A REVIEW OF CONTINUOUS TIME MATH

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Objectives

This course is a review of continuous time math for Finance. The overall approach is towards understanding and applying the relevant mathematical tools rather than becoming more proficient in the technical details. The course uses Mathematica as a tool in financial analysis. The main emphasis will be on the following:

- Discuss and interpret *Arbitrage Theorem*
- See financial applications of stochastic differential equations and of partial differential equations.
- Develop some familiarity with *stochastic* calculus.
- Learn the use of *martingales* and their applications to arbitrage-free asset valuation.
- Discuss some fundamental models of asset pricing using Martingale methods.
- Learn the uses of Kolmogorov's backward and forward equations.
- Learn recent techniques: Levy processes and transform methods.

1 References

1. Shereve (2005), Continuous time methods, second volume.
2. Neftci (2000), Intro to Math.

OUTLINE

2 The Background

1. Arbitrage Theorem.
   - Application to interest rate derivatives
   - Understanding Risk-Neutral and Forward measures
2. Martingale Methods: Major results
3. Equivalent Martingale Measures and how to use them in asset pricing.
4. Computer example: Introduce Mathematica, Obtain risk-neutral and Forward measures numerically for a pre-selected set of securities of interest.

3 Tools of Stochastic Calculus

1. The important notion of Wiener Process.
2. Why Wiener process is "very general"?
3. How can one capture fat tails with Wiener processes?
4. Derivative in Stochastic Calculus.
5. Stochastic Taylor series expansions and their uses.
6. ITO's Lemma.
   - Math Examples
   - Finance examples
7. Stochastic Differential Equations
   - Construction of SDE's
   - Comparison with Ordinary Differential Equations

- Major SDE's used in finance

8. How to use SDE's in practical situations?
   - Case of Stock prices
   - Case of short-term spot rate modeling
   - Case of forward rate modeling

4 PDE methods and Uses of PDE's

1. What is a Partial Differential Equation?
2. Why are they useful in finance?
3. Why are tree-models and PDE-methods the same.
4. An important example: A PDE for bond prices.
5. Numerical solution of PDE's.
6. An important relation between PDE's and Martingale pricing methods.
7. Feynman-Kac Formula and its uses.

5 Girsanov Theorem, Martingale Representation Theorem

1. The Theorem and its interpretation.
2. Uses of Girsanov theorem in finance.
3. Examples from Exotic options.
4. Recent examples to the use of Girsanov Theorem.
5. Martingale representation theorem and its uses.

6 Backward and forward Kolgorov equations

1. The Backward Kolmogorov Equations and its interpretation.
2. Uses of the Backward Equation in finance: Asset Prices.
3. The Forward Kolmogorov Equations and its interpretation.
4. Uses of the Forward Equation in finance: Probabilities and the implied volatilities.
5. Other uses of Kolmogorov equations.

7 Jumps and Transform Methods in finance

1. Levy processes and Jumps.
2. Examples of Levy Processes.
3. Poisson Process and Merton's model.
4. Extending results obtained thus far to Levy processes
5. Why Jumps are needed?
6. IPDE in Jump-Diffusions: How to solve them?
7. Fourier transform: A quick review
8. Uses of Fourier transform in solving IPDEs.
9. Other uses of Fourier transforms.