{dc} = [1 - (1 - p)f(1 - c, c)] - c(1 - p) \left[\frac{\partial f}{\partial c} - \frac{\partial f}{\partial x} \right] = 0$$

It is easy to verify that $c(p) \geq c_0$ and $c(p)$ is increasing in p .

Q.E.D.

B Proof of Lemma 4

Suppose channel 1's first break appears earlier: $x_1 \leq x_2$. Since the length of each break is fixed at c , let $f(x) = f(x, c)$. Depending on whether the first breaks on the two channels overlap or not, there are two cases:

Case (1) There is no overlap. The joint payoff is:

$$\begin{aligned} \pi = & c[4 - 2(1-p)f(x_1) - (1-p)f(1-2c-x_1) \\ & - (2-p)f(x_2) - (1-p)f(1-2c-x_2)] \end{aligned}$$

The first order condition on x_1 is:

$$\frac{\partial \pi}{\partial x_1} = 0 \quad \implies \quad 2f'(x_1) = f'(1-2c-x_1) \quad (3)$$

On the other hand, when the two channels arrange simultaneous timing x , the payoff is:

$$\pi_0 = 2c[2 - 2(1-p)f(x) - (1-p)f(1-2c-x)]$$

The first order condition for x in simultaneous timing is exactly the same as that for x_1 in (3). We call the solution in both equations x_0 . It is the unique optimal timing of a monopolist. Now $x_1^* = x_0$.

$$\begin{aligned} \pi_0 - \pi &= (2-p)f(x_2) + (1-p)f(1-2c-x_2) - (1-p)[2f(x_0) + f(1-2c-x_0)] \\ &= (1-p)\{[2f(x_2) + f(1-2c-x_2)] - [2f(x_0) + f(1-2c-x_0)]\} + pf(x_2) \end{aligned}$$

Since x_0 is the solution to $\min [2f(x) + f(1-2c-x)]$, the term in $\{\}$ is positive, therefore $\pi_0 > \pi$. In other words, the joint payoff could have been bigger when both breaks happened simultaneously at x_0 .

Case (2) There is an overlap between the first breaks. The payoff is:

$$\begin{aligned} \pi = & c[4 - 2(1-p)f(x_1) - (1-p)f(1-2c-x_1) - (2-p)f(x_2)] \\ & - c(1-p)f(1-2c-x_2) + p(x_1 + c - x_2)f(x_2) \end{aligned}$$

The first order conditions are:

$$\begin{aligned} \frac{\partial \pi}{\partial x_1} &= c[-2(1-p)f'(x_1) + (1-p)f'(1-2c-x_1)] + pf(x_2) = 0 \\ \frac{\partial \pi}{\partial x_2} &= c[(-2+p)f'(x_2) - (1-p)f'(1-2c-x_2)] + p(x_1 + c - x_2)f'(x_2) - pf(x_2) = 0 \end{aligned}$$

Let $G(x_2) = \partial \pi / \partial x_2 = 0$. Then $G(x_1) = -p[f(x_1) + f(x_2)] < 0 = G(x_2)$. By the second order condition, $G'' < 0$, so $x_1 > x_2$, thus the desired contradiction.

Q.E.D.

C Proof of Lemma 5

Fix the total length of commercials at $2c$ on either channel. We want to prove that it could never be an equilibrium for the two channels to choose different number of breaks. Not losing generality, suppose that channel 1 has one break and that channel 2 has two breaks with equal length c and a timing x . Then the payoffs are:

$$\begin{aligned} \pi_1^1 &= 2c[1 + pf(x, c) - f(1-2c, 2c)] + pc f(1-2c-x, c) \\ \pi_2^2 &= c[2 - 2f(x, c) + pf(1-2c, 2c) - f(1-2c-x, c)] \end{aligned}$$

π_i^j is channel i 's payoff with j breaks, $i, j = 1, 2$. We further define channel 0 as a monopolist, whose payoffs of having one or two breaks are:

$$\begin{aligned}\pi_0^1 &= 2c[1 - f(1 - 2c, 2c)] \\ \pi_0^2 &= c[2 - 2f(x, c) - f(1 - 2c - x, c)]\end{aligned}$$

Define the difference between a monopolist's two payoffs to be d_0 :

$$d_0 = \pi_0^2 - \pi_0^1 = c[2f(1 - 2c, 2c) - 2f(x, c) - f(1 - 2c - x, c)]$$

Given channel 2's choice, if channel 1 had chosen two breaks with timing x ,

$$\begin{aligned}\pi_1^2 &= c[2 - 2(1 - p)f(x, c) - (1 - p)f(1 - 2c - x, c)] \\ \implies d_1 = \pi_1^2 - \pi_1^1 &= c[2f(1 - 2c, 2c) - 2f(x, c) - f(1 - 2c - x, c)]\end{aligned}$$

Obviously $d_1 = d_0$. When $d_0 > 0$, i.e. a monopolist prefers two breaks with timing x to one break, channel 1 should also prefer to have two breaks, i.e. its choice of one break is not optimal. On the other hand, when $d_0 \leq 0$,

$$\pi_2^1 = 2c[1 - (1 - p)f(1 - 2c, 2c)] \implies$$

$$\begin{aligned}d_2 &= \pi_2^1 - \pi_2^2 \\ &= c[-2f(1 - 2c, 2c) + 2f(x, c) + f(1 - 2c - x, c) + pf(1 - 2c, 2c)] \\ &= pcf(1 - 2c, 2c) - d_0 > 0\end{aligned}$$

π_2^1 is channel 2's payoff when it chooses one break, given that channel 1 has already chosen one break. When $d_0 \leq 0$, $\pi_2^1 - \pi_2^2 > 0$. That is, whenever a monopolist prefers one break to two, channel 2's choice of two breaks is not optimal.

Combining the above two results, when the two channels choose different number of breaks, either channel 1's choice is not optimal, or channel 2's choice is not optimal. It could not be an equilibrium.

Q.E.D.

D Proof of Lemma 6

The drop function is $f(x_i, c_i)$. Since c_i is fixed at c , f contains only one variable x_i , which we will also call $f(x_i)$. f is strictly decreasing and strictly convex: $f' < 0$, $f'' > 0$.

Each channel has only one choice variable: the length of the news interval before the first break, x_i , $i = 1, 2$. As a benchmark we want to find a monopolist's choice of the timing. Suppose the news interval before the first break is x . The monopolist's payoff is:

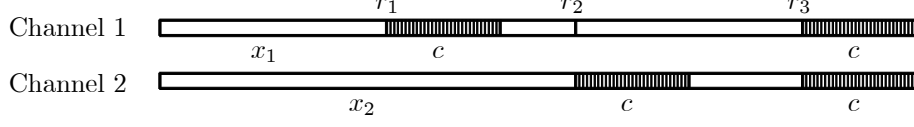
$$\pi(x) = c[2 - 2f(x) - f(1 - 2c - x)]$$

The optimal timing x_0 is characterized by the first order condition:

$$2f'(x_0) = f'(1 - 2c - x_0) \tag{4}$$

Now we want to find the best response for channel 1. There are three cases:

Case (1) Channel 1's first break is ahead of channel 2's and there is no overlap between the two breaks: $0 \leq x_1 \leq x_2 - c$.



$r_1 = 1 - f(x_1)$ is the audience size of channel 1's first break. $r_2 = r_1 + pf(x_2)$ is channel 1's audience size when channel 2 shows its first break. And

$$r_3 = r_2 - f(1 - 2c - x_1) + pf(1 - 2c - x_2)$$

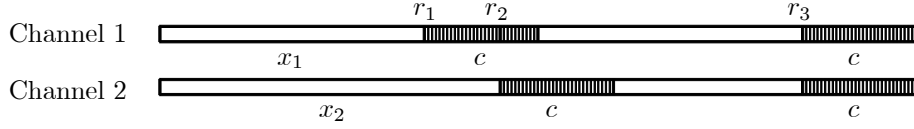
is channel 1's audience of its second break. Channel 1's payoff is:

$$\pi_1(x_1) = c(r_1 + r_3) = c[2 - 2f(x_1) - f(1 - 2c - x_1) + pf(x_2) + pf(1 - 2c - x_2)]$$

Channel 1 chooses $x_1 \in [0, x_2 - c]$ to maximize π . The first order condition turns out to be exactly the same as in equation (4), therefore $x_1^* = x_0$. Considering the constraint $x_1 \leq x_2 - c$, the final result is:

$$\begin{aligned} &\text{when } x_2 \geq x_0 + c, x_1^* = x_0; \\ &\text{when } x_2 \leq x_0 + c, x_1^* = x_2 - c. \end{aligned}$$

Case (2) Channel 1's first break is ahead of channel 2's and there is an overlap between the two breaks: $\max\{0, x_2 - c\} \leq x_1 \leq x_2$.



r_1, r_2, r_3 are exactly the same as in case (1). Channel 1's payoff is:

$$\begin{aligned} \pi_2(x_1) &= r_1(x_2 - x_1) + r_2(c + x_1 - x_2) + cr_3 \\ &= 2c + 2pcf(x_2) - px_2f(x_2) + pcf(1 - 2c - x_2) \\ &\quad + px_1f(x_2) - 2cf(x_1) - cf(1 - 2c - x_1) \end{aligned}$$

The first order condition is:

$$\frac{d\pi}{dx_1} = pf(x_2) - 2cf'(x_1) + cf'(1 - 2c - x_1) = 0 \quad (5)$$

Let $x_1^* = z(x_2)$ be the solution to the above equation. Let $G(x_1) = d\pi/dx_1 = 0$. Then $G(x_0) = pf(x_2) \geq 0 = G(z)$. The first equality comes from equation (4). Now $G'(x) < 0$ by the second order condition, so $z \geq x_0$. It is an equality only when $p = 0$.

It is easy to verify that z is decreasing in x_2 : rewrite equation (5) as:

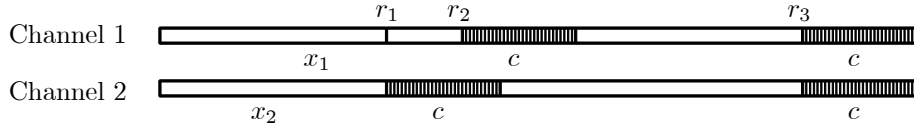
$$G(z(x_2), x_2) = pf(x_2) - 2cf'(z) + cf'(1 - 2c - z) = 0$$

Then $\frac{\partial G}{\partial z} z'(x_2) + \frac{\partial G}{\partial x_2} = 0$. By the second order condition $\partial G / \partial z = d^2 \pi / dx_1^2 < 0$. Then $z'(x_2)$ has the same sign as $\partial G / \partial x_2$, which is negative (remember f' is negative).

Now $x_1^* = z(x_2)$. Remember the constraint: $x_2 - c \leq x_1 \leq x_2$. Let w be the solution of x_2 to $x_2 - c = z(x_2)$, and let y be the solution of x_2 to $x_2 = z(x_2)$. Note that because z is decreasing in x_2 , $w > y$. The final result in case (2) is:

- when $x_2 \leq y$, $x_1^* = x_2$;
- when $y \leq x_2 \leq w$, $x_1 = z(x_2)$;
- when $x_2 \geq w$, $x_1^* = x_2 - c$.

Case (3) Channel 1's first break is behind channel 2's. It does not matter whether the two breaks overlap or not. $x_2 \leq x_1 < 1 - 2c$.



$$r_1 = 1 + pf(x_2), \quad r_2 = r_1 - f(x_1), \quad r_3 = r_2 - f(1 - 2c - x_1) + pf(1 - 2c - x_2),$$

$$\pi_3(x_1) = c(r_2 + r_3) = c[2 + 2pf(x_2) + pf(1 - 2c - x_2) - 2f(x_1) - f(1 - 2c - x_1)]$$

Like in case (1), the optimal choice of x_1 does not depend on x_2 : $x_1^* = x_0$. Combined with the constraint $x_1 \geq x_2$, this result leads to the conclusion:

- when $x_2 \leq x_0$, $x_1^* = x_0$;
- when $x_2 \geq x_0$, $x_1^* = x_2$.

Finally, summarizing the optimal choice of x_1 in the three cases, the following best response for channel 1 has been reached:

- when $0 \leq x_2 \leq x_0$, $x_1^* = x_0$;
- when $x_0 \leq x_2 \leq y$, $x_1^* = x_2$;
- when $y \leq x_2 \leq v$, $x_1^* = z(x_2)$;
- when $v \leq x_2 \leq 1 - 2c$, $x_1^* = x_0$,

where $v \in [y, w]$ is the solution of x_2 to $\pi_2(z(x_2)) = \pi_3(x_0)$. Since channel 1 and 2 are symmetric, all of the pure strategy equilibria are characterized by $x_1 = x_2 \in [x_0, y]$.

Q.E.D.

E Proof of Lemma 7

We study the monopoly first. The monopolist's payoff is $\pi(c) = c[1 - f(1 - c, c)]$. Let the optimal choice of c be c_0 . It is characterized by the first order condition:

$$1 - f(1 - c_0, c_0) = c_0 \left(\frac{\partial f}{\partial c} - \frac{\partial f}{\partial x} \right) \quad (6)$$

We want to find the best response for channel 1 in a duopoly game. Depending on which break is longer, there are two cases:

Case (1) $c_1 \leq c_2$. Channel 1's payoff is:

$$\pi(c_1) = c_1 [1 + pf(x_2, c_2) - f(1 - c_1, c_1)]$$

The first order condition is:

$$\frac{d\pi}{dc_1} = 1 + pf(x_2, c_2) - f(1 - c_1, c_1) - c_1 \left(\frac{\partial f}{\partial c_1} - \frac{\partial f}{\partial x_1} \right) = 0$$

Let the solution be $c_1^* = y(c_2)$. Let $G(c_1) = d\pi/dc_1 = 0$. Then $G(c_0) = pf(x_2, c_2) \geq 0 = G(y)$. Because $G'(c_1) = d^2\pi/dc_1^2 < 0$ by the second order condition, we know $y(c_2) \geq c_0$. It is easy to verify that y is increasing in c_2 .

Now $c_1^* = y(c_2)$. Remember the constraint $c_1 \leq c_2$. Let z be the solution of c_2 to $y(c_2) = c_2$. Then z is characterized by:

$$1 - (1 - p)f - z \left[\frac{\partial f}{\partial c_1} - \frac{\partial f}{\partial x_1} \right] = 0$$

where $f = f(1 - z, z)$. Since $y(c_2)$ is greater than x_0 and z is a special point of y , $z > x_0$. The final result in case (1) is:

$$\begin{aligned} &\text{when } 0 \leq c_2 \leq z, c_1^* = c_2; \\ &\text{when } z \leq c_2 \leq 1, c_1^* = y(c_2). \end{aligned}$$

Case(2) $c_1 > c_2$. Channel 1's payoff is:

$$\pi(c_1) = c_1 [1 - f(1 - c_1, c_1)] + pc_2 f(x_2, c_2)$$

Obviously the optimal choice of c_1 does not depend on c_2 . The first order condition is exactly the same as equation (6), therefore $c_1^* = c_0$.

Taking into consideration the constraint $c_1 > c_2$, the final result in case (2) is:

$$\begin{aligned} &\text{when } 0 \leq c_2 \leq c_0, c_1^* = c_0; \\ &\text{when } c_0 \leq c_2 \leq 1, c_1^* = c_2. \end{aligned}$$

Finally, combining the result of the two cases together, channels 1's best response is: (remember $z > x_0$.)

$$\begin{aligned} &\text{when } 0 \leq c_2 \leq c_0, c_1^* = c_0; \\ &\text{when } c_0 \leq c_2 \leq z, c_1^* = c_2; \\ &\text{when } z \leq c_2 \leq 1, c_1^* = y(c_2). \end{aligned}$$

Obviously only symmetric choice can be supported as an equilibrium. The two breaks must have the same length. The range of the equilibrium length is $[x_0, z]$.

Q.E.D.

F Proof of Lemma 8

Suppose there is a non-symmetric equilibrium. We have already shown in Lemma 6 that when $c_1 = c_2$, any equilibrium must have $x_1 = x_2$. So in any non-symmetric equilibrium $c_1 \neq c_2$. Not losing generality, assume $c_1 < c_2$. Depending on whether there is an overlap between the two channels' first breaks, and which channel's break comes first, there are four cases:

Case (1) There is overlap and channel 1 airs first: $x_1 \leq x_2 \leq x_1 + c_1$. The payoffs are:

$$\begin{aligned}\pi_1 &= c_1 [2 - 2f(x_1, c_1) - f(1 - 2c_1 - x_1, c_1) + pf(x_2, c_2) + pf(1 - 2c_2 - x_2, c_2)] \\ &\quad + p(x_1 + c_1 - x_2)f(x_2, c_2) \\ \pi_2 &= c_2 [2 - 2f(x_2, c_2) - f(1 - 2c_2 - x_2, c_2) + 2pf(x_1, c_1)] + pc_1f(1 - 2c_1 - x_1, c_1)\end{aligned}$$

The first order conditions on x_i are:

$$\begin{aligned}H(x_1, c_1) &= \frac{\partial \pi_1}{\partial x_1} = c_1 \left[\frac{\partial f}{\partial x}(1 - 2c_1 - x_1, c_1) - 2\frac{\partial f}{\partial x}(x_1, c_1) \right] + pf(x_2, c_2) = 0 \\ \frac{\partial \pi_2}{\partial x_2} &= c_2 \left[\frac{\partial f}{\partial x}(1 - 2c_2 - x_2, c_2) - 2\frac{\partial f}{\partial x}(x_2, c_2) \right] = 0\end{aligned}$$

$H(x_1, c_2) > H(x_2, c_2) = pf(x_2, c_2) > 0 = H(x_1, c_1)$. The first equation has made use of the first order condition $\partial \pi_2 / \partial x_2 = 0$, while the first inequality comes from the fact that $\partial H / \partial x_2 = \partial^2 \pi_2 / \partial x_2^2 < 0$ by the second order condition, and the fact that $x_1 < x_2$. However,

$$\begin{aligned}\frac{\partial H(x_1, c_1)}{\partial c_1} &= -2\frac{\partial f}{\partial x}(x_1, c_1) + \frac{\partial f}{\partial x}(1 - 2c_1 - x_1, c_1) - 2c_1 \frac{\partial^2 f}{\partial x^2}(1 - 2c_1 - x_1, c_1) \\ &= -\frac{p}{c_1}f(x_2, c_2) - 2c_1 \frac{\partial^2 f}{\partial x^2}(1 - 2c_1 - x_1, c_1) < 0\end{aligned}$$

The assumption that $f(x, c)$ is additive and separable in x and c is needed in the first equation in order to get rid of the cross derivatives.²¹ The second equality comes from the first order condition $\partial \pi_1 / \partial x_1 = 0$. By now we have $\partial H(x_1, c_1) / \partial c_1 < 0$ and $H(x_1, c_2) > H(x_1, c_1)$. Therefore $c_2 < c_1$, which contradicts the assumption.

Case (2) There is overlap and channel 2 airs first: $x_2 \leq x_1 \leq x_2 + c_2$. The payoffs are:

$$\begin{aligned}\pi_1 &= c_1 [2 - 2f(x_1, c_1) - f(1 - 2c_1 - x_1, c_1) + 2pf(x_2, c_2) + pf(1 - 2c_2 - x_2, c_2)] \\ \pi_2 &= c_2 [2 - 2f(x_2, c_2) - f(1 - 2c_2 - x_2, c_2) + pf(x_1, c_1)] \\ &\quad + pc_1f(1 - 2c_1 - x_1, c_1) + p(x_2 + c_2 - x_1)f(x_1, c_1)\end{aligned}$$

The first order conditions are:

$$\begin{aligned}\frac{\partial \pi_1}{\partial c_1} &= 2 - 2f(x_1, c_1) - f(1 - 2c_1 - x_1, c_1) + 2pf(x_2, c_2) + pf(1 - 2c_2 - x_2, c_2) \\ &\quad + c_1 \left[2\frac{\partial f}{\partial x}(1 - 2c_1 - x_1, c_1) - 2\frac{\partial f}{\partial c}(x_1, c_1) - \frac{\partial f}{\partial c}(1 - 2c_1 - x_1, c_1) \right] = 0 \\ \frac{\partial \pi_2}{\partial c_2} &= 2 - 2f(x_2, c_2) - f(1 - 2c_2 - x_2, c_2) + pf(x_1, c_1) \\ &\quad + c_2 \left[2\frac{\partial f}{\partial x}(1 - 2c_2 - x_2, c_2) - 2\frac{\partial f}{\partial c}(x_2, c_2) - \frac{\partial f}{\partial c}(1 - 2c_2 - x_2, c_2) \right] = 0 \\ \frac{\partial \pi_1}{\partial x_1} &= c_1 \left[-2\frac{\partial f}{\partial x}(x_1, c_1) + \frac{\partial f}{\partial x}(1 - 2c_1 - x_1, c_1) \right] = 0 \\ \frac{\partial \pi_2}{\partial x_2} &= c_2 \left[-2\frac{\partial f}{\partial x}(x_2, c_2) + \frac{\partial f}{\partial x}(1 - 2c_2 - x_2, c_2) \right] + pf(x_1, c_1) = 0\end{aligned}$$

²¹This assumption could be dropped if we assume appropriate sign on the cross derivatives. My guess is, so long as the second order conditions are satisfied to guarantee an interior solution, the desired sign on $\partial H(x_1, c_1) / \partial c_1$ can always be obtained. For simplicity, let's stick to the original assumption.

Using the same tactics as in case (1), we are able to produce a contradiction. For example, let $G(x_2, c_2) = \partial\pi_2/\partial c_2 = 0$. Then

$$\begin{aligned} G(x_1, c_1) &= p[2f(x_1, c_1) - 2f(x_2, c_2) - f(1 - 2c_2 - x_2, c_2)] \\ &< 0 = G(x_2, c_2) < G(x_2, c_1) \end{aligned}$$

The first inequality comes from the fact that $x_1 \geq x_2$, $c_1 < c_2$ and f is decreasing in x_i and increasing in c_i . The second inequality comes from the fact that $c_1 < c_2$ and $\partial G/\partial c < 0$ by the second order condition. Since $\partial G/\partial x > 0$, we have $x_1 < x_2$, thus the contradiction.

Case (3) There is no overlap and channel 1 airs first: $x_1 + c_1 \leq x_2$. The payoffs are:

$$\begin{aligned} \pi_1 &= c_1 [2 - 2f(x_1, c_1) - f(1 - 2c_1 - x_1, c_1) + pf(x_2, c_2) + pf(1 - 2c_2, x_2, c_2)] \\ \pi_2 &= c_2 [2 - 2f(x_2, c_2) - f(1 - 2c_2 - x_2, c_2) + 2pf(x_1, c_1)] + pc_1 f(1 - 2c_1 - x_1, c_1) \end{aligned}$$

x_1 and x_2 must satisfy the following first order conditions:

$$\begin{aligned} 2\frac{\partial f}{\partial x}(x_1, c_1) &= \frac{\partial f}{\partial x}(1 - 2c_1 - x_1, c_1) \\ 2\frac{\partial f}{\partial x}(x_2, c_2) &= \frac{\partial f}{\partial x}(1 - 2c_2 - x_2, c_2) \end{aligned}$$

They are the same functions except in the c_i . In other words, $x_i = g(c_i)$ for $i = 1, 2$. It is easy to verify that x_i is decreasing in c_i . Now $c_1 < c_2$. So $x_1 > x_2$, thus the desired contraction.

Case (4) There is no overlap and channel 2 airs first: $x_2 + c_2 \leq x_1$. The payoffs are:

$$\begin{aligned} \pi_1 &= c_1 [2 - 2f(x_1, c_1) - f(1 - 2c_1 - x_1, c_1) + 2pf(x_2, c_2) + pf(1 - 2c_2, x_2, c_2)] \\ \pi_2 &= c_2 [2 - 2f(x_2, c_2) - f(1 - 2c_2 - x_2, c_2) + pf(x_1, c_1)] + pc_1 f(1 - 2c_1 - x_1, c_1) \end{aligned}$$

The first order conditions are the same as in case (3). $x_i = g(c_i)$ for $i = 1, 2$. It is easy to verify that $y_i = 1 - 2c_i - x_i = h(c_i)$ is also decreasing in c_i . But $c_1 < c_2$. So $y_1 > y_2$, again the desired contradiction.

Combining all of the four cases, we reach the conclusion that non-symmetric situation could never be supported as an equilibrium.

Q.E.D.

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