

Auctions in Corporate Finance *

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March, 2006.

Keywords: Bidders, Targets, Private Value, Common Value, Winner's Curse, Auctions, Bidding, Takeovers, Mergers and Acquisitions, Toeholds, Bankruptcy Auctions, IPO Auctions

*This survey is to appear in B. E. Eckbo (ed.), 2006, *Handbook of Corporate Finance: Empirical Corporate Finance* (North-Holland/Elsevier, Handbooks in Finance Series), Chapter 3. We thank Parimal K. Bag, Espen Eckbo and Mike Fishman for comments and suggestions. Author email addresses: dasgupta@ust.hk, Robert.G.Hansen@dartmouth.EDU

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Abstract

This paper reviews the applications of auction theory to corporate finance. It starts with a review of the main auction theory frameworks and the major results. It then goes on to discuss how auction theory can be applied, in the context of the market for corporate control, not only to “inform” a company’s board or regulators, but also to understand some of the observed empirical evidence on target and bidder returns. It then considers the role of preemptive bidding, stock versus cash offers, the effect of toeholds on bidding behavior, the effect of bidder heterogeneity and discrimination in auctions, merger waves, bankruptcy auctions, share repurchases and “Dutch” auctions, IPO auctions, and the role of debt in auctions. It concludes with a brief discussion of the econometrics of auction data.

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1 Introduction

This paper reviews developments in auction theory, with a focus on applications to corporate finance. Auctions, viewed broadly, are economic mechanisms that transfer control of an asset and simultaneously determine a price for the transaction.¹ Auctions are ubiquitous across the world. Formal auctions are used to buy and sell goods and services from fish to mineral rights and from logging contracts to lawyers' services in class action lawsuits. In the world of finance, auctions are used to buy and sell entire firms (in bankruptcy and out of bankruptcy) as well as securities issued by governments and companies. In the most recent and public example of auctions in corporate finance, the internet search firm Google sold its shares via a Dutch auction method in its initial public offering.

Auction theory has developed to explore a variety of issues, with the most important ones relating to pricing, efficiency of the allocation, differential information, collusion, risk aversion, and of course a very large topic, the effects of different auction rules (sealed-bids versus open auctions, reserve prices, entry fees etc.) on the revenue to the seller.² Concomitant with theoretical work, there has been significant work in applications of auction theory, with many of these being related in some way to corporate finance. On one level, application of auction theory to corporate finance is very natural, for corporate finance sometimes directly involves auctions (e.g., auctions in bankruptcy). At another level, though, auction theory should serve to inform corporate finance because the underlying primitive issues are the same: pricing of assets, exchange of control, uncertainty especially in regard to asset valuation, heterogeneity of agents, asymmetric or disparate information, and strategic behavior. Given this similarity in the underlying frameworks, one should expect auction theory to have significant influence, both direct and indirect, on corporate finance research. There has also been, particularly in recent years, much work in the estimation of auction models. The econometrics of this work is very sophisticated, utilizing structural estima-

¹Throughout this survey, we will normally consider auctions where an item is being sold. Reverse auctions, where an auction is used to purchase an item, can generally be modeled by simply reversing the direction of payment.

²Krishna (2002) provides an excellent, comprehensive review of all existing auction theory. Klemperer (2000) is a shorter, recent review of auction theory, while McAfee and McMillan (1987) is thorough but a bit dated by now.

tion methods that can retrieve estimates of the underlying distribution of bidders' valuations from bid data. While these techniques have not yet been applied to data from finance-related auctions, there would seem to be room for application to, for instance, corporate bankruptcy auctions. The broad lesson from these econometric studies is also very relevant for empirical work in financial auctions: use the restrictions from the theory to learn more from the data than non-structural methods will reveal. For this reason, empirical finance researchers studying auctions should have a good knowledge of auction theory.

A careful review of the literature shows that auction theory has had a significant but not overwhelming influence on corporate finance. Perhaps the more insightful applications have been in the context of corporate takeover bidding: pre-emptive bidding, means of payment (takeover auctions are not always financed with cash), bidder heterogeneity, and discrimination amongst bidders. Application of auction theory to these contexts has at times produced new insights. Overall, however, while the applications have extended our understanding of the inefficiencies that are due to the underlying primitive construct of private information, they have not changed that understanding in any fundamental way.

The survey proceeds as follows. Section 2 reviews the simplest auction setting, that of independent private values. Many key insights can be developed from this simplest model: the basic pricing result that an auction's expected price equals the expected second-highest value; general solution methodology; effects of more bidders, risk aversion, reserve prices; revenue equivalence of the different auction forms; revenue enhancement from ex-post means-of-payment; and the solution of auction models via the Revelation Principle. Section 3 considers the interdependence amongst bidders valuations (including the special case of a "common value" for the object) and reviews Milgrom and Weber's (1982) generalized auction model. Critical insights in this section pertain to the effects of the winner's curse; that lack of disclosure by the seller can lower expected prices; and that the different auction forms are no longer revenue-equivalent. With the basic theory developed in these sections, Section 4 turns to the applications most relevant to corporate finance. Section 5 ends with some thoughts about future applications and further development of auction theory that would make it more relevant for corporate finance.

2 The Most Basic Theory: Independent Private Values

2.1 Initial Assumptions

Auction theory begins with assumptions on how bidders value the asset for sale; the model then shows how an auction converts valuations into a price and an exchange of control. Valuation assumptions are absolutely key to auction theory. However, as we will argue later, the existing paradigms are not complete as they do not consider certain sets of valuation assumptions that are particularly relevant in corporate finance.

Independent preference (sometimes called independent private values) assumptions are straightforward: each bidder is simply assumed to know her value for the asset. For bidder i , denote this value as v_i . While each bidder knows her own value, to make the situation realistic and interesting, we assume that a bidder does not know other bidders' values. To model this uncertainty, we assume that each bidder believes other bidders' values to be independent draws from a distribution $F(v)$. We have therefore introduced a degree of symmetry in the model, that of symmetric beliefs.³

Fix a particular bidder, and focus on the highest value among the remaining $N - 1$ values from the other $N - 1$ bidders, and denote this value as v_2 . Since v_2 is the highest among $N - 1$ independent draws from the same distribution, its probability distribution $G(v_2)$ (i.e. the probability that $N - 1$ independent draws are less than a value v_2) is

$$G(v_2) = F(v_2)^{N-1}. \quad (1)$$

Notice that the distribution $G(v_2)$ has a density function $g(v_2) = (N - 1)F(v_2)^{N-2}f(v_2)$.

If $F(v)$ is uniform over the unit interval, i.e., $F(v) = v$ for $0 \leq v \leq 1$, then note that

$$G(v_2) = F(v_2)^{N-1} = v_2^{N-1} \quad (2)$$

³Several papers examine the effects of asymmetric beliefs, for example, Maskin and Riley (2000). See also Krishna (2002).

2.2 First-Price Sealed-Bid Auctions

We are now in a position to evaluate any specific set of auction rules. Turn first to the common first-price sealed-bid auction, where bidders submit sealed bids and the highest bidder wins and pays the amount of her bid (hence the “first-price” qualifier). For now we assume a zero reserve price (a price below which the seller will keep the asset rather than sell).

In placing a bid b , bidder i has expected profit of

$$E(\pi_i) = \Pr(\text{win})(v_i - b) \tag{3}$$

where one can note that in the case that bidder i loses, her profit is zero. While (3) does not make it explicit, $\Pr(\text{win})$ will be a function of b , normally increasing. This creates the essential tension in selecting an optimal bid: increasing one’s bid increases the chance of winning, but the gain upon winning is less.

To solve this model, we need just a bit more structure. Let us use an intuitive version of the so-called Revelation Principle. Fix a bid function $b(v)$, and think of bidder i as choosing the v she “reports” rather than choosing her actual bid. So long as $b(v)$ is properly behaved, we have not restricted bidder i ’s choice in any way, for she could get to any bid b desired by simply “reporting” the requisite v .

Looking ahead, we are searching for a symmetric Nash equilibrium in bidding strategies. In terms of our $b(v)$ function, symmetry means that all bidders use the same $b(v)$. Nash equilibrium requires that, given other bidders’ strategies, bidder i ’s bid strategy is optimal. In terms again of our $b(v)$ formulation, equilibrium requires each bidder to report $v = v_i$, i.e., “honest” reporting. Our requirement for Nash equilibrium will therefore be as follows: Suppose that the other bidders are using $b(v)$ and honestly reporting, so that bidder j ’s bid is $b(v_j)$. If $b(v)$ represents a (symmetric) bidding equilibrium, then bidder i ’s optimal decision will be to report $v = v_i$, so that her bid is $b(v_i)$.

In the situation where the other $N - 1$ bidders are both using $b(v)$ and reporting honestly, we can re-write (3) as

$$\begin{aligned} E(\pi_i) &= \Pr(\text{win})(v_i - b) \\ &= G(v)(v_i - b(v)) \end{aligned} \tag{4}$$

where we assume bidder i is using $b(v)$ but not requiring $v = v_i$. Note that bidder i wins if all other $N - 1$ values are less than the v that bidder i reports, hence the conversion of $\Pr(\text{win})$ into $G(v)$, the distribution for the highest value among the remaining $N - 1$ values.

Now we simply require that bidder i 's optimum decision is also honest reporting. Taking the first derivative of (4) with respect to v , we have

$$\frac{dE(\pi_i)}{dv} = g(v)(v_i - b(v)) - G(v)\frac{db(v)}{dv} = 0. \quad (5)$$

The first term of (5) shows the marginal benefit of bidding higher while the second term shows the marginal cost. Re-arranging, we have

$$G(v)\frac{db(v)}{dv} = g(v)(v_i - b(v)). \quad (6)$$

For equilibrium, we require that (6) hold at $v = v_i$. Hence we get

$$G(v)\frac{db(v)}{dv} = g(v)(v - b(v)). \quad (7)$$

Equation (7) is a standard first-order differential equation that can be solved via integration-by-parts.⁴ Doing this yields

$$b(v) = \frac{1}{G(v)} \int_0^v yg(y)dy. \quad (8)$$

Equation (8) can be easily interpreted. As $G(x)$ is the distribution for the highest value among the remaining $N - 1$ values, $g(x)/G(v)$ is the density of that value conditional on it being lower than v . Equation (8) tells a bidder to calculate the expected value of the highest value among the remaining $N - 1$ bidders, conditional on that value being less than bidder i 's own, and to bid that amount. This is about as far as intuition can take us: the expected value of the second-highest value is in some sense bidder i 's real competition, and equilibrium bidding calls for her to just meet that competition. (One other intuitive approach involves marginal revenue; we will turn to this view below.)

⁴Rewrite (7) as $G(y)db + bdG = ydG$ or $d(G(y)b(y)) = ydG$. Integrating, and using the fact that $G(0) = 0$, we get $b(v) = \frac{\int_0^v ydG}{G(v)} = \frac{\int_0^v yg(y)dy}{G(v)}$.

If beliefs on values are governed by the uniform distribution, then $G(v) = v^{N-1}$, $g(v) = (N-1)v^{N-2}$, and (8) becomes

$$\begin{aligned}
 b(v) &= \frac{1}{v^{N-1}} \int_0^v y(N-1)y^{N-2} dy \\
 &= \frac{N-1}{v^{N-1}} \int_0^v y^{N-1} dy \\
 &= \frac{N-1}{N} v
 \end{aligned} \tag{9}$$

In the particular case when $N = 2$, (9) implies that equilibrium bidding calls for bidding half of one's value - a significant "shading" of one's bid beneath true value. Note that in this case, however, the lowest the competitor's value could be is zero. If the distribution of values was instead uniform over $[8, 10]$, the equilibrium bid would be $(8 + v)/2$ - halfway between the lower bound and one's own valuation.

To see that in general, there is bid shading, notice that we can write

$$\begin{aligned}
 b(v) &= \frac{1}{G(v)} \int_0^v yg(y) dy \\
 &= \frac{1}{G(v)} \int_0^v y dG(y) \\
 &= \frac{1}{G(v)} [yG(y) \Big|_0^v - \int_0^v G(y) dy] \\
 &= v - \int_0^v \frac{G(y)}{G(v)} dy \\
 &= v - \int_0^v [F(y)/F(v)]^{N-1} dy
 \end{aligned}$$

where we have used integration-by-parts in the third line ⁵. Notice that while $b(v) < v$, since $F(y) < F(v)$ within the integral, as $N \rightarrow \infty$, $b(v) \rightarrow v$. In other words, intense competition will cause bidders to bid very close to their true values, and be left with little surplus from winning.

How does the seller fare in this first-price auction? We can construct the seller's expected revenue by calculating the expected payment by one bidder

⁵Since $d(uv) = u dv + v du$, we can write $\int u dv = uv - \int v du$. This handy trick is used very commonly in the auction literature.

and then multiplying that by N . Sticking to the uniform $[0, 1]$ distribution for clarity, we have the expected payment by bidder i as

$$\begin{aligned} E(\text{Payment}_i) &= \int_0^1 \text{Pr.}(\text{win})b(y)dy \\ &= \int_0^1 y^{N-1}\left(\frac{N-1}{N}\right)ydy \\ &= \frac{N-1}{N(N+1)} \end{aligned} \tag{10}$$

Multiplying this by N gives the sellers expected revenue as

$$E(\text{Revenue to seller}) = \frac{N-1}{N+1}. \tag{11}$$

Intuitively, since each bidder is bidding her expectation of the highest value among the remaining $N-1$ bidders, conditional on her value being the highest, the expected payment received by the seller should be the unconditional expected value of the second-highest value. In general, the density for the second-highest value is, from the theory of order statistics,⁶

$$f_2(y_2) = N(N-1)(1-F(y_2))F(y_2)^{N-2}f(y_2). \tag{12}$$

In the case of the uniform $[0, 1]$ distribution, the expected value of the second-highest value is then

$$\begin{aligned} E(y_2) &= \int_0^1 xN(N-1)(1-x)x^{N-2}dx \\ &= \frac{N-1}{N+1} \end{aligned} \tag{13}$$

as expected.⁷ To reiterate and emphasize, the seller's expected revenue from the auction is exactly the expected value of the second-highest value. This result, of course, extends beyond the uniform distribution.

⁶To see this, first note that the distribution function $F_2(y_2)$ of the second-highest value y_2 is the probability that either: (a) all N values are less than or equal to y_2 or (b) *any* $N-1$ values are less than y_2 and the remaining value is greater than y_2 . Note that this latter event can happen in N possible ways. Thus, the probability is $F_2(y_2) = F^N(y_2) + NF^{N-1}(y_2)(1-F(y_2))$. Differentiating this expression with respect to y_2 , we get the expression for $f_2(y_2)$.

⁷This interpretation of the expected revenue holds for any distribution. Notice

2.3 Open and Second-Price Sealed-Bid Auctions

As compared to the first-price sealed-bid auction, the open auction and second-price sealed-bid auctions are considerably easier to solve. For this reason, they are often chosen to model any kind of auction mechanism; the Revenue Equivalence Theorem discussed below ensures that, in many cases, the results for one auction form extend to others.

In an open auction, bidders cry out higher and higher bids until only one bidder, the winner, remains. It is easy to see that “staying in the auction” until the bid exceeds one’s value is a dominant strategy.⁸ Staying in the auction beyond the point of the bid equaling one’s value cannot be rational. Likewise, if the item is about to be won by someone else at a bid less than v_i , then bidder i should be willing to bid a bit higher than the current bid, for if such a bid wins the auction it will yield a profit.

An open auction therefore will quite easily find the second-highest valuation and establish that as the price - for bidding will cease once the bidder with the second-highest valuation is no longer willing to bid more.⁹ The expected price in the open auction is therefore the expected value of the second-highest valuation, the same as for the first-price sealed-bid auction. This result is an implication of the Revenue Equivalence Theorem; we return to a more general statement of that below.

Turn now to a second-price sealed-bid auction: in such an auction, sealed bids are submitted and rules call for the highest bid to win but that the price paid will be the second-highest bid submitted. With these rules it is again a dominant strategy to submit a bid equal to one’s valuation. Bidding more than one’s value would mean possibly winning at a price in excess of value. Bidding less than one’s value will mean possibly forgoing an opportunity to

that the expected payment from any bidder is $\int_0^{\bar{v}} [\text{Prob.}(\text{win}) \cdot \text{Amount Bid}] f(v) dv = \int_0^{\bar{v}} G(v) b(v) f(v) dv = \int_0^{\bar{v}} (\int_0^v yg(y) dy) dF(v)$ from (8). Integrating by parts, this expression becomes $\int_0^{\bar{v}} yg(y) dy - \int_0^{\bar{v}} F(v) v g(v) dv = \int_0^{\bar{v}} yg(y) dy - \int_0^{\bar{v}} F(y) yg(y) dy = \int_0^{\bar{v}} y(1 - F(y))g(y) dy = \int_0^{\bar{v}} y(1 - F(y))(N - 1)F^{N-2}(y)f(y) dy$. N times this expression is the expected revenue to the seller in the auction, and is exactly the expected value of the second-highest valuation.

⁸A (weakly) dominant strategy in game theory is a strategy which does at least as well as any other strategy no matter what strategies other agents use.

⁹This neglects effects (usually unimportant) of a minimum bid increment.

buy the object at a price less than value. The key to understanding the second-price auction is to note that the linkage between one's bid and the price one pays has been severed; bidding equal to value to maximize the probability of profitable wins becomes optimal. Thus, as is the case for the open auction, the second-price sealed bid auction will also yield as a price the second-highest value out of the N values held by the bidders.¹⁰

All three auctions therefore yield the same expected price.¹¹ Note, however, that the first-price and second-price (including the open auction as essentially a second-price auction) auctions have equilibrium strategies that are easy to compute for both the modeller and the bidder. Note also that the first-price auction gives a different (and less volatile) price for any given set of bidders. It is also important that all three auctions are efficient in that the bidder with the highest valuation is the winner. Auctions can be seen as accomplishing two distinct tasks: reallocating ownership of an asset and determining a price for the transfer of ownership. Efficiency is an important characteristic of any sales procedure, and auctions under private value assumptions should get the asset to its most highly-valued use. Reserve prices, considered below, may hamper this efficient transfer. Efficiency of auctions under asymmetric beliefs is also not assured (see Krishna (2002) for further discussion).

2.4 Revenue Equivalence

The result that the second-price auctions and the first-price auction yield the same expected revenue to the seller is a consequence of the so-called "Revenue Equivalence Theorem". What is fascinating about the revenue

¹⁰Therefore, the second-price and the open (also called English or Ascending) auctions are "equivalent" in the sense that they lead the bidders to bid or drop out at their private value for the object. However, this "equivalence" holds only in the private values setting. If the other bidders' signals or valuations are relevant for a given bidder's valuation of the object, this equivalence breaks down, as the open bids by the other bidders conveys additional information.

¹¹The first-price auction is "strategically equivalent" to yet another auction known as the Dutch auction, which is an open descending price auction in which the auctioneer starts with a high price and then gradually lowers the price until some bidder accepts the price. Provided that the object has not been sold yet, a bidder will accept an asking price that equals her bid in the first-price auction. Strategic equivalence is a stronger notion than the equivalence between the open and the second-price auctions.

equivalence of these two auctions is that such sophisticated models confirm a result which is really quite intuitive: different mechanisms all yield what is really a competitive price, that being the second-highest valuation. The seller cannot, under these standard auction rules, extract any more revenue than the valuation of the second-highest bidder.

The revenue equivalence result in this independent private value context can be generalized - not only to encompass a broader class of auctions, but also a more general value environment. Suppose that each bidder i privately observes an informational variable x_i . To simplify notation, we assume $N = 2$. Assume that x_1 and x_2 are independently and identically distributed with a distribution function $F(x_i)$ and density $f(x_i)$ over $[0, \bar{x}]$ for $i = 1, 2$. Let $v_i = v(x_i, x_j)$ denote the value of the object to bidder i , $i = 1, 2$ and $i \neq j$.

Consider a class of auctions in which the equilibrium bid function is symmetric and increasing in the bidder's signal, and let A denote a particular auction form. Let $\Pi_i^A(z, x)$ denote the expected payoff to bidder i when she receives signal $x_i = x$ and bids as if she received signal z . Then

$$\Pi_i^A(z, x) = \int_0^z v(x, y) f(y) dy - P_i^A(z)$$

where $P_i^A(z)$ denotes the expected payment conditional on bidding as if the signal were z , and we have used the assumption that the bidders have symmetric and increasing bid functions, so that i wins if and only if $x_j < z$. Differentiating with respect to z , we get:

$$\frac{\partial \Pi_i^A(z, x)}{\partial z} = v(x, z) f(z) - \frac{dP_i^A(z)}{dz}.$$

In equilibrium, $\frac{\partial \Pi_i^A(z, x)}{\partial z} = 0$ at $z = x$, and hence

$$\frac{dP_i^A(y)}{dy} = v(y, y) f(y).$$

Integrating, we get

$$P_i^A(x) = P_i^A(0) + \int_0^x v(y, y) f(y) dy.$$

Notice that $P_i^A(0)$ is the expected payment made by bidder i with the lowest draw of the signal. Since the seller's expected revenue is simply 2

times $\int_0^{\bar{x}} P^A(x)f(x)dx$, it follows that all auctions in which the bid functions are symmetric and increasing, and in which the bidder drawing the lowest possible value of the signal pays zero in expected value, are “revenue equivalent”.¹²

The model considered here is one in which the values of the bidders are “interdependent” in the sense that one bidder’s signal affects the value (estimate) of the other bidders. The signals themselves, however, are statistically independent. An example of the value function we considered here would be, for example, $v_1 = \alpha x_1 + (1 - \alpha)x_2$ and $v_2 = \alpha x_2 + (1 - \alpha)x_1$, where $1 \geq \alpha \geq 0$. Clearly, the independent private values model is a special case, in which $\alpha = 1$. The case of $\alpha = 1/2$ corresponds to a case of the “pure common value” model, for which $v(x, y) = v(y, x)$, i.e. the bidders have identical valuations of the object as a function of both bidders’ signals.

2.5 Reserve Prices

As reserve prices have figured in some of the corporate finance literature, it is worthwhile to consider analysis of reserve prices in auctions. Sticking with independent private values, consider an open auction with two bidders. Suppose that bidder 1 has valuation $v_1 > 0$ and bidder 2 has valuation $v_2 = 0$. Then the open auction will yield a price of zero. Better in this case would be for the seller to have a reserve price set in-between 0 and v_1 so that bidder 1 would still win but pay the reserve. Of course, the problem with a reserve price is that if it is set above v_1 no sale will result.

To understand how the reserve price is chosen,¹³let us return to the independent private values model with N bidders. Consider any auction form A in the class of auctions with symmetric increasing bid functions. As above, denote by $P^A(z)$ the expected payment by a given bidder in auction A when she bids $b^A(z)$. If the bidder’s private value of the object is v , her expected profit is

$$\Pi^A(z, v) = G(z)v - P^A(z).$$

¹²Absent reserve prices, the bidder drawing the lowest possible signal will typically be indifferent between bidding and not bidding.

¹³Our treatment of the problem here follows that in Riley and Samuelson (1981).

where $G(z) = F^{N-1}(z)$. As above, in equilibrium, it must be optimal for the bidder with valuation v to bid $b(v)$, which requires that $\Pi^A(z, v)$ is maximized at $z = v$. This implies that

$$g(y)y = \frac{dP^A(y)}{dy}. \quad (14)$$

Let us suppose now that a bidder with private value v^* is indifferent between bidding and not bidding. For such a bidder (known as the “marginal bidder”), by definition $\Pi^A(v^*, v^*) = G(v^*)v^* - P^A(v^*) = 0$. Now from (14), integrating, we get for $v \geq v^*$

$$\begin{aligned} P^A(v) &= P^A(v^*) + \int_{v^*}^v yg(y)dy \\ &= G(v^*)v^* + \int_{v^*}^v ydG(y) \\ &= vG(v) - \int_{v^*}^v G(y)dy \end{aligned} \quad (15)$$

where in the last step, we used integration by parts.

The expected revenue for the seller from a single bidder is $\int_0^{\bar{v}} P^A(v)f(v)dv$. Again, using integration by parts, this can be written as

$$\begin{aligned} E(R_i^A) &= \int_0^{\bar{v}} P^A(v)f(v)dv \\ &= \int_{v^*}^{\bar{v}} P^A(v)f(v)dv \\ &= \int_{v^*}^{\bar{v}} vG(v)f(v)dv - \int_{v^*}^{\bar{v}} \left[\int_{v^*}^v G(y)dy \right] dF \\ &= \int_{v^*}^{\bar{v}} vG(v)f(v)dv - \int_{v^*}^{\bar{v}} G(y)dy + \int_{v^*}^{\bar{v}} F(v)G(v)dv \\ &= \int_{v^*}^{\bar{v}} [vf(v) - (1 - F(v))]G(v)dv \end{aligned} \quad (16)$$

Given equal treatment of all N buyers, the expected revenue to the seller is simply N times the above expression.

Notice that what we have shown is that all auction forms in the class of auctions being considered must provide the seller with the same expected

revenue if the marginal bidder is the same. The reserve price will determine the marginal bidder. If no bidder has a valuation above that of the marginal bidder, the seller keeps the object. Assume that the seller values the object at v_0 . Then for any auction, the seller should choose the marginal bidder to maximize

$$\int_{v^*}^{\bar{v}} [vf(v) - (1 - F(v))]G(v)dv + F(v^*)v_0.$$

From the first-order condition with respect to v^* , we get

$$v^* = v_0 + \frac{1 - F(v^*)}{f(v^*)}. \quad (17)$$

Since the optimal marginal bidder is the same in all auctions - all auctions in the class of auctions we are considering provide the seller with the same expected *profit* as well as revenue. The revenue equivalence result survives when a reserve price is introduced.

It remains to characterize the reserve prices in different auction settings. Suppose the reserve price is r . Notice that in both the first-price and the second-price auctions, no bidder with a value less than r can make any positive profit, as they have to bid at least r to win the object. On the other hand, the profit of a bidder with value greater than r must be strictly positive (in the second price auction, if no other bidder bids higher than r , the bidder pays r). Thus, by continuity, the marginal bidder must have a value $v^* = r$.

Note from (17) that the optimal reserve price exceeds the seller's own valuation and is independent of the number of bidders. This latter point makes sense given that the optimal reserve price is only aimed at making the high bidder pay more in the instance when all other valuations are beneath the reserve price. Note also that a reserve price destroys the assurance of an efficient allocation; in the case where the highest valuation among the bidders is less than v^* but greater than v_0 , the seller will retain possession even though one of the bidders has a valuation greater than the seller.

Notice also that *entry fees* are an alternative way of implementing a positive reserve price. By setting an entry fee equal to the expected profit of a

bidder with value r when the reserve price is 0,¹⁴ the seller can ensure that a bidder participates if and only if her value exceeds r .

2.6 Optimal Selling Mechanisms

Auctions are best thought of as “selling mechanisms” - ways to sell an object when the seller does not know exactly how the potential buyers value the object. There is obviously a very large number of ways in which an object could be sold in such a situation: for example, the seller could simply post a price and pick one bidder randomly if more than one buyer is willing to pay that price; post a price and then negotiate; use any one of the common auctions; use any of the *less* common forms of auction such as an “all pay” auction in which all bidders pay their bids but only the highest bidder gets the object; impose non-refundible entry fees; use a “matching auction” in which one bidder bids first and the other bidder is given the object if he matches the first bidder’s bid, and so on. The search for an optimal selling scheme in a possibly infinite class of selling schemes would indeed seem like a daunting task. The major breakthrough, however, was the insight that without loss of generality, one could restrict attention to selling mechanisms in which each buyer is induced to report her valuation (often called “type”) truthfully. This is the so-called “Revelation Principle” (Myerson (1981), Dasgupta, Hammond and Maskin (1979), Harris and Raviv (1981)), and it greatly simplified the formulation of the problem.

Armed with the Revelation Principle, one can attack the problem in a more general setting than we have discussed so far. While we will still remain within the confines of the independent private values framework,¹⁵ we can dispense with the assumption that all bidders’ valuations are drawn from identical distributions, i.e. one can accommodate asymmetries among bidders. Asymmetries are important in many real world situations - for example, in procurement, when both domestic and foreign bidders participate, and especially in corporate finance, in the context of takeover bidding.

¹⁴From (15), this is $\int_0^r G(y)dy$.

¹⁵Myerson’s (1981) framework is slightly more general in that he allows the value estimate of a bidder as well as the seller to depend on the signals of all other bidders, i.e. his model is one in which the signals are independent and private, but the valuations are interdependent.

Before proceeding further, however, we need to introduce some notation. Let $\mathbf{v} = (v_1, v_2, \dots, v_N)$ denote the set of valuations for bidders $1, \dots, N$ and let $\mathbf{v} \in \mathbf{V} \equiv (\times V_i)_{i=1}^N$, where V_i is some interval $[0, \bar{v}_i]$. Likewise, let $\mathbf{v}_{-i} = (v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_N)$, and $\mathbf{v}_{-i} \in \mathbf{V}_{-i} \equiv (\times V_j)_{j=1, j \neq i}^N$. Let $f(\mathbf{v})$ denote the joint density of the values; since the values are independently drawn, we have $f(\mathbf{v}) = f_1(v_1) \times f_2(v_2) \times \dots \times f_N(v_N)$, and $f_{-i}(\mathbf{v}_{-i}) = f_1(v_1) \times \dots \times f_{i-1}(v_{i-1}) \times f_{i+1}(v_{i+1}) \times \dots \times f_N(v_N)$ is similarly defined.

The seller picks a mechanism, i.e. an allocation rule that assigns the object to the bidders depending on messages sent by the latter. By appealing to the Revelation Principle, we can restrict attention to *direct* mechanisms, i.e., mechanisms that ask the bidders to report their values v_i . Thus, the mechanism consists of a pair of functions $\langle Q_i(\mathbf{v}'), P_i(\mathbf{v}') \rangle_{i=1}^N$ for each i which states the probability Q_i with which the object would go to bidder i and the expected payment P_i that bidder i would have to make for any vector of *reported* values of the bidder valuations. Of course, the mechanism has to satisfy two conditions: (i) it must be Incentive Compatible, i.e. it must be (weakly) optimal for each bidder to report her value truthfully given that all others are doing the same, and (ii) it must be Individually Rational, i.e., the bidders must be at least as well off participating in the selling process than from not participating.

Thus, the probability that bidder i gets the object when she reports her value to be z_i and all other bidders report truthfully is

$$q_i(z_i) = \int_{\mathbf{V}_{-i}} Q_i(z_i, \mathbf{v}_{-i}) f_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}$$

and the expected payment he makes is

$$p_i(z_i) = \int_{\mathbf{V}_{-i}} P_i(z_i, \mathbf{v}_{-i}) f_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}.$$

It can be shown¹⁶ that (i) Incentive Compatibility is equivalent to the requirement that the $q_i(v_i)$ functions are non-decreasing, i.e. the probability that a bidder gets the object is non-decreasing in her reported value of the object, and (ii) Individual Rationality is equivalent to the requirement that the $p_i(v_i)$ functions satisfy $p_i(0) \leq 0$, i.e. the bidder with zero value has non-positive expected payment. It can also be shown that in the optimal

¹⁶For details, please see Myerson (1981) or Krishna (2002).

selling mechanism, the $Q_i(\mathbf{v})$ need to be chosen to maximize the following expression

$$\begin{aligned} \sum_{i=1}^N p_i(0) &+ \sum_{i=1}^N \int_{\mathbf{V}} \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right) Q_i(\mathbf{v}) f(\mathbf{v}) d\mathbf{v} \\ &= \sum_{i=1}^N p_i(0) + \int_{\mathbf{V}} \left(\sum_{i=1}^N J_i(v_i) Q_i(v_i) \right) f(\mathbf{v}) d\mathbf{v} \end{aligned} \quad (18)$$

and the payment made by bidder i needs to satisfy

$$P_i(\mathbf{v}) = Q_i(\mathbf{v}) v_i - \int_0^{v_i} Q_i(z_i, \mathbf{v}_{-i}) dz_i. \quad (19)$$

The quantities $J(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$ are known as “virtual valuations” for reasons that will become clear below. Notice that $\frac{1 - F_i(v_i)}{f_i(v_i)}$ is the inverse of the hazard rate $\frac{f(v_i)}{1 - F_i(v_i)}$. If the hazard rate is increasing, then the virtual valuations are increasing in v_i . This is known as the “regular case” in the literature.

Ignoring the Incentive Compatibility and Individual Rationality constraints for the moment, it is clear that the objective function (18) is maximized pointwise if $Q_i(\mathbf{v})$ is set equal to the maximum value (i.e. 1, since it is a probability) when $J_i(v_i)$ is the highest for any realized \mathbf{v} , and zero otherwise. Two implications immediately follow.

First, notice that the allocation rule implies that if the bidders are symmetric (i.e the private values are drawn from the same distribution $F(v_i)$ for all i), then the bidder with the highest value gets the object with probability one. Moreover, from (19), any two selling procedures that have the same allocation rule must also result in the same expected payment made by the bidders and thus result in the same expected revenue for the seller. In particular, when the bidders are symmetric, all the standard auctions - since they result in the highest value bidder getting the object with probability 1 - are optimal selling mechanisms and result in the same expected revenue for the seller.

Second, if the bidders are not symmetric, then the object need not go to the bidder with the highest v_i . For example, suppose $f_i(v_i) = \frac{1}{b_i - a_i}$.

Then $J_i(v_i) = 2v_i - b_i$. Thus, $v_i > v_j \Rightarrow J_i(v_i) > J_j(v_j)$ if and only if $v_i - v_j > (b_i - b_j)/2$. In other words, the high-value bidder may not get the object if the upper bound on her value for the object is sufficiently high. The intuition is that the potential for such a bidder to under-represent her value is high; thus, by discriminating against her in terms of the likelihood of being awarded the object, the seller induces her to report truthfully when her valuation is high. The basic message here is of considerable importance, as we will see in more detail later: when bidders are asymmetric, it may pay to discriminate against the stronger bidder.¹⁷

2.7 Interpreting the Optimal Auction: The Marginal Revenue View

Bulow and Roberts (1989) provide an intuitive interpretation of the “virtual valuations” $J_i(v_i)$ according to which the object is allocated in the optimal selling scheme. Interpret v_i as a “price” and $1 - F_i(v_i)$ as a demand curve: if a price p is set as a take-it-or-leave-it price, $1 - F_i(p)$ gives the probability of a sale, i.e., the “quantity” $q(p)$ sold at price p . We can then calculate a marginal revenue curve in the usual way, but using $1 - F_i(v_i)$ as the demand curve:

$$\begin{aligned}
 \text{Total Revenue} &= v_i q(v_i) \\
 \Rightarrow (\text{Marginal Revenue}) &= \frac{d(\text{Total Revenue})}{dq} \\
 &= v_i + q(v_i) \frac{dv_i}{dq} \\
 &= v_i + (1 - F_i(v_i)) \frac{1}{dq/dv_i} \\
 &= v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \tag{20}
 \end{aligned}$$

¹⁷Notice that in the regular case, since the virtual valuations are non-decreasing, the q_i 's are non-decreasing as well. Moreover, it is easily checked that $P_i(0, \mathbf{v}_{-i}) = 0$ for all \mathbf{v}_{-i} ; hence $p_i(0) = 0$ for all i . Thus, incentive compatibility and individual rationality conditions are satisfied.

Thus, the virtual valuations are marginal revenues, and the optimal mechanism awards the good to the bidder with the highest marginal revenue. Bulow and Roberts (1989) in fact provide the following “second marginal revenue” auction interpretation of the optimal selling scheme. Each bidder is asked to announce her value, and the value is converted into a marginal revenue. The object is awarded to the bidder with the highest marginal revenue (M_1), and the price she pays is the lowest value that she could have announced without losing the auction (i.e. $MR_1^{-1}(M_2)$).¹⁸

Why does the “second marginal revenue” auction call for the winner to pay the lowest value she could announce without losing the auction? This is, in fact, a property of the optimal selling mechanism discussed in the previous section. To see this, define $s_i(\mathbf{v}_{-i})$ as the smallest value (more precisely, the infimum) of v_i for which i 's virtual valuation (marginal revenue) would be no less than the highest virtual valuation from the rest of the values. Clearly, $Q_i(z_i, \mathbf{v}_{-i}) = 1$ if $z_i > s_i(\mathbf{v}_{-i})$ and 0 otherwise. Thus $Q_i(z_i, \mathbf{v}_{-i})$ is a step function, and this implies that $\int_0^{v_i} Q(z_i, \mathbf{v}_{-i}) dz_i = v_i - s_i(\mathbf{v}_{-i})$ if $v_i > s_i(\mathbf{v}_{-i})$ and 0 otherwise. Since $v_i > s_i(\mathbf{v}_{-i})$ implies $Q_i(\cdot, \cdot) = 1$ and $\int_0^{v_i} Q(z_i, \mathbf{v}_{-i}) dz_i = v_i - s_i(\mathbf{v}_{-i})$, from (19) we get $P_i(v_i, \mathbf{v}_{-i}) = s_i(\mathbf{v}_{-i})$ for the winning bidder. Thus, the bidder with the highest marginal revenue pays the lowest value that would win against all other values when the object is allocated according to the marginal revenue rule.

3 Common-Value Auctions

3.1 Common Value Assumptions

To this point we have mostly considered auctions where bidders' preferences were described by the independent private values assumptions. Clearly, in this framework, given their signals, bidders have complete information about the value of the object to themselves. We turn now to another class of models where each bidder has information that, if made public, would affect the remaining bidders' estimate of the value of the object. The general model

¹⁸If no bidder has positive marginal revenue, the seller keeps the object; if only one bidder has a positive marginal revenue, then she pays the price at which her marginal revenue is zero. It is easy to check that truthful reporting is a dominant strategy in this auction.

could be described as each bidder having a value $V_i = v_i(t_1, t_2, \dots, t_N)$, where t_i represents bidder i 's signal. However, before we turn to the general model, it is useful to focus on a particularly important special case - the case of the “pure common value” model. In this scenario, every bidder has the same valuation for the item, hence the phrase “common value”. In other words, we have

$$V_i = v(t_1, t_2, \dots, t_N) \tag{21}$$

for each bidder i . Such an assumption is reasonable for auctions of many assets. The sale of a company, for instance, is sure to exhibit common-value characteristics, for the company’s underlying cash flows will be uncertain but, at least to the first consideration, will be the same for all potential acquirers.¹⁹

Common or interdependent-value auctions involve a certain form of adverse selection, which if not accounted for by bidders, leads to what has been called the “winner’s curse”. Auctions are wonderful at selecting as winner the bidder with the highest valuation. However, the highest of several value estimates is itself a biased estimate, and this fact would cause the winner to adjust downward her estimate of the value of the object. For example, suppose that there are two bidders, the object is worth $v = t_1 + t_2$ to each, where each t_i is an independent draw from the uniform $[0, 1]$ distribution. Based on her signal alone, each bidder’s estimate of the value is $t_i + 1/2$. However, if the bidders are symmetric, after learning that she is the winner in a first-price auction, bidder i 's estimate of the value will change to $t_i + E(t_j | t_j < t_i) = t_i + t_i/2 < t_i + 1/2$.

The point to emphasize here is that under almost any reasonable bidding scenario, the high bidder will be the one with the highest value estimate. While each bidder’s estimate is an unbiased ex-ante estimate of the common value, the highest of those estimates is biased high. Or to put it another way, winning an auction gives a bidder information that they had the highest

¹⁹The classroom “wallet game” mimics this particular common value auction model. In this game, two students are picked and each is asked to privately check the amount of money in his wallet. The teacher then announces that a prize equal to the combined amount of money in the wallets will be auctioned. The auction method is a standard ascending auction in which the price is gradually raised until one student drops out. The winner then gets the prize by paying that price. See Klemperer (1998).

estimate of value. If one respects the fact that the other bidders are as good at estimating value as oneself, then the information that $N - 1$ other bidders thought the item is worth less should give one pause for reflection (and of course this pause should have been taken before the bid was submitted).

3.2 Optimal Bidding With a Common Value

We begin with the illustrative example introduced above, and show how the principles apply.

Suppose there are only two bidders and the value to each bidder is given as

$$v = t_i + t_j \tag{22}$$

where t_i and t_j are each bidder's privately known signals. We will suppose that the signals are independently distributed according to a uniform distribution on $[0, 1]$.

Consider first a second-price auction. It is easy to show that in this auction, it is optimal for each bidder to bid $2t_i$. Suppose bidder j is following this strategy, and bidder i bids b . Then bidder i wins the auction if $2t_j = b_j < b$, i.e. $t_j < b/2$. Her expected gain is $\int_0^{b/2} (t_i + \tilde{t} - 2\tilde{t}) d\tilde{t} = t_i \frac{b}{2} - \frac{1}{2} \frac{b^2}{4}$. Maximizing with respect to b , one gets $b_i = 2t_i$, as claimed.

With two bidders, the second price auction is equivalent to an ascending auction. Thus, it should be no surprise that the equilibrium bidding strategies in an ascending auction are identical to the one derived above. To see this, suppose bidder j has a bidding strategy of $b_j = 2t_j$. If bidder i continues to be in the auction at a price $b > 2t_i$, her profit if j ended the auction by dropping out would be $t_i + b/2 - b = t_i - b/2 < 0$, and thus it cannot be optimal for her to be in the auction at that price. Similarly, if $b < 2t_i$ her profit if j ends the auction would be $t_i + b/2 - b > 0$, and thus it cannot be optimal for her to quit at that price. Consequently, she must stay in the auction until the price reaches $2t_i$.

Notice that the bidders do take into account winner's curse in equilibrium. If the price reaches a level $b = 2t_i$, the value of the object is at least $t_i + b/2 = 2t_i$, since j is still in the auction. Thus, the expected value is strictly higher than $2t_i$. However, i would still quit at this price, because *if the auction had*

ended at this price because j quit, she would be breaking even. As we saw above, she would lose if the auction ends at any higher price and she is the winner.

A first-price auction is more complicated, but similar results hold. One can think of the optimal bid in a first-price auction as being the result of a two-stage process: first, adjust one's expected value for the bias associated with being the highest out of N signals; and second, further lower the bid to account for the strategic nature of an auction.

3.3 Milgrom and Weber's (1982) Generalized Model

3.3.1 Core Assumptions

While both the independent private value and the pure common value model capture many key aspects of real auctions, they are obviously polar cases. Many real auctions will contain both private value and common value characteristics. In an auction of a company, for instance, the company's "core" cash flow will be a common value for all bidders, but synergies will likely differ across bidders and therefore contribute an element of independent private values. In a seminal paper, Milgrom and Weber (1982) developed analysis of a generalized valuation model for auctions. The key valuation assumption in Milgrom and Weber's general *symmetric* model is that the value of the item to bidder i is given by

$$v_i = u(t_i, \mathbf{t}_{-i}) \tag{23}$$

In (23), t_i is the signal privately observed by bidder i , and \mathbf{t}_{-i} denotes the vector of signals $(t_i, t_2, \dots, t_{i-1}, t_{i+1}, \dots, t_N)$. The function $u(\cdot, \dots)$ is nondecreasing in all its variables. The model is symmetric in the sense that interchanging the values of the components of \mathbf{t}_{-i} does not change the value of the object to bidder i . In this symmetric model, note that both the private and pure common value models are special cases: if $v_i = u(t_i)$ for all i , we have the private value model, and if $v_i = u(t_1, t_2, \dots, t_N)$ for all i (i.e. $u(\cdot, \dots)$ is symmetric in *all* the signals), then the model is a common value model. The interdependent values model with independent signals discussed earlier is also obviously a special case, in which the signals are i.i.d.

The symmetric model assumes that the joint density of the signals, denoted by $f(\cdot, \dots, \cdot)$ is defined on $[0, \bar{t}]^N$, and is a symmetric function of its arguments. The density functions are also assumed to have a statistical property known as “affiliation”, which is a generalized notion of positive correlation among the signals.

It will be convenient to work in terms of the expected value of the object to bidder i conditional on her own signal t_i and the highest among the remaining $N - 1$ signals. Without any loss of generality, we will focus on bidder 1, and accordingly, let us define

$$v(t, y) = E[v_i(\dots)|t_1 = t, Y_1 = y] \quad (24)$$

where Y_1 is the highest signal among the remaining $N - 1$ signals of bidders $2, \dots, N$. We will denote the distribution function of Y_1 by $G(y)$ and its density by $g(y)$. Notice that because of symmetry, it does not matter who among the remaining bidders has the highest signal, and moreover, by virtue of symmetry with respect to the way in which a bidder’s own signal affects the value of the object to the bidder, the function is the same for all bidders. Because of affiliation, it follows that $v(\cdot, \cdot)$ is non-decreasing function in t and y .

3.3.2 Equilibrium Bidding

It is convenient to begin with the second-price auction. Generalizing the example in section 3.2, we shall show that the symmetric equilibrium bid function is given by $v(t, t)$. Recall that the function $v(t, t)$ is the expected value of the bidder’s valuation, conditional upon the bidder having signal t and on the bidder with the second-highest signal also having signal t .

To see that $v(t, t)$ is the symmetric equilibrium bid function, notice that if bidder 1 bids b_1 assuming that all other bidders are following the proposed equilibrium bidding strategy, then her expected payoff is

$$\int_0^{b^{S^{-1}}(b_1)} (v(t, y) - v(y, y))g(y|t)dy.$$

Differentiating, it is immediate that the first-order condition is satisfied if $b_1 = b^S(t)$ so that $b^{S^{-1}}(b_1) = t$.

Turning now to the ascending auction, suppose that the bidding is at a stage where all bidders are still active. Suppose bidder with signal t has the strategy that she will remain in the bidding until the price $b^N(t) = u(t, t, \dots, t)$ is established, provided no bidder has dropped out yet. If the first bidder to drop out does so at the price p_N , let t_N be implicitly defined by $b^N(t_N) = p_N$. Then suppose every remaining bidder with signal t has the strategy of staying until the price reaches $b^{N-1}(t, p_N) = u(t, t, \dots, t, t_N)$. Let p_{N-1} be the price at which the next bidder drops out. Then let t_{N-1} be implicitly defined by $b^{N-1}(t_{N-1}, p_N) = p_{N-1}$. Now every remaining bidder has a strategy of remaining in the bidding until the price reaches $b^{N-2}(t, p_{N-1}, p_N) = u(t, t, \dots, t_{N-1}, t_N)$. Proceeding in this manner, the bidding strategies of the bidders after each round can be written down until two bidders remain. Clearly, these strategies entail that each bidder drops out at that price at which, given the information revealed by the bidding up to that point, the expected value of the object would be exactly equal to the price if all remaining bidder except herself were to drop out all at once at that price.

We shall argue that these strategies constitute an equilibrium of the ascending auction. If bidder 1 wins the auction, then t_1 must exceed all other signals. Now, from the construction of the bidding strategies, it is clear that the bidder with highest signal among the remaining bidders quits at a price $u(y_1, y_1, y_2, y_3, \dots, y_{N-1})$, where y_i denotes the value of the i -th highest signal among the rest of the bidders, i.e., excluding bidder 1. Thus, bidder 1 gets $u(t, y_1, y_2, y_3, \dots, y_{N-1}) - u(y_1, y_1, y_2, y_3, \dots, y_{N-1})$, which is strictly positive. Quitting earlier, she would have obtained zero, and any other strategy that makes her drop out after the bidder with signal y_1 cannot give her any higher payoff. Consider now a situation in which bidder 1 does not have the highest draw. For her to win the auction, she must have to pay $u(y_1, y_1, y_2, y_3, \dots, y_{N-1})$; however, this exceeds the value of the object to her, which is $u(t, y_1, y_2, y_3, \dots, y_{N-1})$. Thus, she cannot do better than drop out as prescribed by the equilibrium strategy.

To find the equilibrium bid in the first-price auction, assume that each of the other $N - 1$ bidders follow a bidding strategy $b^F(z)$, and that bidder 1 bids as though her private signal were z . Since the bids are increasing, the expected profit for bidder 1 whose signal is t is

$$\Pi(z, t) = \int_0^z (v(t, y) - b^F(z))g(y|t)dy.$$

The derivative of this expression with respect to z is

$$(v(t, z) - b^F(z))g(z|t) - b^{F'}(z)G(z|t)$$

which should be zero at $z = t$. Thus, we get

$$(v(t, t) - b^F(t))\frac{g(t|t)}{G(t|t)} = b^{F'}(t) \quad (25)$$

Since $v(0, 0) = 0$, we have the boundary condition $b^F(0) = 0$. The differential equation can then be solved²⁰

$$b^F(t) = \int_0^t v(y, y)dL(y|t) \quad (26)$$

where $L(y|t) = \exp(-\int_y^t \frac{g(x|x)}{G(x|x)}dx)$.

It is easy to check that $L(\cdot|t)$ is in fact a probability distribution function on $[0, t]$, so that the expression for the equilibrium bid is an expected value with respect to some probability measure.

3.3.3 Revenue Ranking and the Linkage Principle

With affiliated signals, revenue equivalence no longer holds. The ascending auction generates at least as much expected revenue to the seller as the second-price auction, which in turn generates at least as much expected revenue as the first-price auction. While a direct comparison is possible, the so-called ‘‘Linkage Principle’’ provides a fundamental insight. Consider an auction A in which a symmetric equilibrium exists, and suppose that all bidders are bidding in accordance with this symmetric equilibrium except possibly bidder 1, who has a signal t but bids as though her signal were z (z could equal t). Suppose $W^A(z, t)$ denotes the expected price that is paid by that bidder if she is the winning bidder. Then the Linkage Principle says that of any two auctions A and B with $W^A(0, 0) = W^B(0, 0)$, the auction for which $W_2^i(t, t)$ (i.e., the partial derivative with respect to the second argument evaluated with both arguments at t) is higher will generate the higher expected revenue for the seller.

²⁰The first-order condition is only a necessary condition. It can be shown that $P_i(z, t)$ is indeed maximized at $z = t$ if the signals are affiliated.

With the benefit of the Linkage Principle, it is easy to see why the first-price auction generates higher revenue than the second-price auction. In the first-price auction, a bidder with signal t bidding as if the signal were z would pay $b^F(z)$ conditional on winning, i.e. $W_2^F(z, t) = 0$ for all t and z . On the other hand, in the second-price auction, the corresponding expected payment is $E[b^S(Y_1)|t_1 = t, Y_1 < z]$, where Y_1 is the highest signal among the other $N - 1$ bidders. It can be shown that given that $b^S(\cdot)$ is an increasing function, affiliation implies that $E[b^S(Y_1)|t_1 = t, Y_1 < z]$ is increasing in t . Hence, the second-price auction generates higher expected revenue.

An important implication of the Linkage Principle - especially for corporate finance purposes - is that the seller can raise her expected price (revenue) by committing to release to all bidders any information relevant to valuations. More formally, if the seller releases an informative variable that is affiliated with the other variables, then the expected equilibrium price (for all auction forms) is at least as high as when the information is not released.

3.4 Limitations of the Common-Value and General Symmetric Auctions

For corporate finance situations especially, issues of information and efficiency in auctions should be important. Existing models do not allow for full consideration of some of these issues.

In the independent private values auction, efficiency has only one dimension: whether the item is sold to the bidder with the highest valuation. In the pure common value model, there is no real allocation problem so that from an efficiency standpoint, one might as well allocate the item randomly. While a random allocation may not provide optimal revenue for the seller, one should be suspicious of a model focused only on wealth-transfer and not efficiency considerations. One can imagine a variety of economic forces outside of the auction process itself that will tend to cause efficient processes to develop (competition between auctioneers, or even the law). Models that assume away any possibility of inefficiency may cause us to lose sight of the true economic issues in comparing alternative selling mechanisms.

The Milgrom and Weber (1982) model brings an allocation problem back into the picture, in that bidders' valuations differ, so there are efficiency im-

plications of the allocation. On another level, though, this relatively general model still fails to permit a complete role for economic efficiency. As pointed out by Hirshleifer (1971), information can have both private and social value. For information to have social value, it must have the capability to affect the allocation of resources. One would expect that in an auction context, information would not only allow bidders to refine their estimates of value, but since the bidders do have inherently different valuations, one would also expect that information would possibly change relative valuations. That is, with one information set, bidder i might have the highest expected value; but with a different information set, bidder j might have the highest expected value.

The Milgrom and Weber model does not permit this kind of role for information. A simple example suffices to show this as well as to illustrate why it is important to allow information to play an efficiency role. Consider the following two-bidder, two-state model:

	State	
	A	B
Bidder 1	100	200
Bidder 2	200	100

In State A, the asset is worth 100 to bidder 1 and 200 to bidder 2, with the valuations reversing for State B. Recall that a major result from the Milgrom and Weber (1982) model is that the expected price increases upon the seller's release of additional information. In the example above this result does not hold. Consider an open auction, and let the information on state initially be diffuse, with each state believed to be equally likely. Then each bidder has an expected value of 150, and an open auction will yield a price of 150. Now let the seller release public information which discloses precisely which state prevails. In either state, an open auction will yield a sale price of only 100, the second-highest valuation. Release of information therefore lowers the expected price, contrary to the Milgrom and Weber findings. Interestingly, there is also now a tension between the seller's objective and economic efficiency: additional information improves efficiency

by allocating the asset to its highest-valued use, but it lowers the sellers revenue. Little work has been done on the relative efficiency of auctions under circumstances such as this, but see Krishna (2002) for an excellent summary of efficiency in auctions. In corporate finance, it would seem that the issue of information and efficiency will be closely related: does additional information increase the efficiency of an auction (bearing in mind the cost of producing the information, possibly by multiple bidders) and does this create a conflict between revenue maximization and efficiency?

4 Applications of Auction Theory to Corporate Finance

4.1 Introduction

We now turn to survey the more important applications of auction theory to corporate finance. We begin with the market for corporate control and auctions in bankruptcy, which are the two largest areas of application. Then we turn to share repurchases, IPOs, and a limited review of corporate finance issues in the Federal Communication Commission's auction of radio spectrum. We do not cover applications of auctions to capital markets finance, for instance to models of the stock trading process or to auctions of bonds by governments and companies. Our intent in this survey is to go beyond a simple review and to point out how well auction theory can actually be used to "inform" corporate finance.

4.2 Applications to the Market for Corporate Control

Auctions of one form or another typically occur in the market for corporate control. The field has proved fruitful for a variety of auction-based models to be constructed that explain many aspects of the market. One aspect is to explain the wealth gains to bidders and targets, as well as the combined wealth gains, on announcements of acquisitions.

4.2.1 Returns to Bidders and Targets

Many studies have documented the evidence on stock returns to bidders and targets in corporate acquisitions, and the overall evidence is that returns to targets are large and positive, while returns to acquirers are generally negative but statistically insignificant. Jarrell, Brickley and Netter (1988) provide evidence prior to 1988; Andrade, Mitchell, and Stafford (2001) provide a recent update: over the period 1973-1998, with a database of 3688 acquisitions, the average two-day abnormal return around the announcement of an acquisition was 16 % (statistically significant at the 5 percent level) for the target; -0.7% for the acquirer (statistically insignificant); and the combined gain was 1.8% (statistically significant at the 5% level). Boone and Mulherin (2003) further update the recent evidence; they find for a sample of acquisitions between 1989 and 1999 that target returns were on average 21.6%, and that the return to acquirers was an insignificant -0.7%.

Further cuts on the data provide interesting results on the returns to bidders. Returns to bidders are generally more negative the more is the competition from other bidders (although see Boone and Mulherin (2006b) discussed below). All-stock offers generally yield lower returns to bidders than do all-cash offers (see discussion below). Returns to bidders are generally more positive when the acquisition is large relative to the acquirer's size (Loderer and Martin (1990), Eckbo and Thorburn (2000), Moeller, Schlingemann, and Stulz (2004)). One strong empirical regularity is that the total profit to bidders and targets (as measured by the event studies) is greater for auctions than for merger negotiations. This is true for both bidders and targets. This may point to a particular measurement problem: merger bids are often a more drawn-out and partially anticipated takeover process than auctions - which means profits in auctions are more easily measured. It is also possible that tender offers are more profitable because they tend to remove old management (to a greater extent than mergers).

The most recent evidence come from the large-sample studies of Betton, Eckbo and Thorburn (2005, 2006). They study more than 12,000 publicly traded targets of merger bids and tender offers over the period 1980-2004. Following the approach of Betton and Eckbo (2000), bids are organized sequentially to form contests for a given target, and they focus in particular on the first and on the winning bidder (which need not be the same). Since

the surprise effect of the initial bid is greater than that of subsequent bids, and since the initial bidder starts the contest, studying abnormal returns to the initial bidder yields additional power to test hypotheses concerning the sign and magnitude of bidder gains. Moreover, since bids are studied sequentially in calendar time, they present a natural laboratory for testing auction-theoretic and strategic bidding propositions (toehold bidding, bid preemption, bid jumps, target defenses, etc.).

Initially, Betton, Eckbo and Thorburn follow the tradition and report average abnormal returns for samples of offer outcomes, including "successful" and "unsuccessful" bids. In the traditional analysis, abnormal returns to "success" (AR_s) is found by cumulating abnormal returns from the first bid announcement through completion of the takeover process which may take several months. The lengthy cumulation adds noise to this estimate of AR_s . Therefore, Betton-Eckbo-Thorburn also report *ex ante* estimates of AR_s using the more precisely measured market reaction to the initial bid announcement only. To illustrate, let x denote a set of offer characteristics (e.g. bid premium, the payment method, toehold purchases), and $p(x)$ the probability that the bid will succeed as a function of x . The market reaction Γ in response to the initial announcement of bid i is

$$\Gamma_i(x_i) = AR_s p(x_i) + AR_u (1 - p(x_i)), \quad (27)$$

where AR_u is the average abnormal return conditional on the offer being unsuccessful. Here, AR_s and AR_u are estimated as regression parameters in a cross-sectional regression involving *all* sample bids, whether ultimately successful or not.²¹ Using the right-hand-side of equation (27), they conclude that the expected value of the initial bid (conditional on x) is statistically indistinguishable from zero. As in the earlier literature, targets expected returns are positive and significant, as is the value of the sum of the gains to targets and bidders. Thus, the data do not support theories predicting value-destruction.

Betton, Eckbo and Thorburn also report that the magnitude and distribution of abnormal returns to bidders and targets depends significantly

²¹The estimation is in three steps: (1) estimate AR_i using time series of returns to the bidder up to the first bid announcement, (2) estimate $p(x)$ using the cross-section of bids, and (3) run regression (27) to produce AR_s and AR_u .

on whether they are private or publicly traded companies. Bidder gains are larger, and target premiums smaller, when the bidder is public but the target is a private firm. Moreover, private bidder firms have a significantly lower probability of succeeding with their bids for public targets. They also report that, in contests where no bids succeed, the target share price reverts back to the level where it was three calendar months prior to the initial bid in the contest. As noted by Bradley, Desai and Kim (1983) as well, this share price reversal is what one would expect if the market conditions the initial target stock price gain on a control change in fact taking place (where control may be acquired by either the initial or some rival bidder).

Overall, the evidence suggests that auctions tend to yield great results for targets but that competition in the auction (or something else) tends to ensure that gains to bidders are at best minimal. From the standpoint of auction theory, this is surprising: certainly in a private values context, and even in a common value context, the strategic equilibrium of an auction should still yield an expected profit for the winning bidder. The fact that gains to bidders are minimal suggests that the pure auction models do not capture the richness of the process, and that other forces are likely at play. As Boone and Mulherin (2003) suggest, the evidence is in favor of two-stage models such as that of French and McCormick (1984) which analyze costly entry. While pure auction models imply an expected surplus for participating bidders, entry of additional bidders will cause that expected surplus to be dissipated through costly entry.

Roll (1986) first used the idea of the winner's curse to explain the empirical evidence that acquiring firms appear to over-bid for targets in that acquiring firms' stock prices fall (or stay at best constant) upon announcement of acquisitions. If bidders ignore the winner's curse, they may well over-pay (in a common value setting, which is not unreasonable in the corporate acquisition market). The problem, of course, is that equilibrium theory does not permit expected over-bidding, so Roll is relying upon acquirers making mistakes. Proponents of behavioral finance will find it quite convincing to think that bidders may not properly adjust their strategy for the pitfalls inherent in common value auctions, for avoidance of the winner's curse takes some careful analysis. Those inclined towards rational, equilibrium based models of behavior will be wary of models that assume incomplete strategic adjustment. Boone and Mulherin (2006b) use unique data that allows

them to characterize sales of companies as either auctions or negotiations, and for the auctions, to say how many potential bidders were contacted in the sales process and how many actually submitted bids. Finding no relationship between bidder returns and these measures of competition, Boone and Mulherin conclude that their findings do not support the existence of a winners curse.

A large literature attributes the acquirer wealth losses to managerial agency problems or “empire building” tendencies. For example, in a sample of 326 U.S. acquisitions between 1975 and 1987, Morck, Shleifer and Vishny (1990) find that three types of acquisitions have systematically lower and predominantly negative announcement period returns to bidding firms : diversifying acquisitions, acquisitions of rapidly growing targets, and acquisitions by firms whose managers performed poorly before the acquisition. The authors argue that these results are consistent with the view that managerial objectives may drive acquisitions that reduce bidding firms values. Lang, Stulz and Walking (1991) present related results. Jensen (2003) provides a new angle to this argument by hypothesizing that high market valuations increase managerial discretion, making it possible for managers to make poor acquisitions when they have run out of good ones.

Another recent approach to overbidding is based on the idea that when bidders own initial stakes or “toeholds” in the target firm, they are essentially wearing two hats - that of a buyer for the target’s remaining shares, and that of a seller of their initial stakes to the rival bidder. We review the theory-based work in this area more fully below. For now, we note that in an independent private values model, Burkart (1995) and Singh (1998) show that a bidder with toehold will bid above her private value in a second-price auction. Similar results are also obtained in alternative value environments and under alternative auction procedures (Bulow et al. (1999), Dasgupta and Tsui (2003)). Evidence on the empirical relevance of toeholds, however, is mixed. In Jennings and Mazzeo’s (1993) sample of 647 tender offers and mergers, the mean toehold is 3%, but only about 15% of the bidders own an initial stake. Betton and Eckbo (2000) study toeholds for initial and rival bidders in a sample of 1,250 tender offer contests over the period 1972-1991. They find that toeholds increases the probability of single-bid success and lowers the price paid by the winning bidders.

Betton et al. (2005) delve more deeply into the subtleties of various facts about toeholds. In their sample of 12,723 bids for control (3,156 tender offers and 9,034 mergers), 11% of the bids involved toeholds. The percentage was significantly higher for tender offers than for mergers, both for non-hostile targets (21% and 6% respectively for tender offers and mergers) and for hostile targets (62% and 31%, respectively). The mean and median toehold sizes conditional on being positive were 21% and 17%, respectively, for the overall sample. However, a majority of these toeholds were “long-term toeholds”, i.e. acquired before 6 months prior to the bid. The percentage of bids involving short-term toeholds for the entire sample was only about 2%. Betton et al. (2005) argue that since toeholds are likely to deter competition, the target might turn hostile if the bidder acquires toeholds when private negotiations might be going on. Thus, it is unclear to what extent toeholds are used strategically in bidding contexts. It is worth recalling in this context Shleifer and Vishny’s (1986) analysis of the role of large shareholders in the target firm: even when they are not bidders, the presence of large shareholders in the target firm who are willing to split the gains on their shares with a bidder has the same effect as the bidder having an initial stake in the target.

Another approach to reconciling the existing findings on loss of value to acquirers, the gains to targets, and joint value losses is presented by Jovanovic and Braguinsky (2004), even though their model is not explicitly auction-based. The model incorporates uncertainty over the skill of corporate managers, the value of projects that companies have, and the takeover market. In equilibrium, the takeover market facilitates the exchange of “good” projects from firms with “bad” managers to firms with “good” managers but “bad” projects. Ex ante values of firms represent investors’ knowledge of management type but uncertainty over project type. If a firm puts itself up for sale, which it does only if its project is good and its management is bad, then investors learn that the firm does have the property right to a good project and its value increases - hence the positive return to targets. A firm becomes an acquirer only if its own project is bad. Upon learning that a firm will be an acquirer, investors learn that the firm’s own project is bad -hence the negative return to acquirers. For reasonable parameter values, including a cost incurred in the takeover process, joint values of the target and acquirer fall. Even so, the mergers in the model are welfare-enhancing.

4.2.2 The Auction Process in the Market for Corporate Control

As our previous discussion shows, takeover models help understand some of the observed empirical evidence on bidder and target returns. Another major role of auction theory, in so far as it facilitates our understanding of the takeover bidding process, has been to “inform” a company’s board or regulators about the impact of selling processes or rules on shareholder wealth, efficiency and welfare. However, here, for the prescriptions to be useful, the auction models must at least reasonably mimic the takeover bidding environment. The question we address now is the extent to which this is the case.

First, it is important to note that auction theory has developed in the spirit of mechanism design, or the design of optimal selling schemes. Any auction model assumes a degree of commitment power on the part of the seller. There are clear “rules of the game” that the seller and the bidders are required to abide by. For example, in a first-price auction, in which bidders shade their bids, the losing bidders might want to submit a bid higher than the winning bid after the latter is disclosed. The seller must be able to commit not to entertain such bids. A similar argument applies to the reserve price. Casual observation, however, suggests that many bids (even when they are friendly) are not seller initiated. It might appear that many control contests are not really formal auctions, in which the seller is trying to secure the best price for the firm’s shareholders by committing to a selling mechanism.

This perspective is misleading, for several reasons. First, the board has a formal responsibility to be an “auctioneer”. Under Delaware law,²² a company’s board must act as “auctioneers charged with getting the best price for the stock-holders at a sale of the company”. In several well-publicized cases, after potential bidders had indicated their interest in acquiring the company, the board of directors of the target company have conducted an auction.²³ Although procedures similar to the ascending auction are most commonly used, boards have also held single, and sometimes even multiple, rounds of

²²The Delaware law is significant because many U.S. public companies are incorporated in Delaware.

²³For example, in the takeover battle for Paramount between Viacom and QVC, the Paramount board eventually conducted an auction in an effort to “select the bidder providing the greatest value to shareholders”.

sealed-bid auctions (e.g. in the well-documented case of RJR Nabisco).

The commitment issue discussed above may influence the board's choice of auction mechanism. For example, the board might have a preference for ascending auctions because, under alternative auction rules such as the sealed-bid auction, should a losing bidder offer a higher subsequent bid, it may be difficult to reject that bid if the board is required to obtain the "best price for the shareholders". In other words, it may be difficult to commit to a single round of bidding.

Legal scholars, however, have taken the view that whether or not it is feasible for the board to pursue a particular auction mechanism depends, ultimately, on how the courts view it. If, in a given context, the courts consider that a particular auction mechanism can generate higher revenue for the shareholders *ex-ante* than the more commonly used ones, there is no reason why a board cannot adopt it as a selling scheme. Further, if the shareholders do not perceive a particular selling scheme to be against their interests *ex-ante*, there is no reason why a board cannot secure shareholder approval prior to conducting a sale. It is exactly in this spirit that legal scholars have looked at alternative selling procedures (see, for example, Cramton and Schwartz, 1991). The focus of this literature has very much been on what one can learn from economic theory (in particular, auction theory) to "inform" takeover regulation or selling practices.

Second, the board's commitment power is sometimes underestimated. Boards can commit to awarding an object to a "winner" from a given round of bidding even when better bids might subsequently emerge - thereby undermining the auction - in a variety of ways. The most common practice is to enter into a lock-up arrangement²⁴ with the declared winner, together with an agreement to pay a break-up fee should the sale be terminated.²⁵

²⁴Lockups are "agreements that give the acquirer the right to buy a significant division, subsidiary or other asset of the target at an agreed (and generally favorable) price when a competing bidder acquires a stated percentage of the target's shares." (see Herzel and Shepro, 1990). They may also involve options to buy a block of target shares from the target that may make acquisition by a competing bidder more difficult. Lockup agreements are quite common in takeover contests. The legal status of lockups is unclear, as some courts have upheld them, while others have not. For an account of the legal literature on lockups, see Kahan and Klausner (1996).

²⁵For example, Viacom's initial offer for Paramount in 1993 was associated with (a) an

Another possibility is for the target board to refuse to rescind poison pills for any but the declared winning bidder. While it is unclear whether the courts will allow such poison pills to stand, the legal costs of challenging the poison pills and the possibility that the board might switch to an ascending auction (so that the challenger is by no means assured of winning the contest) may deter further challenge from a losing bidder.

Third, formal or informal auctions are much more common than is usually assumed. Boone and Mulherin (2006a) analyze a sample of 400 takeovers of US corporations in the 1989-1999 period and find evidence consistent with the idea that boards act as auctioneers to get the best price for the shareholders in the sale of a company. Based on information from the SEC merger documents, the authors provide new information on the sale process. The most important evidence is that there is a significant *private* takeover market prior to the public announcement of a bid. The authors document that almost half of firms in their sample were auctioned among multiple bidding firms, and the rest conducted negotiations with a single bidder. A third of the firms in the former category went through a formal auction, in which the rules were clearly laid out. In all cases, the process usually began with the selling firm hiring an investment bank and preparing a list of potential bidders to contact. After the bidders agreed to sign a confidentiality/standstill agreement, they received non-public information. Subsequently, a subset of the bidders indicating preliminary interest was asked to submit sealed bids.²⁶

Another issue relevant for the applicability of auction models to control contests concerns the complexity of the environments in which takeovers are conducted, compared to the standard auction environments. Auction models are nicely classified as belonging to different value environments, and results differ depending on which value environment is under consideration. The takeover environment is considerably more complicated. The motives for takeover bids could be varied. The early takeover models (e.g. Grossman and Hart (1980)) assumed that the benefit from a takeover comes from an improvement in the operational efficiency of the target company. As the

option to buy 20% of Paramount's outstanding shares and (b) a termination fee of \$150 million plus expenses, should the transaction not be concluded.

²⁶Betton et al.(2005) show that of the 12,000 contests, about 3,000 (25%) start out as tender offers (which subsequently turn into auctions). Some initial merger bids also end up in auctions, so the overall percentage of auctions maybe closer to 30%.

authors showed, this could lead to a “free-rider” problem and the market for corporate control could fail. However, later models have focused on “merger synergies” as the source of gain from takeovers. If the synergies accrue to the bidding firm, then the standard auction environment is more applicable. Here, however, there are issues about whether “private values” or “common values” assumptions are more relevant. Since bidders are different and the synergies are likely to have idiosyncratic components, a private values model does not appear unreasonable. However, common value elements will also undoubtedly exist. Synergies can have common value components if their magnitudes depend on the quality of the target’s assets, or if the bidders plan to bundle these assets with other assets that they own and eventually sell these assets.^{27,28}

Other complexities also arise when applying the auction framework to the analysis of takeover bidding. Bidders could bid for the company, or they could bid for a fraction of the company’s shares. Different regulatory regimes permit different types of bids. Bids could be exclusionary, discriminatory, conditional, and so on. Bids can be in cash, or in shares of the target company. Bidders may have different toeholds, and they might have different degrees of expertise in the target industry (a factor that could affect the degree of information conveyed by their bids in common value environments). Finally, if the bidders are competing with each other in the same industries, then the outcome of the auction may impose externalities on the bidders. As we will argue below, while existing takeover models, drawing on auction theory, have evolved to deal with these many of these complexities, significant gaps still exist in the literature.

²⁷Models of takeover bidding, when making common values assumption about the target’s “true worth”, have often tended to assume away the free-rider problem. If a bidder obtains a large majority of the shares, she may be able to “freeze out” the remaining minority shareholders. Also, the loss of liquidity on any remaining shares can have the same effect as “dilution” (see Grossman and Hart, 1980) that reduce the post-takeover value of the minority shares.

²⁸Betton and Eckbo (2000) show that the average number of days from the initial tender offer bid to the second bid is 15 days (counting only auctions with two or more bids). They suggest that this very short period is evidence of correlated values. Of course, the vast majority of all cases develop a single bid only, which may be taken as evidence of private (uncorrelated) values, or preemptive bidding (see below).

4.2.3 Auctions versus Negotiations

Several papers use auction theory to further refine our theoretical and empirical understanding of the auction process in corporate takeovers. Starting at the most basic level, Bulow and Klemperer (1996) show that in an English auction, it is always better to have $N + 1$ bidders in a formal auction than to have N bidders but with a follow-on (optimal) negotiation between the winning bidder and the seller. If $N = 1$, this shows that it is better to have an auction with two bidders than to sell by posting a reserve price.²⁹ Very simply, the auction process is extremely efficient at extracting value from the high bidder, more so than even an optimally conducted negotiation. This theoretical result does conflict with a stylized fact that companies do frequently avoid auctions and instead negotiate with just one buyer (Boone and Mulherin (2006a)).

4.2.4 Pre-emptive Bidding

Fishman (1988, 1989) considers models where one bidder has incentive to make a “pre-emptive” bid. In the main model of Fishman, a first bidder has incentive to put in a high bid that discourages the second bidder from bidding. The reason for this is that a high bid can signal a high valuation on the part of the first bidder, and a second bidder will then infer that the gain from participating in the auction is low (they are not likely to win in the final English auction). Since participation in the auction requires a bidder to spend resources to determine her own value, the second bidder can be discouraged from even entering the auction. Fishman (1989) extends this initial work by including the possibility of non-cash offers.

Fishman’s (1989) model works as follows. The value of the target assets to the bidders depends in part on the realization of a state of nature which is observed only by the target. Conditional on the target’s information, the value of the assets to the bidders is increasing in the bidders’ independent private signals. The means-of-payment can be either cash or debt that is backed by the target’s assets. Each bidder has to incur some cost to learn the private signal. Bidder 1 identifies a target by accident, and then incurs some cost to learn his signal (bidding is assumed to be not profitable if the

²⁹See Krishna (2002) for an analysis of this case.

true signal is unknown). If bidder 1 submits a bid, the target is “put in play”, and a second bidder is aware of the target. This bidder then decides whether or not to compete for the target and incur the cost of learning her signal.

There is a stand-alone value of the target that is public information, and the target rejects all bids below this value. Since the bidders do not know completely how much the target assets are worth to them (recall that the target privately observes part of this information), bidders could end up overpaying for the target. Paying with debt mitigates the overpayment because the value of the debt is contingent on the value of the target assets (since the debt is backed by these assets). However, if bidder 1 draws a high private signal, a cash offer - though costly - will separate it from a bidder with low signal: the latter will prefer to pay only with debt since his own private signal is not sufficiently high. Thus, by bidding with cash, the first bidder can signal to the second bidder that the latter’s likelihood of winning the ensuing auction is low: hence, the second bidder may decide not to incur the cost of learning her signal. This, then, is a “pre-emptive” bid. On the other hand, if the first bidder’s signal is low, bidding high with a cash offer is too costly. Thus, such a bidder would decide not to preempt and instead bid with debt to mitigate the potential loss from buying a target with low synergy. Notice that one prediction of this model is that more competing offers should be forthcoming with non-cash offers than with cash offers.³⁰

4.2.5 Modelling Auctions of Companies

Hansen (2001) reviews the formal auction process used for selling private companies and divisions of public companies. The model explains the common practices of limiting the number of bidders and limiting the disclosure of information to bidders (Boon and Mulherin, 2005), even though theory suggests that both practices would reduce prices. Hansen argues that some information in a corporate sale is competitive in nature, and that its broad release can destroy value in the selling company. The seller therefore faces a tradeoff between having many bidders and full disclosure versus protecting

³⁰Betton and Eckbo (2000) find that the average offer premium in successful single-bid contests is greater than the average offer premium in the *first* bid in multiple-bid contests. This what consistent with preemptive bidding.

value by limiting disclosure, as well as the number of bidders. While not modeling negotiations formally, the analysis implies that negotiation with a single bidder may be optimal if the “competitive information cost” is high enough. The model also explains the practice of a two-stage auction, with a first stage calling for non-binding “indications of interest” (value estimates for the target) which are used to select bidders for the second round and giving them access to more information on the selling company. If the selling company uses the initial value estimates for the target to set a reserve price that is an increasing function of the estimates, bidders in the initial round will reveal their private valuations honestly and the selling company can select the most highly-valued bidders for the final, binding, round (see the discussion below on the process for pricing IPOs for an earlier similar finding).

4.3 Means-of-Payment

Hansen (1985, 1986) has considered the role of non-cash means of payment in the market for corporate control; this work has now been extended by DeMarzo, Kremer and Skrzypacz (2004). In one model, Hansen shows that ex-post means of payment can increase the seller’s revenue beyond what cash payments can do. Take an independent private values context, where v_i represents bidder i ’s valuation of the target company. An ascending auction with cash as the means of payment will yield v_2 - the second highest value - as the price. Consider, however, bidding using bidders’ stock as the means-of-payment. Let each bidder have a common value, v , of her stand-alone equity. Then each bidder will be willing to bid up to s_i , where s_i is the share of firm i offered (implicitly through an offer of equity) and is defined to make the post-acquisition value of the bidder’s remaining equity equal to its pre-acquisition value:

$$v = (v + v_i)(1 - s_i)$$

which implies

$$s_i = \frac{v_i}{v + v_i} \tag{28}$$

The bidder with the highest valuation of the target will win this auction (s_i is increasing in v_i) and she will have to offer a share defined by v_2 , the

valuation of the second-highest bidder. However, the value of this bid to the target will be

$$s_2 * (v + v_1) = \frac{v + v_1}{v + v_2} v_2 > v_2$$

since $v_1 > v_2$. The stock-based bidding therefore extracts more revenue from the high-bidder than does cash bidding. DeMarzo et al. (2005) generalize this result, showing that expected revenues are increasing in the “steepness” of the security design, where steepness refers, roughly, to the rate of change of a security’s value in relation to the underlying true state. This paper also compares auction formats in a world where bids can be non-cash; it turns out that revenue equivalence does not always hold. Overall the paper concludes that the optimal auction is a first-price auction with call options as the means-of-payment.

Ex-post pricing mechanisms also yield benefits in common-value contexts. The reason for this follows from the return of the adverse selection problem inherent in the winner’s curse: the problem arises because the price for the asset is being determined before the value of the asset is known. Any kind of pricing mechanism that determines all or part of the price ex-post can alleviate the problem. Using the acquiring firm’s stock is an ex-post pricing mechanism, for that stock’s value will depend upon the actual value of the target firm. Hansen (1986) builds on this insight and shows that stock and cash/stock offers can be used efficiently in mergers and acquisitions. However, in offering stock as the means-of-payment, acquiring firms bring in their own adverse selection problem - acquiring firms may offer stock when they have information that their own value is low. Taking into account both the ex-post pricing advantage of stock and the “reverse” adverse selection problem, it turns out that higher-valued acquirers will offer cash while low-valued acquirers offer stock. Fishman (1989) reaches a similar conclusion, in that non-cash offers induce the target firm to make more efficient sell/don’t sell decisions, but that cash offers have an advantage in pre-empting other bids.

³¹

Several studies on US data show results consistent with Hansen and Fishman’s work, that acquirers’ returns are higher for cash offers than for stock

³¹Rhodes-Kropf and Viswanathan (2000) consider a general model of non-cash auctions for a bankrupt firm. We discuss this model later.

offers (see Eckbo et al.(1990) for a brief summary). The first paper to explicitly model the choice of mixed offers is Eckbo et al (1990). These authors prove the existence of a fully separating equilibrium in which the market’s revaluation of the bidder firm is increasing and convex in the proportion of the offer that is paid in cash. Since one can estimate the revaluation, and since the proportion cash is observable, this theory is testable. Using over 250 Canadian takeovers (where tax issues do not confound the choice of payment method), the authors find empirical support for the “increasing” part but not for convexity.

4.4 Toeholds

Recently, a number of theoretical papers have examined how toeholds affect takeover bidding. The main result that emerges from this literature is that the presence of toeholds makes bidders more aggressive, with the result that bidders can bid above the value of the object. The result holds for the second price auction in both the independent private values as well as a common value environment.

Burkart (1995)³² considers a two-bidder and independent private values model. The private values are best interpreted as synergies. The auction form is a second-price auction, which in this context is strategically equivalent to an ascending auction (Lemma 1 in the paper). From standard arguments, it follows that (i) it is a dominant strategy for the bidder with no toeholds to bid exactly her valuation, and (ii) it is a *dominated* strategy for the bidder with positive toehold to bid below her valuation. A general result is that any bidder with positive initial stake will bid strictly above her valuation. The model is then specialized to the case in which one bidder - call her bidder 1 - has an initial stake of θ while the other bidder - bidder 2 - has no initial stake.

Since bidder 2 will bid her value, we have $b_2(v_2) = v_2$. Thus bidder 1’s problem is to choose b_1 to maximize

$$\text{Max}_{b_1} \quad \Pi_1(v_1, b_1, \theta) = \int_0^{b_1} [v_1 - (1 - \theta)v_2] f_2(v_2) dv_2 + \theta b_1 (1 - F_2(b_1)). \quad (29)$$

³²Singh (1998) has essentially similar results.

The first-order condition is

$$(v_1 - (1 - \theta)b_1)f_2(b_1) + \theta(1 - F_2(b_1)) - \theta b_1 f_2(b_1) = 0.$$

Re-arranging, we get

$$b_1 = v_1 + \theta \cdot \frac{1 - F_2(b_1)}{f_2(b_1)} > v_1. \quad (30)$$

If one assumed that the hazard function $\frac{f_2(\cdot)}{1 - F_2(\cdot)}$ is increasing, then a number of results follows immediately. First, bidder 1's equilibrium bid is increasing in her valuation and the size of her toehold. Therefore, the probability that bidder 1 wins the auction is also increasing in her toehold. It is also clear that the auction outcome can be inefficient: since bidder one bids more aggressively than bidder 2, it is clearly possible that $v_1 < v_2 < b_1(v_1)$, i.e. bidder 1 has the lower valuation but wins the auction. This result is similar to the inefficiency in the standard auctions where the seller sets a reserve price. In fact, the intuition for the overbidding result is exactly that of an optimal reserve price from the point of view of a seller. Indeed, with a toehold, a bidder is a part-owner and we should not be surprised to find that she wants to “set a reserve price” in excess of her own value.

It is interesting to note that winning can be “bad news” for bidder 1. Suppose $v_1 = 0$ with probability 1. Then bidder 1 still bids a positive amount (equal to bidder 2's value) but since her bid exceeds the value of the synergy, she always overpays when she wins the auction. By continuity, the same conclusion holds for \bar{v}_1 (the upper bound of the support of the distribution of bidder 1's synergy) sufficiently small., and for bidder 2's valuation in some interval $[v'_2, b_1(\bar{v}_1)]$.³³

Bulow, Huang and Klemperer (1999) examine the effect of toeholds in a pure common value environment. They make a significant contribution to the literature on toeholds by deriving bid functions for both the second and first-price auctions when both bidders have positive toeholds. They examine how (for small positive toeholds) bidder asymmetry affects the takeover outcome

³³Using Burkart's private value setting with two bidders, Betton et al. (2005) also show optimal overbidding when the bidder has a lock-up agreement with the target. Moreover, they show optimal underbidding when the bidder has a breakup fee agreement with the target.

in each auction, and compare expected revenues in the two auctions when the toeholds are symmetric as well as asymmetric. We first discuss their setup in some detail, before discussing the intuition for the main results.

Bulow et al. (1999) consider a “pure common value” model with two bidders where each bidder draws an independent signal t_i from a uniform $[0, 1]$ distribution. The value of the target to each bidder is $v(t_1, t_2)$. Bidder i owns initial stake θ_i in the target, where $1/2 > \theta_i > 0$, for $i = 1, 2$. Each bidder bids for the remaining $1 - \theta_i$ fraction of the shares of the target.

In the second-price auction, bidder i 's problem is to choose b_i to maximize

$$\text{Max}_{b_i} \quad \Pi_i(t_i, b_i) = \int_{t=0}^{b_j^{-1}(b_i)} [v(t_i, \alpha) - (1 - \theta_i)b_j(\alpha)]d\alpha + \int_{b_j^{-1}(b_i)}^1 \theta_i b_i d\alpha. \quad (31)$$

The first-order condition is

$$\frac{1}{b_j'} [v(t_i, b_j^{-1}(b_i)) - (1 - \theta_i)b_j(b_j^{-1}(b_i))] + [1 - b_j^{-1}(b_i)]\theta_i - \theta_i b_i \frac{1}{b_j'} = 0.$$

Let us now define $\phi_j(t_i) = b_j^{-1}(b_i(t_i))$, i.e. this defines the pair of signals for bidders i and j for which they have the same bid, since $b_j(\phi_j(t_i)) = b_i(t_i)$. Similarly, we can define $\phi_i(t_j) = b_i^{-1}(b_j(t_j))$. Using these definitions, we can rewrite the first-order condition as

$$b_j'(\phi_j(t_i)) = \frac{1}{\theta_i} \frac{1}{(1 - \phi_j(t_i))} [b_i(t_i) - v(t_i, \phi_j(t_i))] \quad (32)$$

where we have replaced t_j by $\phi_j(t_i)$.

The corresponding first-order condition for bidder j is

$$b_i'(\phi_i(t_j)) = \frac{1}{\theta_j} \frac{1}{(1 - \phi_i(t_j))} [b_j(t_j) - v(\phi_i(t_j), t_j)] \quad (33)$$

where we have used the fact that $v(\phi_i(t_j), t_j) = v(t_j, \phi_i(t_j))$. Consider a pair of t_i and t_j that in equilibrium bid the same, then we must have $t_i = \phi_i(t_j)$ and $t_j = \phi_j(t_i)$. Using this, the last equation can be rewritten as

$$b_i'(t_i) = \frac{1}{\theta_j} \frac{1}{(1 - t_i)} [b_j(\phi_j(t_i)) - v(t_i, \phi_j(t_i))] \quad (34)$$

Since $b_j(\phi_j(t_i)) = b_i(t_i)$ and $b'_i(t_i) = b'_j(\phi_j(t_i))\phi'_j(t_i)$, dividing (34) by (32), we get

$$\phi'_j(t_i) = \frac{\theta_i}{\theta_j} \frac{1 - \phi_j(t_i)}{1 - t_i}. \quad (35)$$

Integrating, and using the boundary condition $b_i(0) = b_j(0)$ (see Bulow et al. (1999) for a proof), we get

$$\phi_j(t_i) = 1 - (1 - t_i)^{\frac{\theta_i}{\theta_j}}. \quad (36)$$

Since the probability that bidder i wins the object is $\int_0^1 \int_0^{\phi_j(t_i)} dt dt_i = \frac{\theta_i}{\theta_i + \theta_j}$, it is clear that bidder i is more likely to win the auction as her stake increases and that of bidder j decreases. Remarkably, a bidder's probability of winning goes to 0 as her stake becomes arbitrarily small, given that the other bidder has a positive stake. The intuition for this result is that while bidder i with zero stake has no incentive to bid above $v(t_i, \phi_j(t_i))$ given the equilibrium bidding strategy of j , as we shall see below, bidder j with $t_j = \phi(t_i)$ and a positive stake will strictly bid above this value.³⁴

Now, equation (34) can be integrated to give

$$b_i(t_i) = \frac{\int_{t_i}^1 v(t, \phi_j(t))(1 - t)^{\frac{1}{\theta_j} - 1} dt}{\int_{t_i}^1 (1 - t)^{\frac{1}{\theta_j} - 1} dt}. \quad (37)$$

where the boundary condition $b_i(1) = b_j(1) = v(1, 1)$ is used (see Bulow et al. (1999)).

³⁴Klemperer (1998) demonstrates in the context of the "Wallet Game" how a very small asymmetry in a common value model can give rise to very asymmetric equilibria. This is a consequence of the fact that in the standard Wallet Game, there are in fact a continuum of asymmetric equilibria. A small toehold - like a small bonus to one of the players in the Wallet Game - introduces a slight asymmetry that can have a major impact on the equilibrium, i.e., one of the bidders essentially having a zero probability of winning. With a slight advantage, the stronger player bids slightly more aggressively, but that increases the winner's curse on the weaker player. The latter then bids less aggressively, which reduces the winner's curse on the stronger player, who then bids still more aggressively, and so on. With slight entry or bidding costs, this prevents the weaker player/players from entering the auction, so that very low prices result. Klemperer (1998) provides several illustrative examples from Airwaves Auctions.

From (36), we then get

$$b_i(t_i) = \frac{\int_{t_i}^1 v(t, 1 - (1-t)^{\frac{\theta_i}{\theta_j}})(1-t)^{\frac{1}{\theta_j}-1} dt}{\int_{t_i}^1 (1-t)^{\frac{1}{\theta_j}-1} dt}. \quad (38)$$

Bidder j 's bid function is derived similarly. From (37), it is clear that for $t_i < 1$, $b_i(t_i) > v(t_i, \phi_j(t_i))$. Thus, when bidder i wins the auction, she is paying more than the target is worth to her. Moreover, bidder i 's bid is increasing in her stake θ_i , i.e. a higher stake makes the bidder act more like a seller and causes her to bid higher.

Bulow et al. (1999) extend the analysis in two main directions. First, they consider the effect of a more asymmetric distribution of the toeholds and find that subject to an overall constraint on the toeholds of the two bidders that is sufficiently small, a more uneven distribution of toeholds leads to lower expected sale price for the target. This result is a consequence of the fact that as the toeholds become more asymmetric, the bidder with the higher toehold bids more aggressively, i.e. further away from the value. For the bidder with a smaller toehold, this implies that the target is worth less conditional on winning. Exposed to this “winner’s curse”, the bidder with the smaller toehold therefore bids lower. Since in the second-price auction the winner pays the lower of the two bids, the expected sale price is adversely affected when the toeholds become asymmetric.

Bulow et al. (1999) next consider first-price auction and derive the equilibrium bid functions using methods similar to those described above for the second-price auction. In this case, we have $\tilde{\phi}_j(t_i) = t_i^{\frac{1-\theta_i}{1-\theta_j}}$. The probability that bidder i with signal t_i and toehold θ_i wins the auction in this case is given by $\frac{1-\theta_j}{(1-\theta_i)+(1-\theta_j)}$, which is increasing in θ_i . It is easily checked that for $\theta_i < \theta_j$, the probability of bidder i winning the auction is lower in the second-price auction than in the first-price auction. Since in both auctions the probability is exactly 1/2 when $\theta_i = \theta_j$, this implies that the winning probability falls more steeply with a decrease in a bidder’s toehold the second-price auction than in the first-price auction.

The incentive for bidders with toeholds to bid high in the first-price auction are not as strong as in the second-price auction. This is because in the first-price auction (unlike the second-price auction), bidding high does

affect the bidder's cost, although a higher toehold does lower that cost since fewer shares need to be purchased.

Unlike the second-price auction, the expected sale price can increase in the first-price auction as the toeholds become more asymmetric. Revenue comparisons indicate that with *symmetric* toeholds, the expected sale price is higher in the second-price auction. This is because as the winner's curse problem is mitigated with symmetric toeholds, both bidders can bid more aggressively and essentially set a higher reserve price for their stakes in the second-price auction. With asymmetric toeholds, as we saw above, the second-price auction generates low expected sale prices due to the winner's curse.³⁵

4.5 Bidder Heterogeneity and Discrimination in Takeover Auctions

Bidder asymmetry is common in the context of corporate control contests and can take several forms. Asymmetry in initial stakes or toeholds, discussed in the previous section, is one form of bidder asymmetry. Bidder asymmetry can also arise when bidders draw their signals from different distributions, or when (in a common value environment) the bidder signals have asymmetric impact on the value function.

In section 2.6, we saw that when bidders are asymmetric, the optimal mechanism may not allocate the object to the bidder with the highest valuation. For example, in the independent private value context, an allocation rule that discriminates against a stronger bidder may provide a higher expected profit to the seller. Thus, standard auctions are no longer optimal in the presence of various forms of bidder heterogeneity.

To increase the expected sale price when bidders are asymmetric, the seller has essentially two alternative responses. Both involve "levelling the playing field". When the asymmetry is due to differences in toeholds or access to information, the target's board may decide to restore symmetry

³⁵The analysis of toeholds can be extended to models that include the private value model and the common value model of Bulow et al. (1998) as special cases. Dasgupta and Tsui (2004) analyze auctions where bidding firms hold toeholds in each other in the context of such a model.

by allowing the disadvantaged bidder increase his toehold cheaply or provide access to additional information.³⁶ Alternatively, the board may decide to design the auction rules in a way that discriminates against the strong bidder.

An especially simple way to discriminate is to impose an order of moves on the bidders. Since bidding games are price-setting games, there is usually a “second-mover advantage” associated with bidding games (see Gal-Or, 1985, 1987). Thus, to discriminate against the strong bidder, the seller could ask this bidder to bid first. This bid could then be revealed to a second bidder, who wins the auction if she agrees to match the first bid. Otherwise, the first bidder wins. In the context of takeover bidding, this “matching auction” has been studied by Dasgupta and Tsui (2003), who note that since courts are more concerned about shareholder value than whether the playing field is level or not, it is unlikely that the matching auction will run into trouble because it does not treat the bidders symmetrically.³⁷

To see that the matching auction can generate a higher expected sale price than the second-price auction in the independent private value setting, let us return to the private values model introduced in section 4.4. Assume that the private values of both bidders are drawn from the uniform $[0, 1]$ distribution. From equation (30), we get the bid of bidder 1 who has a toehold of θ to be

$$b_1(t_1) = \frac{t_1 + \theta}{1 + \theta}.$$

Thus, the expected bid from bidder 1 is $P_1 = \int_0^1 \int_0^{\frac{t_1+\theta}{1+\theta}} t_2 dt_2 dt_1 = \frac{1}{6} \frac{3\theta^2+3\theta+1}{(1+\theta)^2}$ and that from bidder 2 is $P_2 = \int_0^1 \int_0^{(1+\theta)t_2-\theta} \frac{t_1+\theta}{1+\theta} dt_1 dt_2 = \frac{1}{6} \frac{1-2\theta^2+2\theta}{1+\theta}$. Thus, the expected sale price in the second-price auction is

$$P^S = P_1 + P_2 = \frac{(2\theta + 1)(2 + 2\theta - \theta^2)}{6(1 + \theta)^2}. \quad (39)$$

³⁶Betton and Eckbo (2000) note that when a rival (second) bidder enters the auction with a toehold, the toehold is of roughly the same magnitude as the initial bidder’s toehold (about 5%). This is consistent with the “leveling the playing field” argument of Bulow et al. (1999).

³⁷Herzel and Shepro (1990) note: “opinion in several cases in the Delaware Chancery court has noted that the duty and loyalty [of managers] runs to shareholders, not bidders. As a result, ‘the board may tilt the playing field if it is in the shareholder interest to do so’.”

Now consider the matching auction. Given a bid b_1 from bidder 1, bidder 2 will match if and only if $t_2 > b_1$. Thus, bidder 1 chooses b_1 to maximize

$$\int_0^{b_1} (t_1 - (1 - \theta)b_1) dt_2 + \theta(1 - b_1)b_1.$$

From the first-order condition, one readily gets $b_1(t_1) = (1/2)t_1 + (1/2)\theta$. Thus, the expected sale price in the matching auction is

$$P^M = (1/4) + (1/2)\theta. \quad (40)$$

Comparing (39) and (40), it can be verified that $P^M > P^S$ if and only if $\theta > 0.2899$. Thus, if the toeholds are sufficiently asymmetric, asking the strong bidder to move first increases the expected sale price.

The matching auction's properties in the context of a common value model with independent signals similar to Bulow et al. (1999) have been explored by Dasgupta and Tsui (2003). The authors show that there exists a perfect Bayesian Nash equilibrium in which bidder 1 with stake θ_1 bids

$$b_1(t_1) = v(t_1, F_2^{-1}(\theta_1)) \quad (41)$$

and bidder 2 matches if and only if $t_2 \geq F_2^{-1}(\theta_1)$.³⁸ Here, bidder i 's signal is drawn from the distribution $F_i(t_i)$. Notice that the expected sale price is then

$$P^M = E_{t_1}(v(t_1, F_2^{-1}(\theta_1))). \quad (42)$$

Notice that (i) conditional on her bid, losing is better than winning for bidder 1, since her payoff in the former event is $\theta_1 v(t_1, F_2^{-1}(\theta_1))$, and her payoff in the latter event is at most $v(t_1, F_2^{-1}(\theta_1)) - (1 - \theta_1)v(t_1, F_2^{-1}(\theta_1))$, and (ii) as a consequence, winning is "bad news" for bidder 1, i.e. if she wins, there would be a negative effect on the stock price. In contrast, winning is always "good news" for the second bidder.

It is also immediate that the expected sale price increases in the first bidder's toehold. In contrast, bidder 2's stake has no effect on the expected sale price. The probability of bidder 1 winning the auction is $F_2^{-1}(\theta_1)$ and is

³⁸For a derivation and a complete characterization of the equilibrium, see Dasgupta and Tsui (2003).

therefore increasing in θ_1 . However, the common value feature of the model is apparent in that if bidder 1's toehold is 0, then her probability of winning is also 0; moreover, in this case, she bids $v(t_1, 0)$, i.e., the lowest possible value conditional on her own signal. This is because the bidder who moves first is subjected to an extreme winner's curse problem.

How can the matching auction improve the expected sale price compared to the standard auctions? Recall that in the second-price auction with asymmetric toeholds, the smaller toehold bidder is exposed to an extreme winner's curse problem. The matching auction is a way to shield the low toehold bidder from this extreme winner's curse by asking her to move second. This, of course, imposes a winner's curse on the first bidder. However, if the asymmetry is large, the first bidder with a higher toehold will act more like a seller, and thus the sale price will not suffer as much. Dasgupta and Tsui (2003) show that, for the case of a value function that is symmetric and linear in the signals (i.e. $v(t_1, t_2) = t_1 + t_2$) that are drawn from the uniform distribution, the matching auction generates a higher expected sale price than both the first- and the second-price auctions when the toeholds are sufficiently asymmetric and not too small.

Another type of bidder asymmetry arises in the common value framework if the value function is not symmetric, e.g. $v(t_1, t_2) = \alpha t_1 + (1 - \alpha)t_2$ and $\alpha > 1/2$. Dasgupta and Tsui (2003) show that with symmetric toeholds, the matching auction generates a higher expected sale price than the first-price auction if the value function is sufficiently asymmetric (i.e. α sufficiently close to 0 or 1); and it generates a higher expected sale price than the second-price auction if the value function is sufficiently asymmetric and the toeholds are not too large. Povel and Singh (2005) characterize the optimal selling mechanism for the zero toeholds case and show that discrimination against the strong bidder is optimal. However, to implement the optimal mechanism, the seller needs to know the precise value of α as well as the distribution of the signals. This is not required in the matching auction, for which only the identity of the stronger bidder is needed. In other words, the matching auction is a "detail-free" mechanism. This is an especially appealing property given that for sufficiently large asymmetry, the matching auction does almost as well as the optimal mechanism in extracting the surplus.

4.6 Merger Waves

There is no question that merger and acquisition activity goes in waves. Rhodes-Kropf and Viswanathan (2003) give the following perspective: in 1963-64, there were 3,311 acquisition announcements while in 1968-69 there were 10,569; during 1979 to 1980 and also from 1990 to 1991 there were only 4,000 announcements while in 1999 alone there were 9,278 announcements. The 1980s were generally a period of high merger and acquisition activity, and saw the emergence of the hostile takeover and corporate raiders, but activity dropped off in the early 90s only to rebound again late in the 90s. Holmstrom and Kaplan (2001) review the evidence on merger waves and offer a macro explanation based on changing regulatory and technological considerations which created a wedge between corporate performance and potential performance, along with developments in capital markets which gave institutional investors the incentives and ability to discipline managers.

Rhodes-Kropf and Viswanathan (2004) offer an alternative explanation for merger waves based on an auction-theoretic model rich in informational assumptions. They note that periods of high merger activity tend to be periods of high market valuation, and the means of payment is generally stock. For example, the percentage of stock in acquisitions as a percentage of deal value was 24 % in 1990, but 68% in 1998. They focus on mergers where stock is the means-of-payment. The essence of the argument is as follows: stock values of both targets and acquirers can become over-valued on a market-wide basis. These are economy-wide pricing errors that managers of neither targets nor acquirers have information on, but they do know they occur. Managers of targets know when their own stocks are overvalued; however, they do not know how much of that is due to economy-wide pricing errors and how much is firm-specific. When a stock offer is made *in an overvalued market*, target managers, knowing their own firms are overvalued but not knowing whether this is due to market-wide or firm specific factors, will overestimate potential synergies with acquirers. This is similar to search-based explanations of labor market unemployment, whereby workers think that a decrease in demand for their labor at one firm is firm-specific (when it is in fact business cycle related) and therefore accept unemployment, thinking that their economy-wide opportunities have not been affected. Thus, in times of economy-wide overvaluation, target firms will accept more bids, for they rationally infer that synergy with the bidder is high. Of course, with each

merger, the market should rationally lower the price, taking the possibility of overvaluation into account. However, this does not rapidly lead to an end of a wave: If synergies are correlated, then merger waves can occur, because the market also revises upward the probability that synergies for *all* firms are high. Correlation of synergies can arise out of the sort of considerations that Holmstrom and Kaplan discuss, for example, changes in technology which increase the efficient scale of firms. Thus, a merger wave that begins when the market becomes overvalued may end only when the market realizes that the synergies that were anticipated are actually not there - i.e., the wave end with a market crash.

Rhodes-Kropf and Viswanathan's model is one of an open auction with bidders offering shares of the combined firm, similar to that of Hansen (1985). Multiple bidders and cash offers are possible. High bids by other bidders imply more likely misvaluation in stock offers; however, since synergies are correlated, this does not cause the wave to end. Stock-based deals are also more likely than pure cash deals in times of economy-wide overvaluation because of the valuation errors that targets make given the information structure. Thus, the model explains not only merger waves but also the stylized fact that in times of intense merger activity, stock is more likely to be used as the means-of-payment.

Shelifer and Vishny (2003) propose a theory of mergers and acquisitions which has a similar flavor. They argue that merger activity is driven by the relative valuations of bidders and targets and perceptions of synergies from merger activity. Suppose that acquirer and target have K_1 and K units of capital, respectively. The current market valuations per unit of capital are Q_1 and Q , respectively, where $Q_1 > Q$. The long-run value of all assets is q per unit. If the two firms are combined, then the *short-run* value of the combined assets is $S(K + K_1)$, where S is the "perceived synergy" from the merger. In other words, " S is the story that the market consensus holds about the benefits of the merger. It could be a story about [the benefits of] diversification, or consolidation, or European integration." Suppose P is the price paid to the target in a merger. If the means of payment is stock, it is easily checked that *long-run* benefit to the bidding firms' shareholders is $qK(1 - P/S)$ and that to the target shareholders is $qK(P/S - 1)$.³⁹ Thus,

³⁹Since the synergy is only in the mind of the beholder (the market), the long term

if $S > P > Q$, bidding firms' shareholders benefit in the long run but target shareholders benefit in the short run. Shleifer and Vishny (2003) argue that if target shareholders or managers have shorter horizons, they may be willing to trade off the short run benefits for the long run losses. For example, target management may be close to retirement or own illiquid stock and options.⁴⁰ Shleifer and Vishny (2003) argue that the example of family firms selling to conglomerates and entrepreneurial firms selling to firms such as Cisco and Intel in the 1990s fit this story very well. Alternatively, the bidding firm could simply "bribe" target management - Hartzell et al. (2003) find that target management receive significant wealth gains in acquisitions, and acquisitions with higher wealth gains for target management are associated with lower takeover premia.

Overall, the theory predicts that cash offers will be made when perceived synergies are low but the target is undervalued ($Q < q$). This is likely to be a situation where the firm needs to be split up and/or incumbent management replaced to improve value, and will be associated with target management resistance and poor pre-acquisition target returns. In contrast, stock offers will be made when market valuations are high, but there is also significant dispersion in market values. Finally, for stock offers to succeed, there must be a widely accepted "story" about synergies, and target management must have shorter horizons. Notice that the model also predicts that the short term returns to bidders in stock offers would be negative if the synergies are not extremely high ($S > Q_1$, i.e., the bidder essentially has a money machine) and the long-run returns would also be negative. For cash offers, both short and long-term returns should be positive.

benefit to the bidding firms' shareholders from a cash offer would be $q(K+K_1)-PK < qK_1$ since $P > Q > q$. On the other hand, if the synergy were real, a bidding firm would have no reason to prefer stock over cash. Thus, a large number of stock offer during a particular period should reveal to the market that the synergies are more apparent than real. It is precisely this kind of inference that is carefully modelled in Rhodes-Kropf and Viswanathan's model (2004) discussed above. Shleifer and Vishny (2004) brush aside these issues by assuming that the market is irrational.

⁴⁰Cai and Vih (2006) find that in the cross-section of all firms during 1993-2001, CEOs with higher illiquidity discount are more likely to get acquired. Further, in a sample of 250 completed acquisitions, target CEOs with higher illiquidity discount accept lower premium and are more likely to leave after acquisition. They also put up lower resistance and speed up the process.

Shleifer and Vishny (2003) argue that the three most recent merger waves nicely fall into their framework. The conglomerate merger wave of the 1960s was fuelled by high market valuations and a story about the benefits of diversification through better management. The acquisition of firms in unrelated businesses might have been more attractive because target firms in the same industry would also have high market valuations. The targets were often family firms whose owners wished to cash out and retire. However, since there was really no synergy from diversification, the wave of the 1960s gave rise to the bust-up takeovers of the 1980s - acquisitions that were in cash, hostile, and of undervalued targets. Rising stock market prices ended this wave of takeover activity as undervalued targets became more difficult to find. The most recent wave of the 1990s was ushered in by the rising market valuations. The story of synergy was reinvented: technological synergies, the benefits of consolidation, and the European integration.

4.7 Auctions in Bankruptcy

One of the most fruitful areas for the application of auction theory in corporate finance is in the context of corporate bankruptcy. The theoretical efficiency of auctions in allocating assets to their most highly-valued use has led many scholars to propose auctions as a means to resolve some of the issues in bankruptcy. As an auction also yields a price for the corporation, the question of determining value (for the purpose of settling claims) is also solved. Unfortunately, the informational issues in bankruptcy are quite severe; so any complete auction-based model of the process which will yield predictions on total cost must include the cost of information acquired by bidders. There is also a fairly prevalent view that credit markets may not always allocate financing efficiently to potential buyers of bankrupt companies, so prices may be low because of a dearth of bidders. Some of these issues have been addressed empirically by examining the bankruptcy process in Sweden, where auctions of bankrupt companies are mandatory (see related discussion below).⁴¹

Baird (1986) was one of the first to point out that auctions may be preferable to the court-supervised reorganization process of the United States' Chapter 11 bankruptcy code. Baird, among others, used the auction pro-

⁴¹See Eckbo and Thorburn (2003) and (2005).

cesses and results of the corporate takeover market as an analogy to estimate the gains that may be achieved if auctions were used to transfer control of bankrupt companies' assets. Other researchers, Weiss (1990) in particular, turned to estimating the direct cost of Chapter 11 procedures - with those costs being estimated at between 2.8 and 7.5% of assets. Easterbrook (1990) argues against auctions, maintaining that the costs associated with the IPO process is a good analogy for estimating the costs of determining a firm's value, and calculates IPO direct costs at roughly 14% of proceeds. Hansen and Thomas (1998) argue that Easterbrook's figures need to be adjusted to be put on a total asset, not proceeds, basis, and that the so-called "dealer's concession" built into IPO costs should also be subtracted as it is a cost of distribution, not of the auction process per se. Their resulting figure of 2.7% is then roughly equal to Weiss' estimates of the direct cost of bankruptcy. Thus, auctions and Chapter 11 would seem to have similar direct costs, leaving their relative efficiency to be determined by either theory or further empirical work.

On the empirical side, Thorburn (2000) has exploited the Swedish bankruptcy experience to draw important conclusions on the relative efficiency of cash auctions of bankrupt firms. In Sweden, the typical procedure has been for a bankrupt firm to be taken over by a court-appointed trustee who supervises a cash auction of the firm, either piecemeal or as an ongoing combination. These data therefore allow for direct examination of how auctions work in bankruptcy. Thorburn (2000) finds that three-quarters of the 263 bankrupt firms are auctioned as going-concerns, which compares favorably to Chapter 11 survival rates. As to cost, direct costs average 6.4% of pre-filing assets, with the one-third largest firms experiencing costs of only 3.7% of assets. As to debt recovery, the recovery rates are comparable to Chapter 11 reorganizations of much larger firms: on average, creditors received 35% of their claims, with secured creditors receiving 69% and unsecured creditors only 25%. Thorburn finds that APR is maintained by the auction procedure.

Eckbo and Thorburn (2005) construct an auction-based model to examine the incentives of the main creditor bank in a bankruptcy auction. Their work addresses one fear of bankruptcy auctions, that credit market inefficiencies will sometimes limit credit and cause bankrupt companies to be sold at "fire-sale" prices, possibly to the benefit of the original owner/managers. Eckbo and Thorburn show that the main creditor bank has an incentive to provide

financing to one bidder and to encourage that bidder to bid higher than would be in their private interest. The reason for this follows from the analysis of an optimal reserve price (see also the discussion of toeholds in section 4.4) in an auction, for the main creditor bank is essentially a partial owner of the bankrupt company. Just as an optimal reserve price exceeds the seller's own valuation (see above), the optimal bid for a main-bank financed bidder exceeds that bidder's own valuation. The equation specifying the optimal bid in Eckbo and Thorburn is exactly analogous to the equation for an optimal reserve price. Eckbo and Thorburn, examining again the Swedish data, find strong results for the over-bidding theory and no evidence that auction prices are affected by industry-wide distress or business cycle downturns. They also demonstrate a surprising degree of competition in the automatic bankruptcy auctions, and that auction premiums are no lower when the firm is sold back to its own owners. Overall, their evidence-which is the first to exploit directly the cross-sectional variation in auction prices-fails to support either fire-sale arguments or the notion that salebacks are non-competitive transactions.

Auction theory has also been applied to study the question of optimal bankruptcy procedures. Hart, LaPorta, Lopez-de-Silanes and Moore (1997) propose an ingenious three-stage auction process for bankrupt companies. The first stage solicits cash and non-cash bids for the firm, while the second and third stages determine prices and ownership of so-called "reorganization rights". Reorganization rights are new securities which consolidate all the various existing claims on the firm's assets. This proposal differs from Aghion, Hart and Moore (1992) in that there is a public auction (the third auction) for the reorganization rights. The purpose here is to reduce any inefficiencies caused by liquidity constraints in determining prices and allocations of the new securities which replace the old claims.

Rhodes-Kropf and Viswanathan (2000) extend the limited work done on non-cash bids in auctions discussed previously. While theory such as Hansen (1985) shows that non-cash bids such as equity can increase sales revenue, non-cash bids are themselves subject to uncertain valuation. Building on these basic insights, Rhodes-Kropf and Viswanathan show that in any separating equilibrium, a security auction (the means-of-payment is a security the value of which depends on the bidder's type) generates higher expected revenue to the seller than a cash auction. The reason for this is that in a security auction, the low types have a greater gain from mimicing the high

types, so to separate, the high types have to bid more. However, some securities will not separate the bidders. The authors show that there is no incentive compatible separating equilibrium with stock alone. Debt bids, or a minimum debt requirement, can achieve separation in some cases; in others, cash payments or large non-pecuniary bankruptcy costs are needed to achieve separation (so that the highest value bidder can be identified). However, relative to cash bids, bids that involve debt or equity distort ex-post effort choices. Bids that involve high debt and low equity rank higher because they distort effort less. Convertibles can work better as they give the seller the option to affect the ex-post capital structure of the target firm. The model thus is capable of explaining why debt and convertibles are often part of reorganization plans, and why companies often end up more highly levered than when they were distressed (Gilson (1997)).

Hansen and Thomas (1998) apply the model of French and McCormick (1984) to argue that uncertainty surrounding a bankrupt firm's assets can cause auction prices to be low. Using the French and McCormick model, with free entry of bidders, the auction price will be N^*C less than true value, where N^* is the equilibrium number of bidders and C is the pre-bid cost of entry (which they model as an information acquisition cost). Theoretically, then, the question is whether a court, by having to only obtain one (good) evaluation of the firm's assets, can hold costs below N^*C . They argue that the greater the uncertainty surrounding a firm's assets, the worse an auction will perform. By way of example, Reece (1978) shows that with high uncertainty, a common-value auction yields a price only 70% of true value.

4.8 Share Repurchases

Companies frequently buy back their shares through either fixed-price tender offers or Dutch auction mechanisms. In a Dutch auction repurchase, a company determines a quantity of shares to buy back and asks shareholders to submit bids specifying a price and quantity of shares that they are willing to sell. The bids are ordered according to price (low to high), creating a supply curve. As the Securities and Exchange Commission prohibits price discrimination, a uniform price is set corresponding to the lowest price that enables the firm to buy the pre-determined number of shares.

While there has been little formal modeling of the Dutch auction repurchase process itself (possibly because no real auction-theoretic issues are present) there is considerable empirical study, and their effects relative to fixed-price offers has been studied in a more traditional corporate finance setting. Bagwell (1992) studies 32 Dutch auction repurchases between 1988 and 1991. In one transaction, the highest bid was 14% above the pre-announcement market price, while the lowest bid was only 2% above. Such disparities in bids are documented for the entire sample, showing that the firms did face upward-sloping supply curves for their shares, contrary to naive ideas of a perfect capital market. Bagwell mentions several possible explanations, including differences in private valuations (for example, because of capital gains tax lock-ins), asymmetric information about a common value as in Milgrom and Weber (1982), or differences in opinion (Miller (1977)). While tax considerations could play a large role, it is certainly not a stretch to assume that shareholders will have different information on the value of a company (even though they share the public information embedded in the current price).

Other work has explored signaling aspects of Dutch auction repurchases relative to fixed price tender offers (Persons, 1994) and relative to paying dividends (Hausch and Seward, 1993). Persons (1994) considers a situation in which shareholders demand a premium (perhaps due to capital gains tax frictions) to tender their shares, but this premium varies across shareholders, resulting in an upward sloping supply curve. Repurchases are costly to existing shareholders because the tendering shareholders must be offered a premium. Importantly, the slope of the supply curve is random. In a fixed-price tender offer, the price is fixed, while the quantity of shares tendered adjusts to the random slope of the supply curve; in a Dutch auction, exactly the opposite is the case. If the manager intends to signal the true value by maximizing a weighted average of the intrinsic value and the market value of the shares (as in the dividend signaling model of Miller and Rock (1985)), fixed-price offers are more effective signals of the manager's private information; on the other hand, if the manager needs to buy back a specific number of shares to prevent a takeover threat, a Dutch auction is better as it guarantees that the required number of shares will be tendered.

4.9 Auction Aspects of Initial Public Offerings (IPOs)

In the summer of 2004, the internet search firm Google completed the world's largest initial public offering to be conducted via an auction procedure. Google sold 19.6 million shares at an offering price of \$85 each, for a total of \$1.67 billion raised. The auction method used was a variant of the Wall Street Dutch auction, covered immediately above. Initial public offerings of equity shares would seem to be excellent candidates for an auction procedure: multiple units of the same item for sale, with uncertainty over value and ability of a seller to commit to a sales method.

Interestingly, however, the evidence suggests that formal auctions are not favored as a sales mechanism. Instead, the IPO procedure known as "bookbuilding" attracts most of the market in regions where multiple sales methods can legally exist (Sherman 2005; Jagannathan and Sherman 2006; DeGeorge, Derrien, and Womack 2004). A fair amount of theoretical work has been done to explore differences in sales mechanisms for IPOs as well as issues within any one sales method. There is also a literature examining relative performance of auctions versus other sales methods, for in some countries we do have different sales methods co-existing.

In applying auction theory to IPOs, the place to start is the literature on uniform price, multiple unit auctions. The main initial contributions here are Wilson (1979), and Back and Zender (1993). A recent contribution is by Kremer and Nyborg (2004). The reason this literature is so important is that it shows how simple auction analysis yields the main underpricing result from IPO studies (that is, that the initial stock market returns immediately after setting the IPO price are overwhelmingly positive).⁴² The auction models show in fact that uniform price, multiple unit auctions have a multitude of equilibria with varying degrees of underpricing. The intuition of the underpricing result is quite simple: in a uniform price auction, bidders are asked to essentially submit demand schedules, specifying the number of shares they would be willing to buy at different prices. Wilson (1979) showed that instead of thinking of bidders as selecting a demand schedule to submit, a simple transformation allows us to model a bidder's decision as one of selecting the optimal "stop-out" price after subtracting other bidders' demands

⁴²For a detailed account of various theories of IPO underpricing, see Ljungqvist (2005) in Chapter 12 of this volume.

from the available supply. This makes each bidder a monopsonist over the residual supply and sets up the essential monopsonistic tension: a higher bid increases the quantity of shares purchased, but raises the price paid on all shares. Optimally, a bidder will submit a low stop-out bid, and as this will be the case for all bidders, a Nash equilibrium holds. Interestingly, the literature on underpricing in IPOs has not picked up on this simple explanation, relying instead on more complicated explanations.

While not relying on the Wilson/ Back and Zender insights, Benvenist and Spindt (1989) nonetheless use an auction-based model to explain certain aspects of the IPO process. The basic idea is similar to that of Hansen (2001), as it involves conditions under which bidders reveal truthfully their information through bids that are non-binding “indications of interest”. The model asks under what conditions an investor will reveal her information to the investment banker collecting demand information for an IPO. Underpricing of the IPO guarantees a return to these investors; this is critical for otherwise there could be no incentive to honestly reveal information. Also, those investors who reveal high valuations must receive more of the undervalued shares, or again there would be no payoff from honestly revealing information (and there is a cost to honest revelation as it affects the offering price). Thus, this auction-based model explains two core features of the IPO process, under-pricing and differential allocations of shares.

Biais and Faugeron-Crouzet (2002) present a complex and quite general model of the IPO process that compares auctions to fixed-price offerings. Unfortunately, the authors’ conclusion that the book-building approach dominates the auction method is clouded by the assumption that the auction method will induce collusion between the bidders. It is not at all clear why collusion, if profitable, will occur only in one auction method. This paper also shows why it is extremely difficult to use auction theory to convincingly show that one method is more efficient than another: to do this, one must introduce a myriad of assumptions, covering everything from valuations to costs of information collection. The validity of all these assumptions is difficult to evaluate, and the chances that the ranking of the sales methods would change, or become indeterminate, is high if some of the assumptions were changed.

Sherman (2005) compares bookbuilding to auctions under the very rea-

sonable assumption that entry by bidders is an endogenous decision. Her model yields a result similar in spirit to a core result that emerges from comparing the basic auction methods that while the expected price is the same in sealed-bids versus second-price auctions, the variance of prices is greater for the second-price auction. This result comes about because in the first-price auction, bidders put in their bids using their *expectation* of what other bidders values are, while in the second-price auction, the high-bid is dependent on the *actual* value of the second-highest valuation. Sherman focuses on the uncertainty in the number of bidders caused by a mixed strategy equilibrium in the game of entry into an IPO auction. If bidders are free to enter the IPO auction, then if there is some cost to enter and some classes of bidders are *ex ante* identical, the equilibrium in the entry game has a probability of entry for at least some bidders; the result is uncertainty over the actual number of bidders. This uncertainty over the actual number of bidders causes the IPO price to vary and in particular to vary in its relation to a true underlying value. Sherman observes that this additional uncertainty further worsens the “winners curse and considerably complicates the optimal “bid-shaving calculation that is required when there is winners curse. She also shows that each investor optimally collects less information in a uniform price rather than a discriminatory auction, because of the free rider (moral hazard) problem in the uniform price auction.⁴³

Sherman assumes that in the bookbuilding process, the underwriter can select the number of investors to invite into an information-acquisition process; this makes the bookbuilding process more like the first-price auction in terms of the variance of its outcomes. Jagannathan and Sherman (2006) rely on this model to explain their findings of a worldwide abandonment of IPO auctions in favor of bookbuilding; they also support the theoretical model with evidence on the variance in number of participants for IPO auctions. The issues of number of bidders and information collection would seem to be key in an optimal IPO pricing/allocation mechanism. One wonders, however, if a slight twist on assumptions for the auction models let the auctioneer control somehow the selection of bidders, *a la* Hansen (2002) would bring equivalence back to the two mechanisms. Sherman (2004, pg. 619) does

⁴³In the uniform price auction, since the auction price is set by the actions of bidders who have already paid the information gathering and processing costs, there is an incentive for uninformed bidders to free ride and jump in with a high bid.

note that “If the term “auction is interpreted in a broad sense, it is almost a tautology that an appropriate auction could be designed for IPOs This exemplifies a general difficulty in building theoretical models of two different institutions to explain their empirical performances: one can capture the sense of institutional differences by making clear assumptions (e.g., the underwriter can select the number of potential investors for bookbuilding but not for auctions) but one is left wondering if the assumptions really do justice to what actually happens in practice.

In another recent attempt at comparing bookbuilding to auctions, Degeorge, Derrien, and Womack (2004) show that bookbuilding seems to dominate empirically (they look at France, where for a time auctions and bookbuilding had roughly equal market shares, but now auctions are virtually extinct) and they offer a justification for issuers’ preference for the bookbuilding method that is based not on the price performance of bookbuilding but on the investment bankers’ preference for the method. While one might understand why investment bankers prefer a method that creates more demand for their services, the link to issuers’ interests is less clear. Degeorge et al. hypothesize that bankers agree to provide research coverage for issuers in return for using the bookbuilding method. What is left unstated is that issuers must be unable to buy such research coverage on the open market at prices similar to the costs paid by investment bankers: the authors agree that auctions would yield issuers a better price, so one must wonder why issuers put up with an inefficient procedure simply to get a tied service.

On the empirical side of the auctions/IPO issue, Kandel, Sarig and Wohl (1999) utilize a data set from Israeli IPO auctions to document elasticity of demand and under-pricing. The under-pricing of Israeli IPOs is intriguing, for those IPOs had their prices set by an explicit auction mechanism. In the period 1993-96, Israeli IPOs were conducted much like Dutch auction share repurchases: investors submitted sealed-bids specifying prices and quantities, a demand curve was determined, and a uniform price was set at the highest price for which demand equaled the supply of shares available. Kandal, Sarig and Wohl document some elasticity of demand for the reported bids: the average elasticity at the clearing price, based on the accumulated demand curves, was 37 (relatively elastic). Interestingly, even in these IPO auctions, there was under-pricing: the one-day return between the auction price and the market trading price was 4.5%. Another interesting feature of the Israeli

auctions is that after the auction but before the first day of trading, the underwriters announce the market clearing price corresponding to the offered quantity, as well as the oversubscription at the minimum price stipulated in the auction. This essentially means that the investor can estimate the price elasticity of demand based on two points on the demand curve. The authors find that the abnormal return on the first day of trading is positively related to the estimate of the elasticity. The authors argue that this reflects greater homogeneity in the estimates of value on the part of the participants in the auction; this is “good news” either because it implies greater accuracy of information about future cash flows and thus leads to a lower risk-premium demanded by investors, or because it signifies greater “market depth” and hence greater future liquidity.

Kerins, Kutsuna and Smith (2003) examine IPOs in Japan in the period 1995-1997, a time when Japanese firms had to use a discriminatory (bidders pay the amount of their bid) auction to sell the first tranche of newly issued shares. This first tranche of shares would be relatively small, and the sale by auction was restricted to outside investors only, with further limitations on the amount that could be bought by any investor. These restrictions could be interpreted as limiting the informational advantages of any one bidder. Under that interpretation, it is not surprising that the authors find relatively little “underpricing” of the shares for the auction tranche: for all the issues, the auction proceeds were only 1.6% below what proceeds would have been at the final aftermarket price. The second stage of the Japanese process was a more traditional fixed-price offer, and there was considerable underpricing of shares at this stage. While this might suggest that the auction was a better choice of mechanism, one must recognize that costs of a larger auction (to sell the entire issue) could well be larger than costs of just the first tranche.

4.10 The Spectrum Auctions and the Role of Debt in Auctions

Beginning in 1994, the Federal Communications Commission in the United States auctioned licenses for the use of radio spectrum in designated areas. The licenses were auctioned using a novel auction format involving sequential rounds of sealed-bidding on numerous licenses simultaneously. At the end of each round, complete information on the level of bids for all licenses was

revealed. The auction format was designed by economists, and at least in regard to the vast sums of money raised, was a great success. Numerous articles summarize all aspects of the auctions, including their design and performance: see, for example, McAfee and McMillan (1996), Milgrom (2000) and Salant (1997). For the empirical researcher, FCC auctions provide a wealth of information : for example, the FCC Web site (<http://www.fcc.gov>) lists all the bids in all the auctions. Moreover, many of the participating companies are publicly traded, so that company-specific information is also easily available. We focus here on one analysis which studied the effect of debt on the FCC auctions. Clayton and Ravid (2002) construct an auction model where bidders' debt induces lower bids than would otherwise be the case. In this model, bidders have outstanding debt that is large enough to induce bankruptcy if the auction is not won. Lower bids decrease the probability of winning, of course, but in this case guarantee some residual to the shareholders conditional on winning. In effect, in this model, pre-existing debt holders are "third parties" who have a prior claim of a part of the pie. Thus, pre-existing debt serves to reduce bidders' values and therefore reduces bids.⁴⁴ An empirical analysis of the FCC bidding data produces a negative but generally insignificant effect of a bidder's own debt on their bid but a negative and significant effect on a firm's bid of competitors' debt levels.

Che and Gale (1998) were the first to explicitly study the role of debt in auctions.⁴⁵ They have a result similar to Clayton and Ravid, although the models rely on different effects. In Che and Gale's framework, a second price auction yields lower expected revenue than a first price auction. To see how financial constraints affect revenue comparisons, suppose that due to budget constraints, bidders cannot bid more than a given budget, which is observed only by the bidder. The private valuations and budgetary endowments of each bidder are independently and identically distributed according to some joint distribution function. In this context, since bidders in the second-price auction bid their value, but in the first-price auction they bid *below* their value, bidding is more constrained in the second-price auction because of

⁴⁴On the role of debt holders as "third parties" in the context of bilateral bargaining, see Dasgupta and Sengupta (1993). On the role of "third party" shareholders in the context of bilateral bargaining, see Dasgupta and Tao (1998, 2000).

⁴⁵For a recent contribution on the role of financing in auctions, see Rhodes-Kropf and Viswanathan (2005).

budget constraints, *ceteris paribus*. As a consequence, the first-price auction generates higher expected revenue. Che and Gale (1998) allow for financial constraints that are more general than we have considered here: for example, these could take the form of a marginal cost of borrowing that is increasing in the amount of the loan.

4.11 Advanced Econometrics of Auction Data

There has been considerable progress in the application of econometric techniques to auction data. While the datasets used in these studies do not cover corporate finance directly, the techniques used should be of interest to corporate finance researchers, as they may be applicable to financial datasets and help resolve certain key issues. One broad topic that has been covered in empirical auction studies and that also appears in corporate finance are auctions with one informed bidder and numerous uninformed bidders. In corporate finance, such a situation could reasonably be assumed when current management is allowed to bid for a corporation, either in a takeover or bankruptcy context. Certainly in bankruptcy one concern has been that management, if allowed to bid in an auction, may be able to purchase the corporate assets at less than fair value. Hendricks and Porter (1988, 1993) have studied US government oil lease auctions of so-called “drainage” tracts - tracts that have a neighboring tract currently under lease to one of the bidders. For these drainage tracts, it is reasonable to assume that the owner of the neighboring tract would have better information than other bidders. The authors of several studies have found this assumption, and the related equilibrium bidding theory, to be consistent with the data. The econometrics used relies heavily on the underlying auction theory. For example, equilibrium with one informed bidder imposes restrictions on the distributions of the informed bidder’s bid distribution and the uninformed bidders’ bid distributions. Note that a test of this type requires that data on all bids be available.

Structural models are also being used successfully to examine auction data. The most exciting approach here is to use equilibrium theory in conjunction with data on all bids to estimate the underlying probability distribution of the valuations of bidders. The essence of the idea here is that an equilibrium bid function maps a valuation to a bid. If data on bids are available, then with suitable econometrics one can recover the distribution of

the underlying valuations from the bid data. Li, Perrigne and Vuong (2002) provide a step-by-step guide to structural estimation of the affiliated private value auction model. One aim of this work in the economics literature has been to estimate the optimal selling mechanism for a real auction. For example, if valuations are affiliated, then revenue equivalence no longer holds. Also, the optimal reserve price depends upon the underlying distribution of values, so if that distribution can be estimated, we can also get an estimate of the optimal reserve price. Researchers in empirical corporate finance should be aware of the progress made in structural estimation of auctions, for some of the issues at the heart of finance auctions may be resolved through structural estimation (and in some finance auctions, there should be data on all bids). For example, in the bankruptcy area, questions of reserve prices and informational rents abound, and these are two issues that structural estimation can get at.⁴⁶

5 Conclusion

Upon reflection, the accomplishments of auction theory are really quite amazing. The black box of the Walrasian auctioneer has been opened, studied in depth, and its perfection questioned. We can now say a lot about the process of actual price formation in many real markets. While modelers have been able to explore theoretically important topics such as revenue comparisons across auctions, their work has also enabled economists to consult with governments on the design of optimal auctions to sell public assets. And with only a slight time lag, empirical work in auctions is following in the footsteps of theory, with structural estimation methods setting a new standard for creativity and rigor. Similar to the way that theoretical developments made their way into the real world of auction design, empirical work is focusing on

⁴⁶In the context of takeover auctions, Betton and Eckbo (2000) pursue an interesting line of empirical research. A takeover contest typically associated with an “event tree” beginning with the initial bid, possibly followed by the appearance of rival bidders, until the eventual success or failure of the initial bid. The market reaction to a bid (or indeed, at reaching any node) therefore represents the sum of the product of the probabilities of all subsequent events in the tree emanating from that node, and the associated payoffs. Since the probabilities and market reactions can be estimated, the payoff implications associated with the events (the “market prices”) can be estimated. Betton and Eckbo (2000) find generally significant effects for the target, but less significant effects for the bidders.

real world auctions such as those found on Ebay and other online auctions. There are not too many topics in economics that allow researchers to cover such a broad swath of analytical territory, from the highly theoretical to the highly empirical and practical. In this way, auction theory resembles developments in financial asset pricing, where for instance the development of the option pricing model led to a surge in theoretical and empirical work while at the same time the model was applied in real markets.

The application of auction theory in corporate finance really needs to be seen as the intersection of two fields, that of auction theory and of information-based corporate finance theory. Nobody should have been surprised to see auction theory have a bit of a field day in being applied to topics in corporate finance, and as we think this survey shows, this is clearly what has happened and continues to happen. The question before us, however, must be: what have we learned in the process? That there has been considerable learning cannot be doubted, with the most significant learning being in interpreting the returns to bidders and targets in the market for corporate control, and in understanding the real institutional practices used in financial markets, such as underpricing in the IPO market, non-cash bids in takeover markets, and the role of asymmetries and discrimination against selected bidders. Perhaps the single best measure of auction theory's influence in corporate finance is that most PhD courses in corporate finance will include several papers, if not an entire module, on applications of auctions. As even a superficial study of auctions requires a fair amount of knowledge of game theory, the inclusion of auctions in PhD finance courses reinforces the study of games, itself a critical component of modern finance.

While auction theory deserves much credit for its inroads into corporate finance, two areas of concern do emerge. First, there are some phenomena in corporate finance for which we still lack sufficient understanding, and where one might have expected auction theory to lead the way. Yes, we have increased our understanding of returns to bidders and targets in the market for corporate control, but why are acquirers' returns so small? Any auction with heterogeneity of valuations or information leads to strategic behavior and expected profits for inframarginal bidders. And why do acquirers seem to do better when acquiring private companies? There is still a huge question as to whether auctions in bankruptcy are better than a court-supervised valuation and division of assets. Why are toeholds so seldom taken, if they

lead to a bidding advantage? If auctions really are so good, why are they used so infrequently in the initial public offering market, and why do some sellers of companies bypass an auction in favor of a one-on-one negotiation?

The second unsatisfactory aspect of auctions in corporate finance is simply that no new fundamental insights have emerged. We do understand better how information, values, and strategic behavior combine to yield prices and allocations of assets in real financial markets. There has been no quantum leap forward, just incremental learning at the margin. This should, we suppose, actually be gratifying, for it shows the robustness of our primitive and most cherished assumptions. Unfortunately, at times the models that are developed and that are pushing back the frontier only marginally are incredibly complicated, and one has to wonder if the complexity is worth it. One doubts that quantum leaps in knowledge are going to come from models that need a myriad of questionable assumptions.

Where next for auctions in corporate finance? We would suggest three areas for focus. First, data will be key for further empirical discovery, and this could in turn lead to new theoretical developments. We believe that auctions of private companies and auctions in bankruptcy are two areas that may yield significantly better data in the future and where the returns to clever empirical work would be large. Second, on the theoretical side, it is clear that some of the best work to date has been on what might appear as the second-order institutional practices, such as non-cash bids, toeholds, bidder discrimination, and reserve prices. Much progress has been made in understanding the role of these practices, while at the same time reinforcing the importance and validity of the overall auction-based framework. Third, we would like to see more work done with non-standard informational and valuation assumptions. The general symmetric model is extremely powerful, but does it really capture many of the real settings that we observe? We should expect that heterogeneity of bidders will be manifested in many ways and will turn out to affect the equilibria quite strongly, especially in regard to bidders profits. Efficiency of the allocation will also become inherently more interesting a question, and initial work suggests it will be harder to achieve.

We would confidently make the prediction, though, that auctions in corporate finance will be a much-studied topic for years to come. Our very

strong recommendation would be for all PhD students to get a thorough grounding in auction theory.

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