1. Introduction

The focus of behavioral pricing and revenue management is incorporating realistic consumer behavior theories when designing pricing and inventory policies. These theories depart from strict economic rationality—the cornerstone of the expected utility framework—and introduce behavioral regularities, heuristics, and cognitive emotions into the consumer decision-making process. Yet the input to firms’ pricing models is typically not the individual consumer behavior but rather market demand and how it responds to discounts and surcharges. To bridge this gap, pricing models assume that market demand inherits consumer-level behavior. According to prospect theory, for example, consumers are “loss averse” and so react more strongly to losses than to gains of the same magnitude (Kahneman and Tversky 1979). This behavioral bias is often generalized to conclude that likewise market demand is more sensitive to losses than to gains (see e.g. Fibich et al. 2003, Popescu and Wu 2007, Nasiry and Popescu 2011, Chen et al. 2015). We shall demonstrate that
this approach is not correct and that loss aversion at the consumer level does not imply market demand is more responsive to losses than to gains. In fact there is no general relation between the two, and a market consisting of (say) loss-neutral consumers could be more responsive to gains (or to losses) or be equally responsive.

We consider a market that consists of consumers with heterogeneous valuations for a product. A consumer’s surplus from a purchase includes not only the economic surplus but also a psychological surplus; the latter depends on the difference between the product’s price and a reference price, and it captures how the consumer feels about the transaction. So for a given price and reference price, market demand is the fraction of consumers with a nonnegative surplus. We show that the distribution of the valuations, which captures the market’s heterogeneity, determines how demand responds to surcharges versus discounts. For loss-neutral consumers, we find that overall demand is more sensitive to losses than to gains only when the market consists mainly of high-valuation consumers. When most consumers are bargain hunters, however, overall demand will be more sensitive to gains than to losses. Finally, if the valuations are uniformly distributed then market demand is equally sensitive to gains and losses.

We also characterize the conditions under which the market demand of loss-averse consumers is more sensitive to losses than to gains—that is, when overall demand does inherit the loss aversion property of individual consumer demand. We prove that only uniform valuation distributions are capable of faithfully transmitting the individual-level behavior (whether loss averse, loss-neutral, or gain seeking) to the overall demand function.

Our paper contributes to the literature on behavioral pricing and revenue management by demonstrating that psychological biases are individual-level phenomena and that aggregate market demand need not inherit those biases. This finding cautions against applying behavioral pricing models more generally and suggests that a careful approach is warranted when incorporating behavioral biases at the aggregate level. Such caution is necessary because implications for optimal pricing differ significantly. Fibich et al. (2003), Popescu and Wu (2007), and Nasiry and Popescu (2011) all show that, if market demand is more responsive to losses than to gains, then firms should charge constant long-run prices. Conversely, Hu et al. (2016) show that if demand is instead more responsive to gains then firms should adopt cyclic pricing policies. Our work specifies when each of these models is applicable depending on the market heterogeneity. In its theoretical contribution, our paper is closest to Kallio and Halme (2009); those authors assume a random consumer utility model and develop a conditional logit demand function for each product in a category. They show that a product’s market share determines how demand for the product responds to price changes.

1 For similar modeling approaches see Kőszegi and Rabin (2006), Nasiry and Popescu (2012), and Liu and Shum (2013).
and that the response may be stronger when the price decreases than when it increases. Our core insight resonates with that of Stoker (1993) on “aggregation problem” in empirical economic research that “Neglecting distributional features undercuts the foundation of any equation based entirely on aggregate variables. [. . . ] models need to account for heterogeneity and the composition of the population explicitly” (page 1836).

This paper also adds to a large literature in marketing that attempts to prove or disprove the existence of reference effects and establish whether, at the individual level, there is an asymmetric response to losses and gains. Although the existence of reference effects has been widely confirmed, the evidence for loss aversion is mixed (Mazumdar et al. 2005). Neslin and van Heerde (2009) review the empirical marketing literature on reference prices and conclude “. . . several crucial issues are unresolved. . . . we need to resolve the issue of loss aversion, i.e., do indeed losses loom larger than gains? The evidence is frustratingly mixed” (Section 5.4, p 49-50). Some empirical studies find support for a stronger response to losses (for a review, see Kalyanaram and Winer 1995), but others report a reduced effect or the total absence of loss aversion. Chang et al. (1999), Bell and Lattin (2000), and Kopalle et al. (2012) argue that loss aversion may not exist at the individual level if one accounts for consumer (household) heterogeneity. Krishnamurthi et al. (1992) find no evidence for loss aversion among loyal consumers and also find that non-loyal consumers (“switchers”) seem actually to be gain seeking. Greenleaf (1995) is one of only a few to estimate a market demand function that incorporates a reference price for the whole market, and he finds evidence that gains have a greater effect than losses at the aggregate level; similar findings are reported in Slonim and Garbarino (2009). Our work emphasizes that, whether or not consumers are loss averse, market demand may be either more or less responsive to losses than to gains. In other words, the empirical evidence that overall demand is more responsive to gains does not trouble the loss-averse behavior at the individual level: “[w]hat seem to be phenomena which cannot exist together are in reality phenomena which can exist together” (Davis 1971, p. 324).

In the remainder of the paper, we present the consumer behavior model and discuss the aggregation process to construct the market demand (Section 2). We then show that there is no general relation between how an individual demand function and an aggregate, market demand function respond to gains or losses. We demonstrate how the key messages of two influential pricing papers (Ho and Su (2009) and Popescu and Wu (2007)) change if one takes into account the aggregation process we develop in this paper (Section 3). We conclude the paper in Section 4.

2. Consumer Behavior Model and Market Demand
The market consists of consumers with heterogeneous valuations for a product. A consumer’s privately known valuation is \( v \); the firm knows only that the valuations are continuously distributed
over an interval with density \( f(\cdot) \) and cumulative distribution \( F(\cdot) \). We assume \( F(\cdot) \) to be continuously differentiable on its support. A heavy left tail of the density function implies that the market has many price-sensitive consumers (bargain hunters) whereas a heavy right tail implies that high-valuation consumers make up most of the market.

A consumer’s utility consists of two parts: an economic surplus and a gain/loss surplus. The economic surplus is the utility derived from acquiring or consuming the product; for a given consumer valuation \( v \) and product price \( p \), that surplus is equal to \( v - p \). The gain/loss surplus captures how the consumer feels about the transaction; this surplus is defined as the gap \( x = r - p \) between the price and the reference price \( r \). We use a general value function of the following form:

\[
t(x) = \begin{cases} 
  m(x), & x > 0; \\
  -\lambda m(-x), & x \leq 0.
\end{cases}
\]  

(1)

Here \( m(x) : \mathbb{R}^+ \to \mathbb{R}^+ \) is continuous and strictly increasing with \( m(0) = 0 \). A consumer is loss averse if \( \lambda > 1 \), is gain/loss neutral if \( \lambda = 1 \), and otherwise is gain seeking. The consumer’s overall surplus is \( u(x, p) = v - p + t(x) \); she purchases the product when \( u(x, p) \geq 0 \) or, equivalently, when \( v \geq p - t(x) \). We assume consumers are homogenous in the strength of their gain/loss feelings and share the same reference price. Hence market demand is

\[
D(x, p) = \text{Prob}(v \geq p - t(x)) = 1 - F(p - t(x)),
\]  

(2)

where we have normalized the market size to 1. Following Popescu and Wu (2007), we define the reference effect on demand as

\[
R(x, p) = D(x, p) - D(0, p) = F(p) - F(p - t(x)).
\]  

(3)

The function \( R(x, p) \) measures the change in demand at price \( p \) due to the discount or surcharge \( x \). We shall use this function when defining the notion of demand sensitivity to gains and losses.

**Definition 1.** Assume \( p \) is given. For all \( x > 0 \), demand is (i) more responsive to losses if \( R(x, p) < -R(-x, p) \), (ii) equally responsive to gains and losses if \( R(x, p) = -R(-x, p) \), or (iii) more responsive to gains otherwise.

We use the terms loss averse, gain/loss neutral, and gain seeking to describe individual-level behavior and adopt the terminology stipulated in Definition 1 for the demand function. The distinction is appropriate given that prospect theory describes consumer-level but not aggregate-level responses to gains and losses.

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2 Piecewise linear value functions and power value functions are special cases of (1) with, respectively, \( m(x) = \eta x \) and \( m(x) = \eta x^d \) (\( 0 < d < 1 \)); here \( \eta \geq 0 \) captures the strength of reference effects. These two particular forms are widely applied in the literature (see e.g. Tversky and Kahneman 1992, K"oszegi and Rabin 2006, Heidhues and K"oszegi 2008, Courty and Nasiry 2016).
Definition 1 requires that the demand response satisfy a global condition because the inequalities must hold for any $x > 0$. This requirement is strong, and alternative definitions based on local properties have been proposed. A convenient property of the value function given by (1) is that the global and local definitions of individual demand responsiveness coincide and therefore leave the results in this section unaffected. Local definitions are discussed in Section 2.2.

Finally, we remark that Definition 1 describes the asymmetry of demand response to gains and losses for a particular price $p$. That is, for any $x > 0$ we compare the demand response at two reference prices, $p + x$ and $p - x$. A change in price, on the other hand, would affect demand directly through $D(0, p)$, along with having a reference effect through $R(x, p)$, and so this approach cannot cleanly measure demand sensitivity to gains and losses—which is the focus of our research.

For an illustration of how this framework works, assume that $v$ is uniformly distributed on $[0, b]$ for $b > 0$. By (2) we then have that $D(x, p) = 1 - \frac{1}{b}p + \frac{1}{b}t(x)$ and so, by (3), $R(x, p) = \frac{1}{b}t(x)$. Hence for any $x > 0$, if consumers are gain/loss neutral then $R(x, p) = \frac{1}{b}m(x) = -R(-x, p)$ but if consumers are loss averse then $R(x, p) = \frac{1}{b}m(x) < -R(-x, p) = \frac{1}{b}\lambda m(x)$; similarly, if consumers are gain seeking then $R(x, p) = \frac{1}{b}m(x) > -R(-x, p) = \frac{1}{b}\lambda m(x)$. We conclude that if valuations are uniformly distributed then the characteristics of consumer behavior are inherited by market demand. In fact, uniform distributions are the only class of distributions that transfer individual-level behavior to market demand in this way, as we prove in our first result.

**Proposition 1.** Market demand inherits consumers’ psychological biases if and only if consumer valuations are uniformly distributed.

Proposition 1 implies that market demand will not exhibit individual demand biases if valuations are distributed in any form other than uniform. Our paper’s main result follows: it identifies general conditions under which market demand is more responsive to losses or to gains (or is equally responsive to both).

**Proposition 2.** For all $x > 0$, the following statements hold: (i) if $2F(p) < F(p - m(x)) + F(p + \lambda m(x))$, then demand is more responsive to losses; (ii) if $2F(p) > F(p - m(x)) + F(p + \lambda m(x))$, then demand is more responsive to gains; (iii) if $2F(p) = F(p - m(x)) + F(p + \lambda m(x))$, then demand is equally responsive to gains and losses.

We discuss the implications of Proposition 2 for gain/loss-neutral consumers in Section 2.1 and for loss-averse consumers in Section 2.2. Throughout the paper, we use convexity and monotonicity properties in their weak sense unless indicated explicitly otherwise.
2.1. Reference Effects and Market Demand

Assume that consumers are prone to reference effects but are gain/loss neutral ($\lambda = 1$). In this case, Proposition 2 has an intuitive interpretation that relates the market response to properties of the distribution function.

**Corollary 1.** Suppose $\lambda = 1$. Then demand is: (i) more responsive to losses if and only if $F(\cdot)$ is strictly convex; (ii) more responsive to gains if and only if $F(\cdot)$ is strictly concave; and (iii) equally responsive to gains and losses if and only if $F(\cdot)$ is linear.

Corollary 1 implies that if the market consists mainly of high-valuation consumers then demand is more responsive to losses whereas demand responds more to gains if consumers are instead most bargain hunters. We illustrate this result using some commonly applied valuation distributions.

**Example 1 (Exponential Demand).** Assume that $\lambda = 1$ and that consumer valuations are distributed exponentially with mean $\mu$. Also, let $m(x) = \eta x$; that is, let the value function be piecewise linear (see (1)). Then $D(x,p) = \exp((-p + \eta x)/\mu)$ and $R(x,p) = \exp((-p + \eta x)/\mu) - \exp(-p/\mu)$. Since $F(\cdot)$ is strictly concave (see the right panel in Figure 1), it follows from Corollary 1 that demand is more responsive to gains than to losses. The left panel in Figure 1 illustrates $R(x,p)$ as a function of $x$ for mean valuations 0.25, 1, and 4. This figure shows that demand is generally more responsive when consumers are given the perception of a gain ($x > 0$) in the purchase transaction. As the mean valuation decreases, the market becomes dominated by low-valuation consumers and hence demand becomes less responsive to losses yet highly responsive to gains—and even more so for larger gains. At the extreme, demand is practically unaffected by losses when $\mu = 0.25$. In that case, Assumption 1 in Hu et al. (2016)—that the market demand is insensitive to losses—becomes plausible.

![Figure 1](image-url)
Example 2 (Logit Demand). Suppose $\lambda = 1$, and let valuations be distributed according to the logistic distribution $F(v) = 1/(1 + \exp(-(v - \mu)))$ with mean $\mu^3$. Let $m(x) = \eta x$. Then

$$D(x,p) = \frac{\exp(\mu - p + \eta x)}{1 + \exp(\mu - p + \eta x)} \quad \text{and} \quad R(x,p) = \frac{\exp(\mu - p + \eta x)}{1 + \exp(\mu - p + \eta x)} - \frac{\exp(\mu - p)}{1 + \exp(\mu - p)}.$$

The logit demand accounts for uncertainty in consumer choice behavior and is widely applied in empirical studies to verify reference price models and the loss aversion of consumers (see e.g., Krishnamurthi et al. 1992). The distribution $F(\cdot)$ is neither convex nor concave, so no general statement can be made concerning the reference effect on demand. However, $F(\cdot)$ is strictly convex when $v < \mu$ and strictly concave when $v > \mu$. So given $p$, if $\mu$ is relatively small (i.e., if $\mu < p$) then $F(p)$ lies on the concave part of $F(\cdot)$. Therefore, for $r$ in the vicinity of $p$ where $F(\cdot)$ is concave, it follows from Corollary 1 that demand is more responsive to gains. Likewise, if $\mu$ is relatively large (i.e., if $\mu > p$) then $F(p)$ lies on the convex part of $F(\cdot)$ and demand is more responsive to losses for sufficiently small $x$, that is, for $r$ in the vicinity of $p$. The intuition is similar to our previous example. This point is illustrated in Figure 2, where the price $p = 5$. If $\mu = 1$, then most consumers have valuations lower than the price $p$ and hence demand responds strongly to gains. In comparison, if $\mu = 9$ then a significant fraction of consumers have valuations higher than the price $p$ and so demand tends to be more responsive to losses.

These findings are related to those reported by Kallio and Halm (2009), who examine how a conditional logit demand responds to gains and losses. Their main result is that if the market share of a product is below a certain threshold then its demand is more responsive to gains and that

\[^3\text{We let the scale parameter be 1.}\]
otherwise it is more responsive to losses. Example 2 is consistent with their observations: given price \( p \) if \( \mu = 1 \) then \( F(p) \) is high and hence the choice probability \( 1 - F(p) \) (which captures the market share of the product) is low and so the demand is more responsive to gains. However, our results suggest that market share is not an essential factor in predicting the demand response. Example 1 shows that, if consumer valuations are exponentially distributed then, irrespective of \( 1 - F(p) \), demand will necessarily be more responsive to gains than to losses. What drives the asymmetric nature of demand response is the mass of consumers with valuations higher than \( p \) relative to that with lower valuations. Thus, if there are more consumers with valuations below (resp. above) \( p \)—that is, if \( f \) is strictly decreasing (resp. increasing) around \( p \)—then demand tends to be more responsive to gains (resp. losses).

2.2. Consumer Loss Aversion and Market Demand

In Section 2.1 our analysis established that, when consumers are subject to reference effects but not loss aversion, the heterogeneity in consumer valuations determines the demand response to gains and losses. Yet market demand response is affected also by the the asymmetry in individual consumer’s response to gains and losses when consumers are loss averse—that is when \( \lambda > 1 \). To proceed in this case, we start by formally defining the concept of local demand response to losses and gains. To ensure the local properties are well-defined, we assume in this section that \( m(x) \) is continuously differentiable for \( x \geq 0 \).

**Definition 2.** Let \( \lambda_- \) and \( \lambda_+ \) be the left and right derivatives of \( R(x,p) \) with respect to \( x \) at \( x = 0 \), and put \( \rho = \lambda_- / \lambda_+ \). We say that demand is locally more responsive to losses if \( \rho > 1 \), locally more responsive to gains if \( \rho < 1 \), or locally responsive to gains and losses equally if \( \rho = 1 \).

The local property in Definition 2 follows from the definition of loss aversion at the individual level suggested by Köbberling and Wakker (2005), who refer to \( \rho \) as the *index of loss aversion*: a quantitative measure of how loss averse consumers are. This approach considers how consumers behave only in the vicinity of \( x = 0 \)—that is, for only small deviations of the reference price from the product’s price. Popescu and Wu (2007) also adopt this definition to describe the behavior of market response.

There is ongoing debate over whether a global or rather a local definition of loss aversion is more appropriate for describing consumer behavior (Abdellaoui et al. 2007). In this paper we abstract from that debate because, for the value function in (1), the global and local definitions of loss aversion coincide. However, that coincidence obtains only with respect to consumer behavior and not to aggregate demand behavior. In other words, neither Definition 1 nor Definition 2 implies the

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4 The threshold is determined by assuming that a consumer’s valuation of a product bears an inverse relation to its price; that is, the valuation is lower when the price is higher. In the Kallio and Halme (2009) model, if this assumption is violated then consumer loss aversion always translates into a stronger demand response to losses than to gains.
other. In Example 1, for instance, demand is more responsive to gains (by Definition 1) even though clearly \( \rho = 1 \) and so (by Definition 2) demand is locally responsive equally. Similarly, demand may be locally but not globally more responsive to losses, as will be seen in Example 3.

Our next result shows how behavioral biases of consumers when evaluating gains and losses affect the demand response.

**Proposition 3.** (i) The local demand responsiveness to gains and losses inherits the individual consumers’ responses to gains and losses. (ii) Demand is globally more responsive to losses when \( \lambda > 1 \) and \( F(\cdot) \) is convex.

Part (i) states that consumers’ behavioral biases determine the demand function’s local behavior. That is, if consumers are loss averse (resp., gain seeking) then market demand will be locally more responsive to losses (resp., gains). Part (ii) states that if the market consists of loss-averse consumers of whom most have high valuations, then demand is also globally more responsive to losses. The combination of parts (i) and (ii) leads us to conclude that, if price-insensitive (high-valuation) and loss-averse consumers constitute the majority in a market, then aggregate demand is both globally and locally more responsive to losses.

Our next example will demonstrate that, even if all consumers are loss averse, market demand may not be more responsive to losses. The example first shows that the inequality \( R(x,p) < -R(-x,p) \) does not hold for all \( x > 0 \) and second that the reverse inequality \( R(x,p) > -R(-x,p) \) does hold for most \( x > 0 \) (except for a small neighborhood of \( x = 0 \)). In other words, demand is more responsive to gains than to losses when the magnitudes of gains and losses are sufficiently large. This dynamic motivates the following definition.

**Definition 3.** If there exists an \( \epsilon > 0 \) such that \( R(x,p) < -R(-x,p) \) for any \( x > \epsilon \), then demand is said to be more responsive to losses at large stakes; if \( R(x,p) > -R(-x,p) \) for any \( x > \epsilon \), then demand is more responsive to gains at large stakes; and if \( R(x,p) = -R(-x,p) \) for any \( x > \epsilon \), then demand is equally responsive to gains and losses at large stakes.

Definition 3 is weaker than Definition 1 in the sense that we require the corresponding inequality to hold only for \( x \) larger than some \( \epsilon > 0 \). Our next example illustrates Definition 3 and shows that consumer loss aversion is not sufficient to determine the global responsiveness of demand.

**Example 3.** Suppose that \( \lambda > 1 \) and that valuations are exponentially distributed with mean \( \mu \). According to Example 1, \( D(x,p) = \exp((-p + t(x))/\mu) \) and \( R(x,p) = \exp((-p + t(x))/\mu) - \exp(-p/\mu) \). First, observe that the inequality \( 2F(p) < F(p - m(x)) + F(p + \lambda m(x)) \) or, equivalently, \( 2(1 - \exp(-p/\mu)) > 1 - \exp((-p+m(x))/\mu) + 1 - \exp((-p-\lambda m(x))/\mu) \) implies that \( \exp(m(x)/\mu) + \exp(-\lambda m(x)/\mu) > 2 \); because this expression clearly does not hold for all \( x > 0 \), it follows from Proposition 2 that the demand function is not globally more responsive to losses. Second, because
\(m(\cdot)\) is continuous and strictly increasing, its inverse \(m^{-1}(\cdot)\) exists and is well-defined. Let \(\varepsilon = m^{-1}(\mu \log 2)\). Then for any \(x > \varepsilon\) we have \(\exp(m(x)/\mu) + \exp(-\lambda m(x)/\mu) > \exp(m(\varepsilon)/\mu) = 2\), which means (by Definition 3) that market demand is more responsive to gains at large stakes. As the mean consumer valuation decreases, \(\varepsilon\) also decreases because \(m^{-1}\) is strictly increasing. So regardless of how loss averse consumers are, if their mean valuation is sufficiently low then demand is more responsive to gains for almost all \(x > 0\) (except in a tiny neighborhood of \(x = 0\)).

The left panel of Figure 3 plots the shape of \(R(x, p)\) for \(\mu = 1/2\) and \(\lambda = 2\). The two dashed lines at the origin are \(\lambda_- x\) and \(\lambda_+ x\), indicating the local sensitivity of \(R(x, p)\) with respect to \(x\). In this example, \(\rho = \lambda = 2\) and so the implication is that demand is locally more responsive to losses. Hence for \(x\) sufficiently small, \(\lambda_- x\) and \(\lambda_+ x\) can be used to approximate the change in \(R(x, p)\) and one will have \(R(x, p) < -R(-x, p)\). Yet the left panel of Figure 3 reveals that, at large stakes (say, for \(x \geq 1/2\)), gains have a far greater effect than do losses.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3}
\caption{Actual and estimated reference effect with \(\mu = 1/2\) and \(\lambda = 2\)}
\end{figure}

Compared to the local behavior, the responsiveness of market demand at large stakes has important implications for empirical studies, especially when \(\varepsilon\) is small. The right panel of Figure 3 illustrates the case when one attempts to use a piecewise linear demand function to fit the underlying market response data (Greenleaf 1995). The assumption in such empirical estimation is that observed demand is the mean demand \(D(x, p)\) plus a zero-mean normal random noise. The data points in the right panel of Figure 3 are generated according to the market response function in the left panel plus a normal random noise. Clearly, a piecewise linear function with a downward kink at the origin best fits the data, even though local sensitivity at the origin has an upward kink. So even when all the market’s consumers are loss averse, the result can still be a piecewise linear demand function that is more responsive to gains than to losses. Our results provide formal
support for the conjecture and empirical result of Greenleaf that “this market-level result [demand being more responsive to gains] need not contradict previous results that the reverse relationship holds for purchases at the household level” (1995, p. 90).

3. Applications in Pricing and Revenue Management

We now illustrate how our key insight applies in the settings of two behavioral pricing papers and leads to qualitatively different results.

3.1. Pricing under Fairness

Ho and Su (2009) study the economic implications of peer-induced fairness. In their basic model, a leader plays two consecutive independent ultimatum games with two followers whereby the follower in the second game receives a signal about the offer the leader made to the first follower. The second follower anchors on the first follower’s payoff and is averse to receiving less. The authors then estimate the parameters of the model experimentally and find evidence that peer-induced fairness has a significant effect on whether the second player accepts the leader’s offer.

In one application, Ho and Su discuss the implications of their findings for price discrimination when a firm sells to two markets: a low-value market and a high-value market. They argue that the price set for the low-value market serves as a reference point for consumers in the high-value market if the monopolist enters the low-value market first. They subsequently show that when demand functions in both markets are linear, the monopolist will completely eliminate price discrimination provided that consumers are sufficiently sensitive to fairness.

Fairness is a reference effect and “[t]he aversion of being behind is similar to the notion of loss aversion” (Ho and Su 2009, page 2026). A peer’s payoff is the reference point with respect to which advantageous or disadvantageous comparisons induce feelings of gain or, respectively, loss. Therefore, our results apply to models of fairness.

The main result Ho and Su (2009) put forward is that “[m]any firms charge the same price in different markets, even though the opportunity for price discrimination exists. Peer-induced fairness provides a plausible rational explanation for this phenomenon” (page 2040). However, this insight critically hinges on the assumption that the overall demand functions in both low- and high-value markets are linear which implies that consumers’ valuations in both markets are uniformly distributed. By our Proposition 1, the market demand then inherits the psychological bias of the consumers. If we relax the uniform valuation assumption then the result in Ho and Su (2009) does not hold: uniform pricing may never be optimal with fairness-minded consumers.

To see this, consider a setting similar to Ho and Su (2009). A firm sells in two markets. Consumer valuations in the two markets, $v_1$ and $v_2$, are independent with distributions $F_1(\cdot)$ and $F_2(\cdot)$, probability density functions $f_1(\cdot)$ and $f_2(\cdot)$, and mean valuations $\mu_1$ and $\mu_2$, respectively. We
assume \( v_1 \) is strictly less than \( v_2 \) in hazard rate order, i.e., \( \frac{f_1(x)}{1 - F_1(x)} > \frac{f_2(x)}{1 - F_2(x)} \), \( x \geq 0 \). That is, we can think of market 1 as the low-value market and market 2 as the high-value market. The hazard rate order implies that \( \mu_1 < \mu_2 \). For simplicity, we let the marginal production cost at both markets be zero. In absence of fairness concerns, the optimal prices at the two markets satisfy \( p_i^* f_i(p_i^*) = 1 - F_i(p_i^*) \), \( i = 1, 2 \), and by hazard rate order \( p_1^* < p_2^* \).

In a slightly more general setting than Ho and Su (2009), we assume that a consumer in the high-value market experiences not only a feeling of loss if he pays more than his peers in the low-value market but also a feeling of gain if he pays less. That is, his utility upon a purchase is \( v = p_2 + \lambda (p_1 - p_2)^- + \gamma (p_1 - p_2)^+ \) where \( z^+ = \max\{z, 0\} \) and \( z^- = \min\{z, 0\} \). By prospect theory, \( \lambda > \gamma \), i.e., a disadvantageous comparison with peers has a larger absolute impact on the consumer’s utility that an advantageous comparison. We first show that the uniform pricing observed by Ho and Su (2009) generalizes to this setting if the valuations in the two markets are uniformly distributed.

Let \( v_1 \) and \( v_2 \) be uniformly distributed on \([0, 2\mu_1]\) and, respectively, \([0, 2\mu_2]\). In absence of peer-induced fairness, the optimal prices in the two markets are \( \mu_1 \) and \( \mu_2 \). The firm’s profit maximization problem, when incorporating fairness, becomes

\[
\max_{p_1, p_2} \Pi^\mu(p_1, p_2) = p_1 \frac{2\mu_1 - p_1}{2\mu_1} + p_2 \frac{2\mu_2 - p_2 + \gamma (p_1 - p_2)^+ + \lambda (p_1 - p_2)^-}{2\mu_2}.
\]  
(4)

Problem (4) is not a smooth optimization problem. To proceed, we first define two smooth problems:

\[
\max_{p_1, p_2} \Pi^\gamma(p_1, p_2) = p_1 \frac{2\mu_1 - p_1}{2\mu_1} + p_2 \frac{2\mu_2 - p_2 + \gamma (p_1 - p_2)}{2\mu_2};
\]  
(5)

\[
\max_{p_1, p_2} \Pi^\lambda(p_1, p_2) = p_1 \frac{2\mu_1 - p_1}{2\mu_1} + p_2 \frac{2\mu_2 - p_2 + \lambda (p_1 - p_2)}{2\mu_2}.
\]  
(6)

Assume (5) and (6) are concave\(^5\) and observe that the optimal solutions to (5) and (6) are, respectively,

\[
(p_1^\gamma, p_2^\gamma) = \left( \frac{2\mu_1 + \frac{\gamma}{1 + \gamma} \mu_1}{2 - \frac{\gamma^2}{2(1 + \gamma)} \mu_1}, \frac{2\mu_2 + \frac{\gamma}{1 + \gamma} \mu_2}{2 - \frac{\gamma^2}{2(1 + \gamma)} \mu_2} \right), \quad (p_1^\lambda, p_2^\lambda) = \left( \frac{2\mu_1 + \frac{\lambda}{1 + \lambda} \mu_1}{2 - \frac{\lambda^2}{2(1 + \lambda)} \mu_1}, \frac{2\mu_2 + \frac{\lambda}{1 + \lambda} \mu_2}{2 - \frac{\lambda^2}{2(1 + \lambda)} \mu_2} \right).
\]

The following proposition then characterizes the solution to (4).

**Proposition 4.** (i) If \( \frac{\mu_2}{\mu_1} \leq \gamma + 1 \) then \((p_1^*, p_2^*) = (p_1^\gamma, p_2^\gamma)\); (ii) if \( \frac{\mu_2}{\mu_1} \geq \lambda + 1 \) then \((p_1^*, p_2^*) = (p_1^\lambda, p_2^\lambda)\); (iii) otherwise \( p_1^* = p_2^* = \frac{2\mu_1 \mu_2}{\mu_1 + \mu_2} \).

Proposition 4 confirms the major result in Ho and Su (2009): when \( \lambda \) is sufficiently large, the monopolist charges one price across the two markets. The underlying reason is that consumers

\(^5\)Both functions are concave if \( 4(1 + \lambda) - \frac{\mu_2}{\mu_1} \lambda^2 > 0 \). A similar condition is assumed implicitly in Ho and Su (2009).
in the high-value market are averse to being treated unfairly, and by Proposition 1, this bias is inherited by the aggregate demand under uniform valuations. Therefore, when the demand in the high-value market is sufficiently sensitive to unequal prices, the monopolist finds it optimal to forego price discrimination.

Proposition 4 also implies that under uniform valuations, the optimality of uniform pricing depends on the relative mean valuations $\mu_2/\mu_1$ and not the absolute magnitude of the mean valuations. This is not the case in general as we show next.

Suppose now that consumers’ valuations at the two markets are exponentially distributed. For comparison purposes, we keep the mean valuations unchanged so that $\mu_1$ and $\mu_2$ are still the optimal prices at the two markets in absence of fairness concerns. The monopolist’s profit maximization problem, when incorporating fairness, becomes

$$\max_{0 \leq p_1, p_2 \leq \bar{p}} \sum_{t=0}^{\infty} \beta^t p_t D(r_t - p_t, p_t),$$

where $\bar{p}$ is the upper bound on prices and $\bar{p} > \mu_2/(1 + \gamma)$ to ensure $p_2^* = \mu_2/(1 + \gamma)$.

**Proposition 5.** There exists a $\bar{\mu} > 0$ such that for any $\mu_2 < \bar{\mu}$, the optimal solution to Problem (7) is $p_1^* = \bar{p}$ and $p_2^* = \mu_2/(1 + \gamma)$.

Proposition 5 implies that, instead of uniform pricing, the monopolist should price out the low-value market and fully cater to the high-value market. From Example 3, one can see that when $\mu_2$ is sufficiently small, and regardless of how loss-averse consumers are, the demand in market 2 is more responsive to gains (price markdowns) than to losses (price markups). Hence, it is more profitable for the monopolist to price discriminate and exploit the demand boost from price markdown in the high-value market.

### 3.2 Dynamic Pricing With Loss-Averse Consumers

Suppose a monopolist sells a product over an infinite horizon in a market with loss-averse consumers. The firm’s problem is to set the price in each period, $p_t$, taking into account that the demand is sensitive to the price history through a reference price. Assume the reference price $r_t$ follows an exponential smoothing process $r_t = \alpha r_{t-1} + (1 - \alpha)p_{t-1}$, $t \geq 1$, where $\alpha \in [0, 1)$ is a memory parameter which captures how fast consumers adapt to recent prices. Given the demand in period $t$, $D(r_t - p_t, p_t)$, the dynamic pricing problem of the firm is

$$\max_{p_t \in [0, \bar{p}], \beta \geq 0} \sum_{t=0}^{\infty} \beta^t p_t D(r_t - p_t, p_t),$$

where $\bar{p}$ is the upper bound on prices and $\bar{p} > \mu_2/(1 + \gamma)$.
where $\beta \in [0, 1)$ is a discount factor and $\bar{p}$ is the maximum permissible price.

The dynamic pricing problem in (8) has been examined extensively in the literature (see Kopalle et al. 1996, Fibich et al. 2003, Popescu and Wu 2007, Hu et al. 2016). These papers assume the period-$t$ demand $D(r_t - p_t, p_t)$ consists of a base demand $D(0, p_t)$ and a reference effect $R(r_t - p_t, p_t)$, and impose loss aversion directly on the reference effect $R$, that is, the effect of a price surcharge on $R$ is larger than that of a price discount. Fibich et al. (2003) assume a linear base demand function and a piecewise-linear reference effect and show that the optimal price path (monotonically) converges to a constant steady state price. Popescu and Wu (2007) prove similar results for general base demand and reference effect functions in discrete time. Hu et al. (2016) assume the overall demand is more responsive to gains and demonstrate that the optimal price path cycles.

Based on our approach, in period $t$ and upon a purchase, the surplus of a consumer with valuation $v$ is $v - p_t + \eta(r_t - p_t)^+ + \lambda \eta(r_t - p_t)^-$, and hence from (2), the demand in period $t$ is $D(r_t - p_t, p_t) = 1 - F(p_t - \eta(r_t - p_t)^+ - \lambda \eta(r_t - p_t)^-)$. If consumers are loss averse (i.e., $\lambda > 1$) and their valuations are uniformly distributed then by Proposition 1, the overall demand is more sensitive to losses and therefore the results in the above papers hold.

However, we next illustrate that for an exponential valuation distribution, the key insight in Popescu and Wu (2007)—that the optimal price path converges to a steady state—may no longer hold. The firm’s profit function in period $t$ in this case is 

$$
\Pi(r_t, p_t) = p_t D(r_t - p_t, p_t) = p_t \exp \left( \frac{-p_t + \eta(r_t - p_t)^+ + \lambda \eta(r_t - p_t)^-}{\mu} \right),
$$

and the value function is $V(r_0) = \max_{p_t \in [0, \bar{p}]} \sum_{t=0}^{\infty} \beta^t \Pi(r_t, p_t)$. Because profit per stage is bounded, the value function is the unique solution to the following Bellman equation

$$
V(r) = \max_{p \in [0, \bar{p}]} \{ \Pi(r, p) + \beta V(\alpha r + (1 - \alpha)p) \}. \quad (9)
$$

We denote $p^*(r)$ as the optimal pricing strategy that solves the Bellman equation (9). A state $r$ is a steady state if $p^*(r) = r$.

The function $\Pi(r, p)$ is non-concave, non-differentiable, and nonlinear; finding the optimal policy is then challenging. Here, we assume a simpler problem where $\alpha = 0$ which implies consumers anchor only on the most recent price. We prove that when $\mu$ is not too large, there does not exist a steady state and consequently a long-run constant pricing strategy is not optimal.

**Proposition 6.** Suppose $\alpha = 0$. There exists a $\bar{\mu} > 0$ such that, for any $\mu < \bar{\mu}$, Problem (9) admits no steady state.
Proposition 6 only establishes the non-optimality of a constant pricing strategy. When $\alpha > 0$, our numerical analysis reveals complex patterns for the optimal price paths. Figure 4 compares, for $\alpha = 0.8$, the optimal price path under a uniform valuation and that under an exponential valuation with the same mean. In Figure 4, $\lambda = 2$, i.e., consumers are loss-averse. Under uniform valuations, such bias is inherited by market demand and the optimal prices monotonically converge to a constant price, which is consistent with the results in Fibich et al. (2003) and Popescu and Wu (2007). However, when valuations are exponentially distributed, the optimal price path becomes cyclic. This finding challenges the established understanding that price fluctuations antagonize loss-averse consumers. While this is true at the individual-level demand, our results prove that it does not hold at the aggregate level when the heterogeneity among consumers is accounted for.

4. Conclusion

This paper establishes that psychological biases such as loss aversion are individual-level phenomena and are not necessarily inherited by aggregate market demand. We showed that market demand may be more sensitive to gains than to losses even if consumers are loss averse. As a result, dynamic pricing may antagonize a loss-averse consumer but is not necessarily detrimental to a firm in a market of loss-averse consumers. What drives this result is the heterogeneity in how consumers

\footnote{We observe numerically that for $\alpha = 0$ and $\mu$ sufficiently small, a high-low pricing strategy is optimal. However, we leave a formal proof as future research.}
value a product which is reflected in the convexity properties of the valuation distribution function. Our insights then are not limited to markets with loss-averse consumers and extend as long as the convexity effect on the market demand function dominates the behavioral effect.

Our specification of a consumer’s surplus in Section 2 follows that in Thaler (1985). An alternative specification is provided by Baucells and Hwang (2016). Their MARA (Mental Accounting and Reference price Adaptation) model suggests that a reference price acts as the book value of a purchase and is used to amortize it upon consumption. This differs from our model in that we use the price to amortize the purchase. Adopting MARA does not affect our key insight: convexity properties of consumers’ valuation distribution may dominate the behavioral effect in characterizing the overall demand function.

In light of the research presented here, we caution against generalizing individual-level phenomena to the aggregate market level. Instead we recommend building demand functions by incorporating psychological biases into consumer utility functions and accounting for market characteristics (e.g., valuation heterogeneity) when aggregating individual demand functions. Some prior work shows the practical relevance of our research. Stoker (1993) argues that population heterogeneity cannot be ignored in empirical economic research. Camerer (1987) in a series of experiments shows that despite individuals not following the Bayes rule when updating their beliefs, markets function as if the traders were Bayesians.

Finally, although valuation heterogeneity is the most common form of heterogeneity in pricing models, consumers may also differ in other dimensions. Our work suggests that recommendations for the optimal policy differ significantly depending on how these differences affect the market demand. We suggest that the effect of these heterogeneities and psychological phenomena on aggregate variables such as demand is a worthwhile research direction in decision analysis and behavioral operations.

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References


**Appendix A: Proofs**

We first state the following result from Niculescu and Persson (2006).

**Lemma 1.** Let $I$ be an interval and let $g : I \rightarrow \mathbb{R}$ be a continuous function. Then $g$ is convex if and only if $2g(x) \leq g(x+h) + g(x-h)$ for all $x \in I$ and all $h > 0$ such that both $x+h$ and $x-h$ are in $I$. In addition, $g$ is strictly convex if and only if the inequality holds strictly.
**Proof:** See Corollary 1.1.5 in Niculescu and Persson (2006).

**Proof of Proposition 1.** In the main text we proved that market demand inherits the biases of individual consumer behavior when consumer valuations are uniformly distributed; here we prove the reverse. Since market demand inherits the property of individual consumer demand, it follows that if \( \lambda = 1 \) (i.e., if consumers are gain/loss neutral) then for all \( x > 0 \) we must have \( R(x, p) = -R(-x, p) \) or, equivalently, \( F(p) - F(p - m(x)) = -F(p) + F(p + m(x)) \), which simplifies to \( 2F(p) = F(p - m(x)) + F(p + m(x)) \). The last equality implies that \( F(\cdot) \) is linear. This is because, by Lemma 1, the function \( F(\cdot) \) is both convex and concave and hence is linear.

**Proof of Proposition 2.** We prove part (i); the other two parts follow similarly. By Definition 1, demand is more responsive to losses than to gains if \( R(x, p) < -R(-x, p) \) for all \( x > 0 \). Simplifying the inequality, it must then be that \( F(p) - F(p - t(x)) < -(F(p) - F(p - t(-x))) \) or, equivalently, \( F(p) - F(p - m(x)) < -F(p) + F(p + \lambda m(x)) \).

**Proof of Corollary 1.** We use Lemma 1 to prove part (i). The other two parts follow similarly. Because \( F(\cdot) \) is strictly convex, we have \( 2F(p) < F(p - m(x)) + F(p + m(x)) \) for any \( x > 0 \). We rewrite to obtain \( F(p) - F(p - m(x)) < -(F(p) - F(p + m(x))) \) or \( F(p) - F(p - t(x)) < -(F(p) - F(p - t(-x))) \) (recall that \( \lambda = 1 \) here), which is equivalent to \( R(x, p) < -R(-x, p) \) for any \( x > 0 \). That is, demand is more responsive to losses than to gains. We now show the reverse. Since demand is more responsive to losses and since \( \lambda = 1 \) (by Definition 1), it follows that \( 2F(p) < F(p - m(x)) + F(p + m(x)) \) for all \( x > 0 \). Then, by Lemma 1, \( F(\cdot) \) is strictly convex.

**Proof of Proposition 3.** Let \( m'_+(0) \) denote the right derivative of \( m(x) \) at \( x = 0 \). For part (i), by Definition 2 we have

\[
\begin{align*}
\lambda_+ &= \lim_{x \to 0^+} \frac{\partial R(x, p)}{\partial x} = \lim_{x \to 0^+} \frac{dm(x)}{dx} f(p - \eta x) = m'_+(0) f(p), \\
\lambda_- &= \lim_{x \to 0^-} \frac{\partial R(x, p)}{\partial x} = \lim_{x \to 0^-} \frac{dm(-x)}{dx} f(p - \lambda x) = \lambda m'_+(0) f(p).
\end{align*}
\]

Hence \( \rho = \lambda_- / \lambda_+ = \lambda \) and the result follows. For part (ii), the convexity of \( F(\cdot) \) guarantees that \( 2F(p) \leq F(p - m(x)) + F(p + m(x)) \) for all \( x > 0 \); as a consequence, demand is more responsive to losses than to gains.

**Proof of Proposition 4.** The proof is straightforward and omitted for brevity.

**Proof of Proposition 5.** Define \( \Pi(\mu_1, \mu_2) = p_1 \exp(-\frac{\mu_1}{\mu_1}) + p_2 \exp(-\frac{\mu_2}{\mu_2}) \), and let the optimal solution be \( (p_1^*, p_2^*) \). Because \( \gamma < \lambda \), we have \( \Pi(\mu_1, \mu_2) \leq \Pi(\mu_1^*, \mu_2^*) \). Moreover, if \( p_1^* > p_2^* \) then \( \Pi(\mu_1^*, \mu_2^*) = \Pi(\mu_1^*, p_2^*) \) and hence \( (p_1^*, p_2^*) = (p_1^*, p_2^*) \).

Given \( p_1 \), the function \( \Pi(\mu_1, p_2) \) is strictly quasi-concave in \( p_2 \) and by the first order condition \( p_2^* = \frac{\mu_2}{1 + \gamma} \). We substitute \( p_2^* \) into \( \Pi(\mu_1, p_2) \) and obtain \( \Pi(\mu_1, p_2^*) = p_1 \exp(-\frac{\mu_1}{\mu_1}) + \frac{\mu_2}{1 + \gamma} \exp(-1 + \frac{\mu_2}{\mu_2}) \). The function \( \Pi(\mu_1, p_2^*) \) is neither convex nor concave in \( p_1 \). Our goal is to show that \( \Pi(\mu_1, p_2^*) \) attains its maximum at \( \bar{p} \) when \( \mu_2 \) is not too large. To this end, we first assume the relative valuation \( \frac{\mu_2}{\mu_1} < 1 \) is a constant, and prove that, given \( k \), there exists a \( \bar{\mu}(k) > 0 \) such that \( p_1^* = \bar{p} \) for \( \mu_2 < \bar{\mu}(k) \). We then show that \( \bar{\mu}(k) \) is
bounded away from zero for any $k > 1$. Therefore, if we set $\bar{\mu} \triangleq \inf_{k>1} \bar{\mu}(k) > 0$ then, for any $\mu_1 < \mu_2 < \bar{\mu}$, we will have $p_1^* = \bar{p}$.

To prove the first step, assume $k$ is a constant. Differentiating $\Pi^\gamma(p_1, p_2^2)$ with respect to $p_1$ yields

$$\frac{d\Pi^\gamma(p_1, p_2^2)}{dp_1} = \exp\left(-\frac{p_1}{\mu_1}\right) \left[ 1 - \frac{p_1}{\mu_1} + \frac{\gamma}{1+\gamma} \exp(-1 + \frac{\mu_1 \gamma + \mu_2 p_1}{\mu_1 \mu_2}) \right]$$

$$= \exp\left(-\frac{p_1}{\mu_1}\right) \left[ 1 - \frac{p_1}{\mu_1} + \frac{\gamma}{1+\gamma} \exp(-1 + \frac{\gamma + k}{k \mu_1} p_1) \right].$$

It follows that whether the optimal solution is at the boundary of $[0, \bar{p}]$ depends on whether the following equation (in $p_1$) has a solution:

$$\exp(-1 + \frac{\gamma + k}{k \mu_1} p_1) = \frac{1 + \gamma}{\gamma \mu_1} p_1 - \frac{1 + \gamma}{\gamma}.$$ (11)

By a change of variable $x = -\frac{\gamma + k}{k \mu_1} (p_1 - \mu_1)$, equation (11) can be written as

$$x \exp(x) = \tau, \quad \text{(12)}$$

where $\tau = -\frac{\gamma + k}{k \mu_1} \exp(\frac{\gamma + k}{k} - 1)$. Observe that $\tau$ is a constant independent of $\mu_1$ and that the left hand side of (12) obtains its minimum at $x = -1$ equal to $-\exp(-1)$. We distinguish two cases.

Case 1: $\tau < -\exp(-1)$. Equation (12) admits no solution because $x \exp(x) > \tau$ for any $x \in \mathbb{R}$. Therefore $\frac{d\Pi^\gamma(p_1, p_2^2)}{dp_1} > 0$ for any $p_1$ and $\mu_1$, and hence $p_1^* = \bar{p}$; our claim then holds with $\bar{\mu}(k) = +\infty$.

Case 2: $\tau \geq -\exp(-1)$. Because $\tau < 0$, equation (12) has two roots $x_1 \leq -1 \leq x_2 < 0$. These roots can be computed as the values of the Lambert W function evaluated at $\tau$ (see Corless et al. (1996) for an overview of the Lambert W function). Equation (11) then has two solutions: $p_1^{(1)} = -\frac{\mu_1 x_2 + \gamma}{\gamma + k} + \mu_1$ and $p_1^{(2)} = -\frac{\mu_1 x_1 + \gamma}{\gamma + k} + \mu_1$, with $0 < p_1^{(1)} \leq p_1^{(2)}$. Because at both $p_1^* = 0$ and $p_1 \to +\infty$, we have $\frac{d\Pi^\gamma(p_1, p_2^2)}{dp_1} > 0$, it follows that $\Pi^\gamma(p_1, p_2^2)$ achieves its local maximum at $p_1^{(1)}$ and its local minimum at $p_1^{(2)}$.

We next show that there exists a $\bar{\mu}(k) > 0$ such that for any $\mu_2 < \bar{\mu}(k)$, the inequality $\Pi^\gamma(\bar{p}, p_2^2) > \Pi^\gamma(p_1^{(1)}, p_2^2)$ holds and because $p_1^{(1)}$ is the only local maximum, we then conclude that $p^* = \bar{p}$. To see this, denote $w = -\frac{k}{\gamma + k} x_2 + 1$ and observe that $1 < w \leq \frac{k}{\gamma + k} + 1$. We can then write $\Pi^\gamma(\bar{p}, p_2^2) = \mu_1 M$, where $M = w \exp(-w) + \frac{\gamma + k}{k \mu_1} \exp(-1 + \frac{\gamma + k}{k} w)$ is a constant independent of $\mu_1$.

The inequality $\Pi^\gamma(\bar{p}, p_2^2) > \Pi^\gamma(p_1^{(1)}, p_2^2)$ then reduces to $M < \frac{1}{\mu_1} \Pi^\gamma(\bar{p}, p_2^2) = \frac{\mu_1}{\mu_2} \exp(-\frac{\mu_2}{\mu_1}) + \frac{k}{\gamma + k} \exp(-1 + \frac{\gamma + k}{\gamma + k} w)$. Indeed $\frac{1}{\mu_1} \Pi^\gamma(\bar{p}, p_2^2) > \frac{\gamma + k}{\gamma + k} \exp(-1 + \frac{k}{\gamma + k}) > M$ holds for $\mu_2 = k \mu_1 < \bar{\mu}(k)$, where $\bar{\mu}(k) = \frac{\gamma + k}{\log((\gamma + k)/(\gamma + k - 1))} + 1$.

Combining the two cases above, we conclude that, for a given $k > 1$, there exists a $\bar{\mu}(k)$ such that $p_1^* = \bar{p}$ for $\mu_2 < \bar{\mu}(k)$.

As the last step of the proof, we show that $\bar{\mu}(k)$ is bounded away from 0. By $\frac{1 + \gamma}{k} M = \frac{1 + \gamma}{k} w \exp(-w) + \exp(-1 + \frac{\gamma + k}{k} w)$ and because $k > 1$, we have

$$\frac{1 + \gamma}{k} M \leq \frac{1}{\gamma + k} + \frac{1 + \gamma}{k} + \exp(-1 + \frac{1 + \gamma}{\gamma + k} + \frac{1}{k} < M,$$

where $M = 2 + \gamma + \exp(-1 + \frac{1}{\gamma + k})$ is independent of $k$. Thus, $\bar{\mu}(k) > \frac{\gamma}{\log(M + 1)}$, i.e., it is bounded away from 0 for any $k > 1$.

Now, if we let $\bar{\mu} = \inf_{k>1} \bar{\mu}(k)$ then $\bar{\mu} > 0$, and for $\mu_2 < \bar{\mu}$, we have $(p_1^*, p_2^2) = (\bar{p}, \mu_2/(1 + \gamma))$ and by $\bar{p} > \mu_2/(1 + \gamma)$, we have $(p_1^*, p_2^2) = (p_1^*, p_2^2)$. □
Proof of Proposition 6: We proceed by contradiction. Suppose there exists a steady state \( r^* \) such that \( p^*(r^*) = r^* \). Then there must exist a \( \tilde{\eta} \) such that \( \eta \leq \tilde{\eta} \leq \lambda \eta \) and \( r^* = \frac{\mu}{\beta (1-\beta)} \). To see why, we use a perturbation argument. At \( r^* \), the value function is \( V(r^*) = \frac{t}{\beta (1-\beta)} f(r^*) \), where \( f(r) = r \exp(-\frac{r}{\mu}) \). Consider now a policy where \( p_t = r^* + \Delta \) for some \( \Delta > 0 \) and for any \( t \geq 0 \), that is, a constant price larger than \( r^* \).

The total infinite horizon profit by this policy is

\[
V_1 = (r^* + \Delta) \exp\left(\frac{-r^* + \Delta}{\mu}\right) + \sum_{i=1}^{\infty} (r^* + \Delta) \exp\left(\frac{-r^* + \Delta}{\mu}\right)
\]

\[
= f(r^* + \Delta) \left( \exp\left(\frac{-\eta \Delta}{\mu}\right) + \frac{\beta}{1-\beta} \right).
\]

Because \( V(r^*) \geq V_1 \), we have

\[
\frac{f(r^* + \Delta) - f(r^*)}{\Delta} \leq \frac{(1-\beta)(1-\exp(-\lambda \eta \Delta / \mu))}{\Delta} f(r^* + \Delta).
\]

Letting \( \Delta \to 0 \) on both sides of this inequality yields \( \frac{df(r^*)}{dr} \leq (1-\beta) \frac{\lambda \eta}{\mu} f(r^*) \). A similar analysis with the policy \( p_t = r^* - \Delta \) results in \( \frac{df(r^*)}{dr} \geq (1-\beta) \frac{\lambda \eta}{\mu} f(r^*) \). Therefore, there must exists \( \tilde{\eta} \) with \( \eta \leq \tilde{\eta} \leq \lambda \eta \) such that \( (1-\frac{\tilde{\eta}}{\mu}) \exp(-\frac{\tilde{\eta}}{\mu}) = (1-\beta) \frac{\lambda \eta}{\mu} \exp(-\frac{\mu}{(\lambda - \beta) + 1}) \), which yields \( r^* = \frac{\mu}{\beta (1-\beta)} \) and \( V(r^*) = \frac{\lambda \eta}{(1-\beta)(\lambda - \beta) + 1} \exp(-\frac{1}{(\lambda - \beta) + 1}) \).

Now, consider a high-low pricing policy with \( p_{(2i)} = \bar{p} \) and \( p_{(2i+1)} = p^m = \frac{\mu}{(1+\eta)} \) for \( i = 0, 1, 2, \ldots \). The corresponding total infinite horizon profit then is

\[
V_2 = \sum_{i=0}^{\infty} \beta^i \bar{p} \exp\left(\frac{-\bar{p} + \lambda \eta (r_{(2i)} - \bar{p})}{\mu}\right) + \sum_{i=0}^{\infty} \beta^{2i+1} p^m \exp\left(\frac{-p^m + \eta (\bar{p} - p^m)}{\mu}\right)
\]

\[
\geq \frac{\beta}{1-\beta^2} \frac{\mu}{1+\eta} \exp(-1 + \frac{\eta}{\mu} \bar{p}).
\]

Indeed \( V_2 > V_1(r^*) \) if

\[
\frac{\beta}{1-\beta^2} \frac{\mu}{1+\eta} \exp(-1 + \frac{\eta}{\mu} \bar{p}) > \frac{\mu}{(1-\beta)(\lambda - \beta) + 1} \exp(-\frac{1}{(\lambda - \beta) + 1}). \tag{13}
\]

Denote \( M = \max_{\bar{\eta} \in [\eta, \lambda \eta]} \frac{(1+\beta)(1+\eta)}{\beta (1-\beta)(\lambda - \beta) + 1} \exp(-\frac{1}{(\lambda - \beta) + 1}) \). Then inequality (13) holds if \( \frac{\mu \bar{p}}{\mu} > \log M + 1 \).

Let \( \bar{\mu} = \infty \) if \( \log M + 1 \leq 0 \). Otherwise, let \( \bar{\mu} = \frac{\mu \bar{p}}{\log M + 1} \). We then have \( V_2 > V_1(r^*) \) for \( \mu < \bar{\mu} \); a contradiction. \( \square \)