Wage Transparency and Social Comparison in Salesforce Compensation

Xiaoyang Long
Wisconsin School of Business, University of Wisconsin-Madison, Madison, WI 53706, xiaoyang.long@wisc.edu

Javad Nasiry
School of Business and Management, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong SAR., nasiry@ust.hk

When wages are transparent, sales agents may compare their pay with that of their peers and experience positive or negative feelings if those peers are paid (respectively) less or more. We investigate the implications of such social comparisons on sales agents’ effort decisions and their incentives to help or collaborate with each other. We then characterize the firm’s optimal salesforce compensation scheme and the conditions under which wage transparency benefits the firm. Our results show that the work environment—which includes such aspects as demand uncertainty, correlation across sales territories, and the possibility of help/collaboration—plays a significant role in the firm’s compensation and wage transparency decisions. In particular, wage transparency is more likely to benefit the firm when demand uncertainty is moderate, sales outcomes are positively correlated across different sales territories, and sales agents can collaborate at low cost. We find that, contrary to conventional wisdom, social comparisons need not reduce collaboration among agents. Our study also highlights the importance of providing the right mix of individual and group incentives to elicit the benefits of wage transparency.

Key words: agency theory; salesforce compensation; social comparison; collaboration

1. Introduction

Wage transparency facilitates social comparisons, which may induce strong feelings in employees who are averse to unfavorable comparisons or keen to be ahead. Firms often cite such outcomes as the reason for keeping wage information confidential. For example, Netflix decided against revealing pay data to employees because it would cause “too much disruption, too much emotion” (Financial Times 2016). A 2011 survey shows that half of the companies surveyed either discourage or prohibit employees from discussing salary information (Wall Street Journal 2016). However, some companies have recently adopted different policies. From the supermarket chain Whole Foods Market to technology start-ups such as Buffer and SumAll, an increasing number of companies are making wage information available to all personnel (Business Insider 2014, Seiter 2016, Uzunian 2015,
Long and Nasiry: Wage Transparency in Salesforce Compensation

Wall Street Journal 2013). In government agencies, some states are required to reveal the wages of all public employees (Card et al. 2012). The Equal Pay Act, passed by Germany in 2017, requires companies employing more than 200 individuals to provide them with wage information (Associated Press 2017). Yet it is not obvious whether wage transparency benefits the firm. At Harvard University, wage transparency led to flattened pay for the management team of the university’s endowment fund, which drove away talent and ultimately hurt the fund’s performance (Harvard Business Review 2016). RethinkDB, a database firm, experimented with wage transparency but abandoned the policy after one year (Wall Street Journal 2013).

Social comparison changes workers’ attitudes toward their own tasks, as well as the way they interact with each other. One downside of wage transparency is that it may reduce workplace cooperation (Bierman and Gely 2004, Colella et al. 2007). Hence, we should observe less wage transparency in environments where workers are expected to cooperate. However, many companies that have successfully implemented wage transparency also emphasize cooperation. For example, the Whole Foods chain organizes the employees of each store into teams and rewards them based on both individual and team performances (Whole Foods Market 2016). Similarly, as start-ups, Buffer and SumAll rely heavily on interaction among their employees to process market feedback and improve their products.

We address three research questions in this paper. First, how does social comparison change sales agents’ effort decisions and incentives to help or collaborate with each other? Second, how can a firm incorporate social comparison into an optimal compensation scheme? Third, under what extrinsic conditions (e.g., market uncertainty, outcome correlation, and the possibility of help/collaboration in the workplace) and intrinsic conditions (e.g., agents’ social comparison preferences) does wage transparency benefit the firm?

We consider a firm that employs multiple risk-neutral sales agents. The agents may engage in social comparisons, and the firm can make this tendency stronger by making the pay information salient. Specifically, if wages are transparent, then each agent’s expected utility includes a “social comparison” component—that is, a gain or loss depending on whether his wage is higher or lower than his peer’s. As is common in practice (Ledford et al. 1995), the firm can pay its agents based on both individual performance and group performance. We analyze and compare the firm’s optimal compensation scheme and profit with and without social comparison.

In the base model, we assume that agents decide only their respective individual efforts and that agents’ sales outcomes may be positively or negatively correlated depending on how sales territories are allocated. We then consider the cases when agents have the option to: (a) help, as when one agent assists another in performing his sales task; or (b) collaborate, as when agents make a joint investment to reduce their costs of effort (e.g. sales agents could agree to share client information or
update the IT system in the sales team). These work conditions are commonly found in workgroups and have been studied in various combinations in the literature (see e.g., Caldieraro and Coughlan 2009, Itoh 1991, Siemsen et al. 2007 for discussion and examples).

Social comparison may entail positive or negative effects on sales agents. On the one hand, social comparison can increase agents’ effort levels because it provides additional incentive for agents to outperform each other—a positive motivation effect (Belogolovsky and Bamberger 2014, Blanes Vidal and Nossol 2011, Roels and Su 2014). On the other hand, social comparison could increase agents’ likelihood of quitting and make it more costly for the firm to retain workers if they dislike negative comparisons more than they like positive comparisons—a negative participation effect (Card et al. 2012, Mas 2014). Our study is the first to consider these two effects to identify the conditions under which wage transparency benefits the firm. We are also the first to examine analytically the effect of social comparison on help and collaboration in work groups.

First consider the case when agents exert only individual effort. In this case, we identify new drivers for the firm’s decision to use group incentives. Without social comparison, the firm never uses group incentives. In contrast, with social comparison we find that group incentives are optimal if the profit margin from a unit of sales is sufficiently low. The reason is that, in the presence of social comparison, the firm can use group incentives to reduce wage variance and thereby reduce the impact of both motivation effect and participation effects. When profit margin is low, the positive motivation effect is weaker than the negative participation effect and so the firm prefers to alleviate the latter via offering group incentives. Similarly, we find that the firm is inclined to use group incentives when demand uncertainty is high. This result explains why, in practice, some firms introduce group incentives before a highly uncertain event, such as the launch of a new product (a phenomenon observed but not explained in Caldieraro and Coughlan 2009).

We find that wage transparency is less likely to benefit the firm when the motivation effect is weak (e.g., if profit margins are low or the cost of effort is high) or the participation effect is strong (e.g., if demand uncertainty is high or agents are very behind-averse). Interestingly, our results show that wage transparency benefits the firm if sales outcomes are strongly (and positively) correlated across sales territories. This is because when outcomes are positively correlated, workers anticipate smaller variance in pay and thus less disutility from social comparison, which moderates the negative participation effect but does not affect the positive motivation effect. Our results imply, for example, that transparency is more likely to benefit a firm if it is applied to sales agents in the same physical location or selling complementary products. Conversely, wage transparency is more likely to hurt the firm when applied to sales agents with negatively correlated sales outcomes. Finally, our results have new implications for optimal territory allocation: while a firm has no preference for positive or negative correlation among its agents’ sales outcomes in the absence of
social comparison, it strictly benefits from positive correlation in the presence of social comparison due to lower expected social disutility for the agents.

Now consider work environments in which agents not only exert individual effort but may also help or collaborate with each other. Here, we identify another effect of social comparison: the cooperation effect, i.e., the effect of social comparison on the sales agents’ incentives to help or collaborate with each other. In environments where help is possible, we find that social comparison reduces an agent’s willingness to help because doing so increases the likelihood of a negative comparison. In response, the firm may either (i) increase group incentives to encourage help or (ii) reduce group incentives to promote more individual effort. We identify conditions under which the firm prefers to induce no help among its sales agents, and show that wage transparency is less likely to be optimal for firms that depend on help among agents.

In environments where collaboration is possible, on the other hand, social comparison could decrease or increase the willingness of agents to collaborate with each other. The reason for more collaboration is as follows. Without social comparison, each agent’s individual effort rises as his marginal cost of effort falls. With social comparison, this effect is even stronger, and if compensation depends on group performance, then each agent enjoys a spillover from his peer’s higher effort level. Knowing this, the agents are more willing to collaborate in the first place. We find that wage transparency is more likely to be optimal for firms that depend on collaboration among agents if collaboration is not too costly.

Overall, our results show that social comparison benefits the firm if the firm allocates sales agents to territories with positively correlated demand or designs collaboration processes under which agents can make a joint investment to lower their effort costs. We also highlight the importance of a well-designed compensation scheme—that is, one with an appropriate mix of individual and group incentives—in deriving the benefits of wage transparency.

The rest of our paper is organized as follows. In Section 2 we review the relevant literature. Section 3 presents the base model and solves the firm’s jointly optimal wage transparency and compensation problem when agents exert only individual effort. In Section 4 we consider the cases where the agents can help or collaborate with each other. We conclude in Section 5 by summarizing our results and suggesting avenues for future research.

2. Literature Review
This research is closely related to the literature in marketing and economics that addresses compensation schemes in the principal-agent framework (for a comprehensive review of salesforce compensation, see Albers and Mantrala 2008). Most of the existing studies in salesforce compensation focus on finding the optimal individual-based compensation plan while assuming that sales agents
work independently of each other and generate independent outcomes (e.g., Basu et al. 1985, Chen 2005, Chen et al. 2015, Dai and Jerath 2013, Lal and Srinivasan 1993, Schöttner 2016). Research on sales teams is scarce, despite their increased use over the past decade (Mantrala et al. 2010). A few papers have found that rewarding salespeople based on group performance can be optimal if there are task complementarities (Lazear 1989, Siemsen et al.) or if the sales outcomes are negatively correlated and salespeople are risk-averse (Caldieraro and Coughlan 2009). For example, Siemsen et al. (2007) distinguish between three types of linkages in workgroups (outcome linkage, help linkage, and knowledge linkage) and then examine the effectiveness of individual and group incentives in each case. Caldieraro and Coughlan study the firm’s joint decision on compensation and territory allocation when sales outcomes are correlated across territories. These authors find that assigning agents to negatively correlated sales territories benefits the firm because it can then use group pay to reduce the agents’ outcome risk. Our paper adds to these works by analyzing the effect of social comparison on salespeople’s efforts, their incentives to help or collaborate each other, and the optimal compensation scheme/wage transparency policy.

Several papers have studied optimal compensation schemes for agents with social comparison preferences, with the common finding that individual incentives may be weaker when agents engage in social comparison and are behind-averse (Bartling 2011, Englmaier and Wambach 2010, Itoh 2004, Neilson and Stowe 2010). For example, in a setting similar to ours (but with risk-averse agents), Bartling (2011) finds that group-based payment is optimal if agents are sufficiently behind-averse. Our work differs from these studies in three aspects. First, we consider wage transparency as an *endogenous* decision and identify the firm’s jointly optimal compensation scheme and wage transparency decisions. Second, we show that the firm’s decision to use group incentives under social comparison is driven by extrinsic factors such as profit margins and the level of demand uncertainty—dynamics that are new to the literature. Third, we analyze the effect of social comparison on help and collaboration in sales groups.

Our paper relates to the existence and implications of social comparison in workplaces, which have been studied empirically in various contexts (Ho and Su 2009, Lount and Wilk 2014, Song et al. 2017). Researchers have consistently found that providing public feedback about performance increases worker effort (Blanes Vidal and Nossol 2011, Hannan et al. 2008, Kuhnen and Tymula 2012, Roels and Su 2014). Yet, revealing wage and performance information may lead to excessive employee turnover. For example, Card et al. (2012) conduct an empirical study in which a randomly selected group of employees at the University of California is told about a website listing the salaries of all university employees. The authors find that employees whose pay is lower than the median have less job satisfaction and a greater inclination to switch jobs, whereas the employees with higher-than-median pay are unaffected. Their study concludes that pay secrecy is optimal
Long and Nasiry: Wage Transparency in Salesforce Compensation

because the costs of transparency for lower-paid workers exceed the benefits for their higher-paid peers. Mas (2014) suggests that retaining workers with social comparison concerns may require the firm to increase the base wage or reduce performance-based pay. Our study analytically studies the firm’s trade-off between increased effort and higher compensation costs when it decides whether to make wage information transparent. Our approach is similar to Roels and Su (2014), who study how a social planner can influence the agents’ social comparisons by changing the format of performance feedback. Further, we consider both individual effort and collaborative effort. Traditional social comparison theory asserts that social comparison always induces competitive behavior and reduces collaboration (Festinger 1954, Garcia et al. 2013); however, we identify cases where such comparisons could lead to more collaboration among workers.

3. Base Model with Uncertainty Correlation

In this section, we develop and solve the base model in which agents only exert individual effort. In Section 4, we extend the model to cases when agents can help or collaborate with each other.

3.1. The Model

Consider a firm that employs two sales agents to sell its products. Agent \(i \in \{1, 2\}\) can exert effort \(e_i\) to stochastically improve his sales outcome, \(s_i = e_i + \epsilon_i\). The noise term \(\epsilon_i\) is normally distributed with mean 0 and variance \(\sigma^2\). We assume that the agents’ effort levels are sufficiently high such that the possibility of negative sales is negligible\(^1\). The agent incurs a cost of \(\frac{1}{2}ke_i^2\) to exert effort \(e_i\).

Agents’ sales outcomes may be subject to common shocks. For example, salespeople who work at the same retail store (e.g., a Whole Foods supermarket) face the same uncertainty regarding customer arrivals—a positive correlation in their sales outcomes. Agents assigned to different sales territories/channels may have negatively correlated outcomes. We capture this correlation analytically by the covariance of the error terms \(\sigma_{ij} = \rho\sigma^2\), where \(\rho \in (-1, 1)\) is the correlation coefficient.

Suppose the firm can pay agent \(i\) based on his own sales \((s_i)\) and also on total sales in the group \((s_1 + s_2)\). Specifically, the compensation scheme consists of a base salary \(A\), an individual incentive \(b\), and a group incentive \(q\). The wage for agent \(i\) is given by

\[
w_i = bs_i + q(s_i + s_j + \epsilon_g) + A,
\]

where we assume that the public observable measure for group compensation \((s_i + s_j + \epsilon_g)\) is subject to a random variable \(\epsilon_g\) with mean 0. In practice, companies often tie group compensation to company or division profitability. For example, Joseph and Kalwani (1998) find in a survey of Fortune 500 companies that 38% of respondents use division or company profit to determine

\(^1\) This is a common assumption in the literature; see, e.g., Lal and Srinivasan (1993), Joseph and Thevaranjan (1998), Siemsen et al. (2007).
bonuses, with 20% citing this as the most important measure. Companies such as Caterpillar Inc. tie part of employee pay to the company’s financial performance (Huffington Post 2013). Whole Foods pays its employees based on team profit (Forbes 2016). Company or division profit could be influenced by factors other than total sales such as consumer returns/after-sales guarantees, operational costs, financial environment, etc. The random variable $\varepsilon_g$ captures the uncertainties in these factors. In implementing the contract, the firm would reveal the realized $s_i + s_j + \varepsilon_g$ to the sales agents but refrain from disclosing the specific value of $\varepsilon_g$. For example, Whole Foods reveals team profit to its employees but does not share its total sales, operational costs or other financial specifics (Forbes 2016). Without wage transparency, the agents in our model cannot infer the specific sales outcomes of their peers from the compensation they receive.

We assume that the firm can influence the sales agents’ social comparison tendencies by providing or withholding wage information (i.e., the actual payment that each agent receives). When wage information is provided, sales agent $i$ compares his wage with his peer’s and experiences a positive or negative utility from this comparison. That is, the sales agent’s utility function becomes

$$u_i(w_i, w_j) = w_i - \frac{k^2 e_i^2}{2} - \beta \cdot \max\{w_j - w_i, 0\} + \alpha \cdot \max\{w_i - w_j, 0\},$$

(2)

where $w_i$ is as in (1). The last two terms represent the agent’s utility from social comparison. The behind-aversion parameter $\beta$ captures the extent to which the sales agent dislikes receiving a lower wage, and the ahead-seeking parameter $\alpha$ represents the agent’s gain from receiving a higher wage than his peer. We assume that $\beta \geq \alpha \geq 0$ which implies that losses have a higher impact on utility than gains of the same magnitude. In contrast, when wage information is not provided, then social comparison is not salient and thus each sales agent cares only about his own wage minus his total cost of effort, i.e., $u_i(w_i) = w_i - \frac{k^2 e_i^2}{2}$.

Our approach is consistent with existing literature in that providing feedback can change people’s behavioral tendencies. For example, Roels and Su (2014) investigate how the social planner can “actively influence” workers’ social comparison utilities by manipulating the format of feedback (i.e., providing either the average or full distribution of outcomes). They further conjecture that “the social planner can try to amplify the strengths of social comparison effects.” Itoh (2004) expresses a similar idea in stating that “social preferences can be part of the ‘contract’ designed by the principal.” Lount Jr and Wilk (2014) reason that “posting performance can facilitate a culture of comparison, where performance comparisons are a salient and relevant feature of the

---

2 Our results also hold for $\alpha < 0$ as long as $\beta \geq -\alpha$. A negative $\alpha$ implies that the agent is inequity averse—he dislikes receiving a higher wage than his peer.

3 For detailed discussions of the effect of “salient cues” on human behavioral biases, see Kahneman (1992) and Kahneman (2003). As an example, Zeelenberg and Pieters (2004) find that people express more regret over not participating in lotteries after being told that their neighbors participated and won big prizes.
workplace.” Our model also matches the empirical studies which show that the social comparison effects arise only when wage or performance information is made explicitly available to agents (see, e.g., Blanes Vidal and Nossol 2011, Card et al. 2012, Ho and Su 2009, Lount Jr and Wilk 2014).

We assume that the agents have limited liability (i.e., $b \geq 0, q \geq 0, A \geq 0$). Moreover, the agents have a reservation utility $w_0$, which captures their best outside option. The time-line is as follows. First, the firm decides and announces a wage transparency policy. Then, the firm offers a contract $(b, q, A)$ to each agent. If the agents accept the contracts, they simultaneously decide their respective optimal efforts. The firm then observes the realized sales outcomes and $\epsilon_g$, and rewards the agents based on their contracts.

In the base model, we assume that the agent’s sales outcome depends on his individual effort only (i.e., $s_i = e_i + \epsilon_i$). In Section 4, we consider cases when agents can exert effort to help or collaborate with each other. That is, agent $i$ may exert help effort $e_{ij}$ to increase his peer’s sales outcome, or collaborate with his peer (by paying cost $\psi$) to lower the marginal cost of effort ($k$).

### 3.2. Optimal Compensation Scheme and Wage Transparency Policy

We use backward induction to solve the problem. First, given a contract $(A, b, q)$, agent $i$ chooses an effort level to maximize his expected utility:

$\mathbb{E}u_i(s_i, s_j) = A + \mathbb{E}[bs_i + q(s_i + s_j + \epsilon_g) - \beta b(s_j - s_i)^+ + \alpha b(s_i - s_j)^+] - \frac{k}{2} e_i^2$.  

We write the agents’ effort levels as $\hat{e}_I^1(b, q)$ and $\hat{e}_I^2(b, q)$, where the superscript $I$ signifies “individual effort”.

**Lemma 1.** (i) Given a contract $(A, b, q)$, the effort decisions are

$$\hat{e}_I^1(b, q) = \hat{e}_I^2(b, q) = \hat{e}_I(b, q) = \frac{(1 + \alpha / 2 + \beta / 2)b + q}{k}.$$  

(ii) Social comparison makes individual incentives more effective, i.e., $\frac{\partial^2 \hat{e}_I}{\partial b \partial \alpha} = \frac{\partial^2 \hat{e}_I}{\partial b \partial \beta} = \frac{1}{k} > 0$.

Lemma 1 shows that if compensation depends on individual outcomes ($b > 0$), then sales agents exert more effort under social comparison—a consequence of the additional behavioral incentive of outperforming and earning higher wages than peers. We refer to this as the motivation effect of social comparison.

We now solve the firm’s profit maximization problem. Suppose that the firm’s profit from each unit of sales is $m > 0$. Then the risk-neutral firm chooses contract parameters to maximize its expected profit:

$$\max_{b, q, A} m\hat{e}_I(b, q) - b\hat{e}_I(b, q) - 2q\hat{e}_I(b, q) - A.$$
under the participation constraint
\[
\mathbb{E}u_i = A + (b + 2q)\hat{e}^I(b, q) - \frac{k}{2} \hat{e}^I(b, q)^2 - (\beta - \alpha)b \int_{z \geq 0} zdG(z) \geq w_0. \tag{4}
\]
Here \( z \equiv \varepsilon_j - \varepsilon_i \) is the difference in error terms and \( G(\cdot) \) is the cumulative distribution of \( z \). Observe that the effect of social comparison appears only in (4)’s fourth term, which is nonpositive because \( \beta \geq \alpha \). If \( \beta > \alpha \), then it is clear that social comparison always reduces an agent’s total expected utility. It follows that the firm must pay more (than in the case without social comparison) to ensure participation. We refer to this as the participation effect of social comparison.

In light of Lemma 1 and the participation constraint, we conclude that the higher the individual incentives \( (b) \), the stronger the motivation effect and participation effect. So in identifying a compensation plan’s optimal mix of individual and group incentives, the firm trades off the benefit of higher effort against the greater cost of satisfying the participation constraint. The optimal compensation scheme is described in the following proposition, where \( b^I \) and \( q^I \) denote the optimal individual and group incentives, respectively.

**Proposition 1.** (i) Without social comparison, the firm offers only individual incentives; (ii) In the presence of social comparison, there exist two thresholds \( m_1, m_2 \) such that the firm offers: (a) only group incentives (i.e., \( b^I = 0, q^I > 0 \)) if \( m \leq m_1 \); (b) both individual and group incentives if \( m_1 < m \leq m_2 \); or (c) only individual incentives if \( m > m_2 \).

By Proposition 1, the firm uses group incentives if and only if agents have social comparison preferences and the profit margin is sufficiently low. When agents do not make social comparisons (i.e., \( \alpha = \beta = 0 \), the firm offers individual incentives only. This is because individual and group incentives are equally effective in inducing effort (see Lemma 1) but group incentives may lead to higher payment. With social comparison, however, the firm uses group incentives to balance the positive motivation effect and the negative participation effect. When the profit margin is lower, the motivation effect is less prominent and so group incentives (which reduce both the motivation and the participation effects of social comparison) are more likely to be optimal. In fact, when the profit margin is very low, the firm may eliminate any wage difference among agents by offering group incentives only. Figure 1 illustrates the optimal compensation scheme as a function of the correlation factor \( \rho \) and profit margin \( m \). Observe that as \( \rho \) increases from \(-1 \) to \( 1 \), the firm becomes less likely to use group incentives (both \( m_1 \) and \( m_2 \) decrease).

**Corollary 1** establishes that the optimality of group incentives is affected by firm-related parameters such as demand uncertainty, outcome correlation, and cost of effort.

**Corollary 1.** The thresholds \( m_1 \) and \( m_2 \): (i) increase in the cost of effort \( k \); (ii) decrease in correlation parameter \( \rho \); and (iii) increase in demand uncertainty \( \sigma \).
Corollary 1(i) is intuitive: when the cost of effort is higher, the motivation effect is weaker and so group incentives are more likely to be optimal. It is interesting that, according to Corollary 1(ii), the firm is more likely to use group incentives when the correlation between sales outcomes is lower. This is because the participation effect (as captured by \((\beta - \alpha) b \int_{z \geq 0} z dG(z) = (\beta - \alpha) b \frac{1-\rho}{\pi} \sigma \) in (4)) is stronger as \(\rho\) decreases. At the same time, the motivation effect is not affected by \(\rho\) (see Lemma 1). Similarly, by Corollary1(iii), the firm is more likely to use group incentives when demand uncertainty is higher. This result is consistent with companies implementing group-based compensation in anticipation of highly uncertain events (e.g., launching a new product; see Caldieraro and Coughlan 2009)—a phenomenon that cannot be explained without social comparison.

Our next corollary specifies the sensitivity of group incentives to social comparison parameters.

**Corollary 2.**

(i) The thresholds \(m_1\) and \(m_2\) increase in behind-aversion \(\beta\); and

(ii) The threshold \(m_1\) decreases in ahead-seeking \(\alpha\).

Corollary 2 shows that the agents’ ahead-seeking and behind-aversion preferences affect the firm’s decision to use group incentives in different ways. First consider \(\beta\). Recall from Proposition 1 that the firm’s only reason to use group incentives is to alleviate the negative participation effect of social comparison. As agents become more behind-averse, the negative participation effect becomes stronger. Therefore, the firm is more likely to use group incentives as part of the compensation scheme, and also more likely to use only group incentives (i.e., both \(m_1\) and \(m_2\) increase in \(\beta\)).
Next consider $\alpha$. As agents become more ahead-seeking, the negative participation effect of social comparison falls. Therefore, the firm is more likely to use individual incentives as part of the compensation scheme (i.e., $m_1$ decreases in $\alpha$). However, this does not mean that the firm is willing to give up group incentives altogether (i.e., $m_2$ does not necessarily decrease in $\alpha$). This is because as $\alpha$ increases, agents exert more effort (for the same $b$) and expect to be compensated for their additional effort. When the cost of extra effort outweighs the benefit for the firm, it may wish to limit agent effort by using some group incentives (in additional to individual incentives) in the contract. Therefore, the threshold $m_2$ may decrease or increase in ahead-seeking $\alpha$.

Figure 2 plots the optimal compensation plans and firm profits as functions of behind aversion $\beta$, while assuming that ahead-seeking $\alpha = 0^4$. In this case, if also $\beta = 0$, then there is no social comparison and the firm uses only individual incentives. As $\beta$ increases, the firm first increases the individual incentive to compensate for the negative participation effect, and then switches to increasing the group incentive and decreasing the individual incentive; see Figure 2(a). Figure 2(b) shows that the firm may benefit from social comparison if $\beta$ is sufficiently small. In Proposition 2, we specify the conditions under which the firm’s profit is higher with than without social comparison.

---

4 We assume $\alpha = 0$ because this facilitates easy comparison of results with and without social comparison.
Proposition 2. (i) There exists a threshold $m_1$ such that the firm’s optimal policy is to implement wage transparency if and only if $m \geq m_1$.

(ii) The threshold $m_1$: (a) decreases in ahead-seeking $\alpha$ and increases in behind-aversion $\beta$; (b) decreases in uncertainty correlation $\rho$; (c) increases in demand uncertainty $\sigma$ and cost of effort $k$.

The firm benefits from social comparison if and only if the positive motivation effect dominates the negative participation effect. When $\beta = \alpha$, the participation effect does not exist (see (4)) and thus wage transparency is always optimal, i.e., $m_1 = 0$. When $\beta > \alpha$, wage transparency is optimal if and only if agent effort is sufficiently valuable to the firm, i.e., if the profit margin is sufficiently high ($m \geq m_1 > 0$). The sensitivity results in Proposition 2(ii) follow similar intuitions. In particular, for agents who are more ahead-seeking or less behind-averse, wage transparency is more likely to be optimal. Further, we find that the firm should implement wage transparency when the correlation between outcomes increases. This is because a higher $\rho$ leads to a weaker participation effect but does not change the motivation effect, and so the firm is more likely to benefit from social comparison when $\rho$ is higher. Figure 3 plots the regions where the firm benefits from social comparison and therefore should reveal wage information to its sales agents (along with the optimal compensation schemes).

Proposition 2 implies that the firm may alter its allocation of territory so as to benefit from social comparison among sales agents. In the absence of social comparison, the firm’s profit is not affected by any correlation among risk-neutral agents’ sales outcomes. When agents are risk-averse, Caldieraro and Coughlan 2009 find that a negative correlation among sales territories’ demand strictly benefits the firm; the firm may even prefer to allocate sales agents to a territory with lower sales potential if doing so diversifies the sales territories. However, Proposition 2(ii) reveals a downside to this “diversification” strategy in the presence of social comparison. Namely, negative correlation in sales outcomes may increase differences among agents’ wages, exacerbating the negative participation effect and thus reducing the firm’s profit. Corollary 3 shows that in the presence of social comparison, the firm may prefer to allocate sales agents to positively (rather than negatively) correlated sales territories.

Corollary 3. (i) In the absence of social comparison, the firm is indifferent between negative or positive correlation among its sales territories. (ii) In the presence of social comparison, the firm strictly prefers a positive correlation to a negative correlation.

Corollary 3 provides a rationale for firms not to diversify their sales territories even when they have that option. The result also suggests that a firm which diversifies without wage transparency may wish to change its territory allocation to promote positive correlation after making wage information transparent.
4. Help and Collaboration in Sales Groups

In this section we extend the base model to consider work environments where sales agents have the opportunity to help or collaborate with each other in addition to working on their own tasks. Such opportunities may arise due to complementarities between agents’ tasks. For example, Chan et al. (2014a) find empirically that sales agents are more likely to help their peers when compensation is based on team rather than individual sales. In a related paper, they further find that peer-based learning (e.g., direct teaching) is more important than “learning-by-doing” for an individual’s sales success (Chan et al. 2014b). The increasing complexity of products and customer demand has led to a greater need for sales agents in different territories, channels, or products to share knowledge or to collaborate on innovative sales strategies (Davie et al. 2010). Our models for help and collaboration are similar to those in Siemsen et al. (2007). In order to focus on insights stemming from these new conditions, we assume that agents’ outcome uncertainties are uncorrelated ($\rho = 0$).
4.1. Direct Help

Suppose that agents can exert effort to directly increase their peers’ sales outcomes. For example, a salesperson with some slack time may help cover a sales call for his peer, or a restaurant server may help with the order at another server’s table. Formally, let agent \( i \) choose both individual effort \( e_i \) and help effort \( e_{ij} \); the overall outcome is then given by \( s_i = e_i + he_{ji} + \varepsilon_i \), where \( h \in [0,1] \) represents the extent of substitutability between agents’ efforts. So if \( h = 0 \) then agents cannot help each other and, at the other extreme, if \( h = 1 \) then the agents’ efforts are perfect substitutes. Let the total cost of effort be \( k^2(e_i^2 + e_{ij}^2) \).

Given their contracts, agents simultaneously choose their individual effort and help effort to maximize their expected utilities:

\[
E(u_i(s_i, s_j) = A + E[bs_i + q(s_i + s_j + \varepsilon_g) - \beta b(s_j - s_i)^+ + \alpha b(s_i - s_j)^+] - \frac{k}{2} e_i^2 - \frac{k}{2} e_{ij}^2.
\]

Denote the agents’ effort decisions in equilibrium by \( \hat{e}_H^1(b, q) \), \( \hat{e}_H^2(b, q) \), \( \hat{e}_{12}^H(b, q) \), and \( \hat{e}_{21}^H(b, q) \), where the superscript “H” signifies “help”.

Lemma 2. (i) Given a contract \((A, b, q)\), the effort decisions are

\[
\hat{e}_1^H(b, q) = \hat{e}_2^H(b, q) = \hat{e}^H = \frac{b + q + (\beta - \alpha) b/2}{k}, \quad \hat{e}_{12}^H = \hat{e}_{21}^H = \hat{e}_{ij}^H = \max \left\{ 0, h\frac{q - (\beta - \alpha) b/2}{k} \right\}.
\]

(ii) Without social comparison, individual incentives do not affect help effort; in the presence of social comparison, individual incentives reduce help effort.

By helping a peer, an agent increases his peer’s wage and the likelihood of incurring a disutility due to a negative wage comparison. That disutility is greater when individual incentives are higher. It follows that, when there is social comparison, higher individual incentives reduce the extent to which agents help each other. We refer to this reduction in cooperation effort as the cooperation effect of social comparison.

In this environment, the firm chooses contract parameters to maximize its expected profit:

\[
\max_{b, q, A} m(\hat{e}^H(b, q) + h\hat{e}_{ij}^H(b, q)) - (b + 2q)(\hat{e}^H(b, q) + h\hat{e}_{ij}^H(b, q)) - A,
\]

under the participation constraint

\[
E(u_i = A + (b + 2q)(\hat{e}^H(b, q) + h\hat{e}_{ij}^H(b, q)) - \frac{k}{2} \hat{e}^H(b, q)^2 - \frac{k}{2} \hat{e}_{ij}^H(b, q)^2 - (\beta - \alpha) b \int_{z \geq 0} z dG(z) \geq w_0.
\]

The firm’s optimal compensation plan is described in Proposition 3. We use \( b^H \) and \( q^H \) to denote the optimal individual and group incentives.
Proposition 3. (i) There exists a threshold $h_1$ such that if $h < h_1$, then the firm does not induce help effort (i.e., $e_{ij}^H = 0$) and always uses individual incentives ($b^H > 0$); if $h \geq h_1$, then the firm elicits positive help effort and always uses group incentives ($e_{ij}^H > 0$ and $q^H > 0$).

(ii) In the presence of social comparison, when $h < h_1$ and $m$ is sufficiently low, the firm uses group incentives but does not elicit help effort.

Proposition 3(i) shows that the firm designs the compensation scheme to elicit help effort if substitutability between agents’ efforts is sufficiently high. It is known that, without social comparison, the firm uses group incentives if and only if help effort is preferred (Siemsen et al. 2007, Itoh 1991). Yet in the presence of social comparison, Proposition 3(ii) reveals that the optimality of group incentives does not necessarily imply positive help effort. This is because when there is social comparison, individual incentives have the additional effect of reducing help effort and so the firm must increase the group incentive to induce the same level of help effort. If substitutability between agents’ efforts is low, then the cost of inducing help effort outweighs the benefit, which means that the firm will find it optimal not to induce any help among agents—but may still use group incentives to alleviate the negative participation effect of social comparison.

Figure 4(a) illustrates the optimal compensation structure, as a function of substitutability $h$ and profit margin $m$, in the presence of social comparison. As $m$ increases, the firm shifts from a compensation plan based on group incentives and help effort to one based on individual incentives and only individual effort. This dynamic reflects the motivation effect becoming more important when $m$ increases (as discussed in Section 3) and dominating, when $m$ is sufficiently high, both the negative participation effect and the negative cooperation effect.

Finally, our next result shows that social comparison is less likely to benefit the firm when there is more substitutability between the agents’ efforts. This result follows from the negative effect of social comparison on agents’ incentives to help each other.

Proposition 4. (i) There exists a threshold $m_H$ such that the firm’s optimal policy is to implement wage transparency if and only if $m \geq m_H$.

(ii) The threshold $m_H$ is increasing in $h$; that is, a firm that depends less on help among its agents is more likely to benefit from wage transparency.

Figure 4(b) shows the area in which the firm benefits from social comparison and should therefore implement wage transparency, along with the optimal compensation schemes. We can see that, when profit margin is large and substitutability between agents’ efforts is low, the firm prefers to implement wage transparency and elicit no help effort—thereby encouraging competition, rather than cooperation, among agents. The next corollary summarizes the effect of wage transparency on the agents’ tendencies to help each other.
COROLLARY 4. Wage transparency (as the optimal strategy for the firm) always leads to less help effort among sales agents.

Corollary 4 establishes that when wage transparency is optimal for the firm, it always results in a reduction of help effort.

4.2. Collaboration

We now consider the case where agents can make a joint investment to reduce their cost of effort. For example, sales agents can adopt an information technology system to facilitate scheduling, communication and knowledge sharing among team members (Weinstein and Mullins 2012); they can also work together to sell customized “product bundles” to shared clients (Davie et al. 2010). We refer to this decision as the collaboration decision. We model the collaboration decision as a binary decision $c_i = \{0, 1\}$ that occurs before individual effort decisions are made, and we use $\psi$ to denote the cost of collaboration for each agent. If both agents decide to collaborate, then the cost of individual effort is reduced from $k$ to $k'$, where $0 < k' < k$. If either agent decides not to collaborate, then effort costs do not change.

Given the contract and the collaboration decision, the agents’ individual effort decisions are as given in Lemma 1 in Section 3. Substituting $\hat{e}(b, q)$ in (4), we can write the expected utility of an agent’s expected utility, given $k$, as:

$$
E u_i(k) = A + \frac{(2 + \beta + \alpha)b + 2q((2 - \beta - \alpha)b + 6q)}{8k} - \frac{\sigma}{\sqrt{\pi}(\beta - \alpha)}b,
$$

(5)
where we have put $\rho = 0$. It is clear that, absent social comparison ($\alpha = \beta = 0$), $E_{u_i}(k)$ is always decreasing in $k$. Hence a lower cost of effort always benefits the agents, and they will collaborate if $\psi \leq E_{u_i}(k') - E_{u_i}(k)$. However, the following lemma shows that—in the presence of social comparison—a lower cost of effort may make agents worse-off.

**Lemma 3.** *Given a contract $(A, b, q)$, a lower $k$ strictly benefits agents (i.e., $\frac{dE_{u_i}(k)}{dk} < 0$) if their social preferences are sufficiently weak ($\beta + \alpha < \frac{2b+6q}{b}$), and hurts them (i.e., $\frac{dE_{u_i}(k)}{dk} \geq 0$) otherwise.*

Lemma 3 shows that, when compensation depends on individual outcomes ($b > 0$), a lower cost of effort benefits sales agents if and only if their social preferences are sufficiently weak. In the absence of social comparison, agents care only about their respective sales outcomes and benefit from a lower cost of effort. In the presence of social comparison, however, each agent is motivated to exert more individual effort (motivation effect). If agents are highly ahead-seeking or highly behind-averse, then this motivation effect increases effort so much that the agent’s expected utility declines as $k$ decreases. The implication is that, if social preferences are strong, then agents actually prefer higher costs of effort in the group, and would not collaborate even if collaboration were costless. If possible, agents may even collude to increase $k$ (self-sabotage).

Yet when social preferences are not too strong, Lemma 3 suggests that agents still benefit from a lower $k$. We will show that this benefit may be larger—and the agents more willing to collaborate—with than without social comparison. To see this, consider the agents’ collaboration decision. That decision is positive if and only if $E_{u_i}(k') - E_{u_i}(k) \geq \psi$ or, equivalently,

$$((2 + \beta + \alpha)b + 2q)((2 - \beta - \alpha)b + 6q) \geq \psi \frac{8kk'}{k - k'}.$$  

The next proposition shows that the collaboration decision depends not only on the compensation plan but also on the strength of agents’ social preferences.

**Proposition 5.** (i) *Given a contract with no group incentives ($q = 0$), social comparison always reduces collaboration among agents.*

(ii) *Given a contract with no individual incentives ($b = 0$), social comparison has no effect on collaboration.*

(iii) *Given a contract with both individual and group incentives ($b > 0$, $q > 0$), social comparison increases collaboration among agents if and only if their social preferences are not too strong (i.e., $\beta + \alpha < 4q/b$).*

Figure 5 plots the collaboration outcome as a function of behind aversion $\beta$, collaboration cost $\psi$, and group incentive $q$. From Figure 5(a), we can see that the range of $\psi$ for which agents decide to collaborate first increases and then decreases in $\beta$. Similarly, Figure 5(b) shows that it would be...
cheaper for the firm to induce collaboration among agents who have moderate social preferences. This is because, with social comparison, agents exert more individual effort for the same reduction in effort cost $k$. Hence, their peers benefit through the group incentive $q$, and all agents become more willing to collaborate (in the first place) so as to reduce $k$. If social preferences are strong, however, then the cost of extra individual effort is so high that agents avoid it by not collaborating.

Finally, Proposition 6 shows that the possibility of collaboration in the work environment could make the firm more likely to implement wage transparency and thereby benefit from social comparison.

**Proposition 6.** *If collaboration is not too costly, the firm is more likely to benefit from wage transparency when agents have the opportunity to collaborate with each other.*

Figure 6 illustrates the firm’s profits with and without the opportunity for collaboration. Figure 6(a) depicts the case when agents *cannot* collaborate with each other (which is equivalent to the base model in Section 3). In this case, the firm’s profit under social comparison ($\Pi^I$) is higher than that without social comparison $\Pi^R$ if and only if $m > m_I$. In comparison, Figure 6(b) shows the case when agents *can* collaborate with each other. Once again, the firm’s profit under social comparison ($\Pi^*$) is higher than it is under no social comparison ($\Pi^R$) if and only if $m > m_C$. Observe that $m_C < m_I$, which illustrates that social comparison is more likely to benefit the firm when collaboration is possible.

Figure 6(b) also illustrates the regions where the firm chooses to incentivize collaboration. Specifically, in absence of social comparison, the firm elicits collaboration if and only if the profit margin
is sufficiently high, i.e., \( m > c^R \). Similarly, in the presence of social comparison, the firm elicits collaboration if and only if \( m > c^* \). Observe that \( c^* < c^R \), which shows that the firm is more likely to elicit collaboration with than without social comparison. In particular, when \( m_C < m < c^R \), wage transparency (as the optimal policy) leads to more collaboration among agents.

![Graph](image)

**Figure 6** Firm’s profits with and without collaboration (given \( k = 1, k' = 2/3, \psi = 11/16, \alpha = 0, \beta = 1, \sigma = 1, w_0 = 1 \)).

5. **Discussion and Conclusion**

In this paper, we study the firm’s compensation decisions in the presence of social comparison and also identify the conditions under which it should reveal wage information to its employees. Our results show that the work environment plays a significant role in the firm’s compensation and wage transparency decisions. In particular, wage transparency is more likely to benefit the firm when agents’ outcome uncertainties are positively correlated or when agents have the opportunity to collaborate with each other at low cost. These findings suggest, in order to benefit from wage transparency, the firm may find it optimal to allocate agents to territories that are positively correlated or to create opportunities for low-cost collaboration. We also study the effect of social comparison on how agents interact with each other. We find that social comparison always reduces help among agents, but could actually increase their willingness to make a joint investment to lower the cost of effort. Finally, we highlight the roles of individual and group incentives in delivering these benefits of social comparison.
Our work yields some interesting implications for practice. First, we identify new parameters that drive the firm’s decision to use group incentives. In the presence of social comparison, for example, the firm may switch from individual- to group-based payment when demand uncertainty increases or profit margins decline. Second, our results run counter to the conventional wisdom that wage transparency is most suitable for work environments with no interdependency among workers. These results explain why wage transparency is adopted by firms that value cooperation among their workers; examples include Whole Foods and technology start-ups such as Buffer and SumAll.

There are several directions to extend this study. First, we have focused on individual and group commissions in the firm’s compensation decision. In practice, the firm may choose other compensation structures such as sales tournaments or bonuses. It would be interesting to study the implication of social comparison for the design of these alternative compensation structures and for the corresponding wage transparency policy. Suppose, for instance, that the firm offers each agent a bonus contract consisting of an individual quota, an individual bonus upon reaching that quota, a group quota, and a group bonus. Then social comparison would still have both a motivation effect and a participation effect because an agent may fail to reach his own quota while his peer succeeds—leading to unequal wages. However, the firm may be more likely to benefit from wage transparency because now it has two levers to control social comparison effects—the individual bonus and the individual quota. Furthermore, bonus contracts may strengthen the effect of social comparison in promoting collaboration. That is, agents may become more willing to collaborate because doing so reduces wage variance (and expected social disutility) by increasing the likelihood that both agents reach their quotas. This effect does not exist if the firm implements linear contracts.

Second, our study considers homogenous agents. However, social comparison effects are often most salient in situations where agents have different abilities (e.g., different costs of effort). In this case, both the motivation effect and the participation effect of social comparison would be amplified for the lower-ability agent but reduced for the higher-ability agent; then the wage transparency decision may be driven by differences in ability among workgroup members. Finally, our study yields some interesting hypotheses on the interaction between social comparison and worker collaboration that could be tested in laboratory or field experiments. We leave these topics for future research.

References

Associated Press 2017. German cabinet approves law meant to help ensure equal pay. (January 11).


*Business Insider* 2014. Here’s why Whole Foods lets employees look up each other’s salaries. (March 3).


*Financial Times* 2016. Pay transparency is the last taboo in business. (June 20).

*Forbes* 2016. Why whole foods builds its entire business on teams. (June 8).


Huffington Post 2013. 90 percent of employers tie workers’ pay to company performance. (September 1).


*Wall Street Journal* 2013. Psst...this is what your co-worker is paid. (January 29).


Appendix: Proofs

Remark: Throughout the proofs, we substitute \( x \equiv \beta + \alpha \) and \( y \equiv (\beta - \alpha)\sqrt{\frac{1 - \rho}{\pi}} \) for ease of exposition.

**Proof of Lemma 1.** (i) Agent \( i \in \{1, 2\} \) chooses \( e_i \) to maximize his expected utility:

\[
E u_i = A + E[b s_i + q(s_i + s_j + e_i) - \beta b(s_j - s_i) + \alpha b(s_i - s_j) + \frac{k}{2} e_i^2] \\
= A + (b + q)e_i + q e_j - \beta b \int_{z \geq e_i - e_j} (z + e_j - e_i) dG(z) + \alpha b \int_{z < e_i - e_j} (e_i - e_j - z) dG(z) - \frac{k}{2} e_i^2
\]

where \( z \equiv e_j - e_i \) and \( G(\cdot) \) is the cumulative distribution of \( z \); the last equality follows because \( \int_{z < e_i - e_j} (e_i - e_j - z) dG(z) = \int (e_i - e_j - z) dG(z) - \int_{z \geq e_i - e_j} (e_i - e_j - z) dG(z) = e_i - e_j + \int_{z \geq e_i - e_j} (z + e_j - e_i) dG(z) \).

Observe that

\[
E u_i = A + (b + q)e_i + q e_j - (\beta - \alpha) b \int_{z \geq e_i - e_j} (z + e_j - e_i) dG(z) + \alpha b (e_i - e_j) - \frac{k}{2} e_i^2,
\]

where \( z \equiv e_j - e_i \) and \( G(\cdot) \) is the cumulative distribution of \( z \); the last equality follows because \( \int_{z < e_i - e_j} (e_i - e_j - z) dG(z) = \int (e_i - e_j - z) dG(z) - \int_{z \geq e_i - e_j} (e_i - e_j - z) dG(z) = e_i - e_j + \int_{z \geq e_i - e_j} (z + e_j - e_i) dG(z) \).

Taking the partial derivatives of \( E u_i \) with respect to \( e_i \) yields the first-order conditions (FOCs)

\[
\frac{\partial E u_1}{\partial e_1} = b + q + (\beta - \alpha)b \int_{z \geq e_i - e_j} (e_i - e_j) dG(z) + \alpha b - ke_1 = 0, \quad (7)
\]

\[
\frac{\partial E u_2}{\partial e_2} = b + q + (\beta - \alpha)b \int_{z \geq e_i - e_j} (e_i - e_j) dG(z) + \alpha b - ke_2 = 0, \quad (8)
\]

and the second-order conditions:

\[
\frac{\partial^2 E u_1}{\partial e_1^2} = -(\beta - \alpha)b G'(e_i - e_j) - k < 0,
\]

where the inequality follows because \( \beta \geq \alpha \) and \( G'(\cdot) > 0 \). The optimal effort levels satisfy the FOCs, which—after taking the sum and the difference of (7) and (8)—are equivalent to:

\[
k(e_1 - e_2) = (\beta - \alpha) b [1 - 2G(e_1 - e_2)], \quad (9)
\]

\[
k(e_1 + e_2) = 2(b + q) + (\beta + \alpha)b. \quad (10)
\]

Observe that \( e_1 - e_2 = 0 \) solves equation (9) and is the unique solution because the left-hand side (LHS) of that equation is strictly increasing in \( e_1 - e_2 \) while its right-hand side (RHS) is decreasing in \( e_1 - e_2 \).

Substituting \( e_1 = e_2 \) in (10) then gives the unique equilibrium solution

\[
e_1 = e_2 = \frac{(1 + \alpha/2 + \beta/2)b + q}{k}. \quad (11)
\]

(ii) The results follow directly from (11). \( \square \)

**Proof of Proposition 1.** The firm’s problem is

\[
\max_{b, q, A} \Pi(b, q, A) = \max_{b, q, A} m \hat{e}^f(b, q) - (b + 2q) \hat{e}^f(b, q) - A
\]

subject to agents’ participation constraint

\[
A + (b + 2q) \hat{e}^f(b, q) - \frac{k}{2} \hat{e}^f(b, q)^2 - (\beta - \alpha)b \sigma \sqrt{\frac{1 - \rho}{\pi}} \geq w_0 \quad (12)
\]

and the firm’s liability constraints \( b \geq 0, q \geq 0, A \geq 0 \). Here (12) follows from (4) by substituting \( \int_{z \geq 0} z dG(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{z \geq 0} z e^{-z^2/(2\sigma^2)} dz = \frac{1}{\sqrt{2\pi}} \sigma = \frac{1}{\sqrt{2\pi}} \sigma \sqrt{\frac{1 - \rho}{\pi}} \); the last equality holds because the variance of \( z \) is \( \sigma^2 = 2\sigma^2(1 - \rho) \).

Substituting \( \hat{e}^f(b, q) \) and by definition of \( x \) and \( y \), the firm’s problem becomes

\[
\max_{b, q, A} \frac{(m - b - 2q)(2q + b(2 + x))}{2k} - A,
\]

Long and Nasiry: Wage Transparency in Salesforce Compensation
such that
\[
\frac{(6q + (2 - x)b)(2q + (2 + x)b)}{8k} - yb + A \geq w_0, \quad \text{and} \quad b \geq 0, q \geq 0, A \geq 0.
\]

The Lagrangian of this constrained optimization problem is
\[
L = \frac{(m - b - 2q)(2q + b(2 + x))}{2k} - A + \lambda \left( \frac{(6q + (2 - x)b)(2q + (2 + x)b)}{8k} - yb + A - w_0 \right),
\]
where \( \lambda \geq 0 \) is the Lagrange multiplier. The critical points satisfy the Karush-Kuhn-Tucker (KKT) conditions:
\[
\begin{align*}
\frac{\partial L}{\partial b} &= \frac{(2 + x)m - 2(2 + x)b - 2(3 + x)q}{2} + \lambda \left( \frac{2(4 + x)q + (4 - x^2)b}{4k} - y \right) \leq 0, \\
\frac{\partial L}{\partial q} &= \frac{m - 4q - (3 + x)b}{k} + \frac{6q + (4 + x)b}{2k} \leq 0, \\
\frac{\partial L}{\partial A} &= \lambda - 1 \leq 0, \\
\frac{\partial L}{\partial \lambda} &= \frac{(6q + (2 - x)b)(2q + (2 + x)b)}{8k} - yb + A - w_0 \geq 0, \\
b \frac{\partial L}{\partial b} &= 0, \quad q \frac{\partial L}{\partial q} = 0, \quad A \frac{\partial L}{\partial A} = 0, \quad \lambda \frac{\partial L}{\partial \lambda} = 0, \quad b \geq 0, \quad q \geq 0, \quad A \geq 0, \quad \lambda \geq 0.
\end{align*}
\]

We solve these conditions for two cases: \( \beta = \alpha \) and \( \beta > \alpha \).

(i) When \( \beta = \alpha \), we find three solutions to the KKT conditions, depending on the profit margin \( m \).

Solution 1: When \( x \geq 2 \) or \( x < 2 \) & \( m < \sqrt{\frac{2(2 + x)kw_0}{2 - x}} \), the participation constraint binds while the nonnegativity constraints \( b \geq 0 \) and \( A \geq 0 \) do not. The optimal solution is\(^5\)
\[
b' = \frac{2m}{x + 2}, \quad q' = 0, \quad A' = w_0 - \frac{(2 - x)m^2}{2k(2 + x)}, \quad \lambda' = 1, \quad \Pi' = \frac{m^2}{2k} - w_0.
\]

Solution 2: When \( \{x < 2 \wedge m \leq \sqrt{\frac{2kwm}{2 - x}} \} \) the participation constraint binds as well as the nonnegativity constraints \( q \geq 0 \) and \( A \geq 0 \). The optimal solution is
\[
b' = \sqrt{\frac{8kw_0}{4 - x^2}}, \quad q' = 0, \quad A' = \frac{2(2b' - m)}{(2 - x)b'}, \quad \lambda' = 0, \quad \Pi' = m \sqrt{\frac{2(2 + x)w_0}{k(2 + x)}} - \frac{4w_0}{2 - x}.
\]

Solution 3: When \( \{x < 2 \wedge m \geq \sqrt{\frac{32kw_0}{4 - x^2}} \} \), only the nonnegativity constraints \( q \geq 0 \) and \( A \geq 0 \) bind, and the optimal solution is
\[
b' = \frac{m}{2}, \quad q' = 0, \quad A' = 0, \quad \lambda' = 0, \quad \Pi' = \frac{(2 + x)m^2}{8k}.
\]

In all three solutions, \( q' = 0 \). Since \( \alpha = \beta = 0 \) in absence of social comparison, this completes the proof of (i).

(ii) When \( \beta > \alpha \), the KKT conditions yield five solutions depending on the profit margin \( m \).

Solution 1: When \( m \leq \sqrt{\frac{2kwm}{3}} \), the only binding constraints are the participation constraint and nonnegativity constraint \( b \geq 0 \). The optimal solution is
\[
q' = m, \quad A' = w_0 - \frac{3m^2}{2k}, \quad b' = 0, \quad \lambda' = 1, \quad \Pi' = \frac{m^2}{2k} - w_0.
\]

\(^5\)In this case, the optimal solution is any set of \((b, q)\) that satisfies \( q + (1 + \frac{x}{2})b = m \) and max\( \{0, \frac{3m^2 - 2kw_0}{2(1 + x)m} \} \leq b \leq \frac{2m}{x + 2} \).
Solution 2: When $\sqrt{\frac{2kw_0}{3}} < m \leq m_1$, where $m_1 \equiv \frac{4\sqrt{\frac{2k}{3}w_0} - \frac{(1+x)2kw_0}{(1+x)\sqrt{\frac{2k}{3}w_0 + ky}}}{ky}$, the participation constraint as well as the nonnegativity constraints $b \geq 0$ and $A \geq 0$ bind. The optimal solution is

\[ b' = 0, \quad q' = \sqrt{\frac{2kw_0}{3}}, \quad A' = 0, \quad \lambda' = \frac{4q' - m}{3q'}, \quad \Pi' = m\sqrt{\frac{2w_0}{3k}} - \frac{4w_0}{3}. \]

Solution 3: When $\{x < 2 \& m_1 < m < m_2\}$ or $\{x \geq 2 \& m > m_1\}$, where $m_2 \equiv \frac{4(10 + 5x + x^2)ky(4ky + 2\sqrt{2k(4 - x^2)w_0 + 4k^2y^2})/(4 - x^2) + 8k(1 + x)(2 + x)w_0}{2(1 + x)(4ky + 2\sqrt{2k(4 - x^2)w_0 + 4k^2y^2}) + 4ky(2 - x)}$, the participation constraint and nonnegativity constraint $A \geq 0$ bind, while $b \geq 0$ and $q \geq 0$ do not bind. The optimal solution satisfies

\[ b' = \frac{k(4 - 3\lambda')(1 - \lambda')y - (1 - \lambda')(x + 1)m}{(1 - \lambda')^2(x + 1)^2} > 0, \]

\[ q' = \frac{-k\lambda'y((2 - \lambda')x + 6 - 4\lambda') + (1 - \lambda')(x + 1)(x + 2)m}{2(1 - \lambda')^2(x + 1)^2} > 0, \quad A' = 0, \]

where $\lambda' \in \left[\frac{\sqrt{2kw_0}}{\sqrt{2kw_0 + 3k\sqrt{y/1+x}}}, 1\right]$ solves

\[ \frac{2(1 - \lambda)(1 + x)m - (8 - 9\lambda + 3\lambda^2)ky\lambda}{2(1 - \lambda)^2(1 + x)^2}y - w_0 = 0. \]

We can prove $\lambda'$ is unique by observing that the LHS of (14) has at most two roots (since its slope, i.e., $-\frac{2(1 - \lambda)(1 + x)m}{2(1 - \lambda)^2(1 + x)^2}$, changes signs at most once in $\lambda$) and has negative slope at $\lambda = \lambda'$ (since $b' > 0$). The existence of $\lambda'$ follows because the LHS of (14) is positive at $\lambda = \sqrt{\frac{2kw_0}{3}}/\left(\sqrt{\frac{2kw_0}{3}} + k\frac{1}{1+x}\right)$ and negative as $\lambda \to 1$. The corresponding profit is

\[ \Pi' = \frac{(\lambda')^2y((2 - \lambda')ky - (1 - \lambda')m(1 + x))}{(1 - \lambda')^3(1 + x)^2} \equiv \Pi'_1. \]

Solution 4: When $\{x < 2 \& m_2 \leq m < m_3\}$, where $m_3 \equiv \frac{8ky + 4\sqrt{4k^2y^2 + 2kw_0(4 - x^2)}}{4 - x^2}$, the participation constraint as well as $q \geq 0$ and $A \geq 0$ bind. The optimal solution is

\[ b' = \frac{4ky + 2\sqrt{4k^2y^2 + 2kw_0(4 - x^2)}}{4 - x^2}, \quad q' = 0, \quad A' = 0, \quad \lambda' = \frac{(2 + x)(2b' - m)}{\sqrt{4k^2y^2 + 2kw_0(4 - x^2)}}, \]

\[ \Pi' = \frac{(m - b')b'(2 + x)}{2k} \equiv \Pi'_2. \]

Solution 5: When $\{x < 2 \& m \geq m_3\}$, only the nonnegativity constraints $q \geq 0$ and $A \geq 0$ bind. The optimal solution is

\[ b' = \frac{m}{2}, \quad q' = 0, \quad A' = 0, \quad \lambda' = 0, \quad \Pi' = \frac{(2 + x)m^2}{8k}. \]

\[ \square \]

**Proof of Corollary 1**

(i) This follows because $\frac{d\Pi_1}{dx} = \sqrt{8kw_0\left(4(1+x)y\sqrt{6kw_0 + 6k^2y + w_0(1+x)}\right)} > 0$ and

\[ \frac{d\Pi_2}{dx} = \frac{12ky^2(x^2 + 5x + 10)\left(\sqrt{4k^2y^2 + 2kw_0(4 - x^2)} + w_0(2 - x)(x^2 + 5x + 10)\right)^{2} + \frac{1}{x(1 + x)\sqrt{6kw_0 + 3k^2y}}}{\left(2 - x\right)\sqrt{4k^2y^2 + 2kw_0(4 - x^2)} + \left(x + 4\right)ky} > 0 \]

is positive for all $x < 2$.

To prove (ii) and (iii), we first show that both $m_1$ and $m_2$ increase in $y$. It is easy to see that $m_1$ increases in $y$. Next,

\[ \frac{d\Pi_2}{dy} = \frac{6ky^2(x^2 + 5x + 10)\left(\sqrt{4k^2y^2 + 2kw_0(4 - x^2)} + w_0(2 - x)(x^2 + 5x + 10)\right)^{2} + \frac{1}{x(1 + x)\sqrt{6kw_0 + 3k^2y}}}{\left(2 - x\right)\sqrt{4k^2y^2 + 2kw_0(4 - x^2)} + \left(x + 4\right)ky} > 0 \]

is positive for all $x < 2$. The results in (ii) and (iii) then follow because $y$ decreases in $\rho$ and increases in $\sigma$. \[ \square \]
Proof of Corollary 2 (i) First observe that \( \frac{\partial m_0}{\partial x} = \frac{2(1+2\alpha^2)\sqrt{\pi}}{(\sqrt{\frac{2\alpha}{\beta}}(1+x) + k\beta)^2} > 0 \). Next, we prove \( \frac{\partial m}{\partial x} = \frac{\partial m_0}{\partial x} \). The former inequality follows from the proof of Corollary 1. To establish the latter, observe that
\[
\frac{\partial m}{\partial x} = \frac{8kx(x^2+8x+28)y^3(\sqrt{4k^2y^2+2k/w_0(4-x^2)}+w_0(2-x))+y(2+x)(4k^2y^2+2k/w_0(4-x^2))+2k/y(2+x)(4+80x+29x^2))}{4k^2y^2+2k/w_0(4-x^2)+(x+4)ky}/(2k^2)
\]
is positive for all \( x < 2 \).
(ii) The proof follows because
\[
\frac{\partial m}{\partial x} = -\frac{2(1+2\beta)\sqrt{\pi}}{(\sqrt{\frac{2\beta}{\alpha}}(1+x) + k\alpha)^2} < 0.
\]

Proof of Proposition 2. (i) To determine whether social comparison benefits the firm, we compare the firm’s optimal profit with and without social comparison (denoted, respectively, as \( \Pi^0 \) and \( \Pi^1 \)). By proof of Proposition 1, the firm’s optimal profit without social comparison (\( \alpha = 0, \beta = 0 \)) is
\[
\Pi^0 = \begin{cases} 
\frac{m^2}{2k} - w_0, & \text{if } m < \sqrt{2k}w_0, \\
-\frac{4w_0}{3} - w_0, & \text{if } \sqrt{2k}w_0 \leq m < 2\sqrt{2k}w_0, \\
\frac{m^2}{2k}, & \text{if } m \geq 2\sqrt{2k}w_0.
\end{cases}
\]
When \( \beta > \alpha > 0 \), the firm’s optimal profit is
\[
\Pi^1 = \begin{cases} 
\frac{m^2}{2k} - w_0, & \text{if } x \geq 2 \text{ or } \{x < 2 \text{ & } m < \sqrt{2k}w_0 \}, \\
\frac{4w_0}{3} - w_0, & \text{if } x < 2 \text{ & } \sqrt{2k}w_0 \leq m < \sqrt{4k}w_0, \\
\frac{m^2}{2k} + 1 + \frac{\alpha^2}{8k}, & \text{if } x < 2 \text{ & } m \geq \sqrt{4k}w_0.
\end{cases}
\]
It is easy to see that \( \Pi^1 - \Pi^0 \geq 0 \) for all \( m > 0 \). When \( \beta > \alpha \), the firm’s optimal profit is
\[
\Pi^1 = \begin{cases} 
\frac{m^2}{2k} - w_0, & \text{if } m \leq \sqrt{2k}w_0/3, \\
\frac{4w_0}{3} - w_0, & \text{if } \sqrt{2k}w_0/3 < m \leq m_1, \\
\Pi^0_1, & \text{if } m_1 < m < m_2, \\
\Pi^0_2, & \text{if } x < 2 \text{ & } m_2 \leq m < m_3, \\
\frac{(2+x)m^2}{8k}, & \text{if } x < 2 \text{ & } m \geq m_3,
\end{cases}
\]
where \( \Pi^0_1 \) and \( \Pi^0_2 \) are given in proof of Proposition 1. Note that \( \Pi^1 - \Pi^0 \) is a continuous function in \( m \). It is easy to see that \( \Pi^1 - \Pi^0_R = 0 \) if \( m \leq \sqrt{2k}w_0/3 \), and \( \Pi^1 - \Pi^0_R = \frac{2+x}{8k}m^2 - \frac{m^2}{4k} > 0 \) if \( x < 2 \text{ & } m \geq m_3 \). It remains to prove that in the other regions, as \( m \) increases, \( \Pi^1 \) crosses \( \Pi^0_R \) at most once and from below. We consider three possible regions corresponding to the three different expressions for \( \Pi^1 \).

Region 1: \( \{ \sqrt{2k}w_0/3 < m \leq m_1 \} \). We show \( \Pi^0_1 - \Pi^0_R < 0 \) by considering three possible cases.

R1.1: If \( m < \sqrt{2k}w_0 \), then \( \Pi^0_1 - \Pi^0_R = m\sqrt{2w_0/3w_0} - \frac{w_0}{3} - \frac{m^2}{4k} = -\frac{1}{4k} \left( m - \sqrt{2w_0/3} \right)^2 < 0 \).

R1.2: If \( \sqrt{2k}w_0 \leq m < 2\sqrt{2k}w_0 \), then \( \Pi^0_1 - \Pi^0_R = (\sqrt{3} - 1)\sqrt{2w_0/3} - m \). The inequality holds because \( m \geq \sqrt{2k}w_0 \).

R1.3: If \( m \geq 2\sqrt{2k}w_0 \), then \( \Pi^0_1 - \Pi^0_R = m\sqrt{2w_0/3} - \frac{4w_0}{3} - \frac{m^2}{4k} = -\frac{1}{4k} \left( m - 2\sqrt{2k}w_0 + \frac{4w_0}{3} \right)^2 < 0 \).

Region 2: \( \{ m_1 < m < m_2 \} \). The optimal profit under social comparison is
\[
\Pi^1 = \frac{\lambda^2y(2-\lambda)ky - (1-\lambda)(1+x)m}{(1-\lambda)(1+x)^2},
\]
where \( \lambda = \lambda^1 \) solves (14) and satisfies the inequality in (13). Again we consider three possible cases.
R2.1: If $m < \sqrt{2kw_0}$, we prove $\Pi' - \Pi'_R \leq 0$. Since $\Pi' - \Pi'_R = 0$ at $y = 0$, and $\Pi'_R$ does not change in $y$, it suffices to prove $\frac{d\Pi'}{dy} < 0$. Define the left-hand-side of (14) as $f(\lambda)$ and differentiate both sides of $f(\lambda) = 0$ to yield
\[
\frac{df(\lambda)}{d\lambda} + \frac{df(\lambda)}{d\lambda} \frac{d\lambda}{dy} = 0.
\]
Substituting $\frac{d\lambda}{dy} = -\left(\frac{\lambda(\lambda)^2}{\lambda} / \left(1 + \lambda^2 (1 + \lambda^2)^2\right)\right)$ into $\frac{d\Pi'}{dy} = \frac{d\Pi'}{dx} + \frac{d\Pi'}{dy}$, we obtain $\frac{d\Pi'}{dy} = \lambda^2 ((1-x^2)(x^2) + m \lambda (1-(1-x^2)^2)) / (1+x^2 + (1-x^2)^2) < 0$. The inequality holds by (13).

R2.2: If $\sqrt{2kw_0} \leq m < 2\sqrt{2kw_0}$, we prove that $\Pi' - \Pi'_R = \lambda^2 y (2-\lambda^2) k(1-\lambda^2) (1-x) m - (m \lambda_0 - 2w_0)$ first decreases and then increases in $m$. Similar to R2.1, we first derive $\frac{d\Pi'}{dm} = \frac{d\Pi'}{dx} + \frac{d\Pi'}{dy} = \frac{d\Pi'}{dx} - \frac{d\Pi'}{dy}(\lambda^2 y (2-\lambda^2) k(1-\lambda^2) (1-x) m)$. It follows that $\frac{d(\Pi' - \Pi'_R)}{dm} = \frac{\lambda^2 y}{1+x^2} - \frac{2w_0}{\lambda^2} < 0$ is negative if and only if $\lambda^2 < \sqrt{2kw_0 + ky/1+x^2}$. By (14), this condition is equivalent to $m < \sqrt{2kw_0} (1 + \frac{ky}{\sqrt{2kw_0} + ky/1+x^2})^2$ (note that $\lambda^2$ increases in $m$, i.e., $\frac{d\lambda^2}{dm} = -\left(\frac{\partial^2(\lambda^2)}{\partial y^2}\right) / \left(\partial^2(\lambda^2)/\partial x^2\right) > 0$). Hence $\Pi' - \Pi'_R$ decreases in $m$ if $m < \sqrt{2kw_0} (1 + \frac{ky}{\sqrt{2kw_0} + ky/1+x^2})^2$ and increases in $m$ otherwise.

R2.3: If $m \geq 2\sqrt{2kw_0}$, we prove that $\Pi'$ crosses $\Pi'_R$ at most one time and only from below. This is equivalent to showing that if $m$ exists at which $\Pi' - \Pi'_R = 0$, then $\frac{d\Pi'}{dm}|_{m=m\hat{m}} = \frac{d\Pi'}{dx}|_{m=m\hat{m}}$. To prove this, suppose there exists $m = \hat{m}$ at which $\Pi'(\hat{m}) - \Pi'_R(\hat{m}) = \lambda^2 y (2-\lambda^2) k(1-\lambda^2) (1-x) \hat{m} - \frac{\lambda^2 y}{1+x^2} = 0$, i.e. (after some algebra), $\frac{\lambda^2 y}{(1+x^2)(1-x^2)} - \frac{\lambda^2 y}{(1+x^2)(1-x^2)} = 0$. Since $0 < \lambda^2 < 1$, this equation simplifies to $\frac{\lambda^2 y}{(1+x^2)(1-x^2)} = \frac{\lambda^2 y}{(1+x^2)(1-x^2)} = \frac{\lambda^2}{(1+x^2)(1-x^2)}$. It suffices to prove that equation (15) implies $\frac{d\Pi'}{dx}|_{m=m\hat{m}} > 0$. Substitute (15) into $\frac{d\Pi'}{dm}|_{m=m\hat{m}} = \frac{\lambda^2 y}{(1+x^2)(1-x^2)} = \frac{\lambda^2}{(1+x^2)(1-x^2)}$, which is positive if and only if $\lambda^2 > \frac{1}{2}$. The inequality holds because by (14) and (15), $(\lambda^2 y) / (1+x^2)(1-x^2) = \frac{\lambda^2 y}{(1+x^2)(1-x^2)} = \frac{\lambda^2 y}{(1+x^2)(1-x^2)} < 0$, which implies $\lambda^2 > 0.693$.

**Region 3:** $\{x < 2 \text{ or } m_2 < m < m_3\}$. We prove that $\Pi' - \Pi'_R$ strictly increases in $m$ in this region by considering two possible cases.

R3.1: If $m_2 < m < 2\sqrt{2kw_0}$, then take the derivative of $\Pi' - \Pi'_R$ with respect to $m$ yields $\frac{d\Pi'}{dm}(\Pi' - \Pi'_R) = \frac{4ky + 2\sqrt{4k^2y^2 + 2kw_0(4-x^2)}}{2k(2-x)} - \frac{2w_0}{k} > 0$.

R3.2: If $\sqrt{2kw_0} \leq m < m_3$, then we have $\frac{d\Pi'}{dm}(\Pi' - \Pi'_R) = \frac{1}{2k} \left(4ky + 2\sqrt{4k^2y^2 + 2kw_0(4-x^2)} - m\right) > \frac{1}{2k} (m_3 - m) > 0$.

Combining all the regions, we conclude that there exists a unique threshold $m_t$ such that $\Pi' - \Pi'_R \leq 0$ if $m \leq m_t$ and $\Pi' - \Pi'_R > 0$ if $m > m_t$.

(ii) **Sensitivity Results**

By proof of (i), the threshold at which $\Pi'$ crosses $\Pi'_R$ is

$$m_t = \begin{cases} M_1 = \frac{(2k + 2\sqrt{4k^2y^2 + 2kw_0(4-x^2)})}{(4-x^2)(4-x^2)} & \text{if } x \geq 2 \text{ or } \{x < 2 \text{ or } m_1 \leq M_1 \leq m_2\}, \\ M_2 = \frac{(2k + 2\sqrt{4k^2y^2 + 2kw_0(4-x^2)})}{(4-x^2)(4-x^2)} & \text{if } x < 2 \text{ and } m_2 \leq M_2 > 2\sqrt{2kw_0}, \\ M_3 = 2(2 + x - \sqrt{x(2 + x)}) & \text{if } x < 2 \text{ and } \max\{m_2, 2\sqrt{2kw_0}\} \leq M_3 < m_3, \end{cases}$$

where we define $Z \equiv \sqrt{2k(4-x^2)w_0 + 4k^2y^2}$ for ease of exposition, and $M_1$, $M_2$, and $M_3$ are the respective solutions to $\Pi'(m) - \Pi'_R(m) = 0$ in Region 2, R3.1, and R3.2. Note that $m_t$ is a continuous function in parameters $a, b, r, k, \sigma$ because it is the unique solution to $\Pi' - \Pi'_R = 0$ and both $\Pi'$ and $\Pi'_R$ are continuous.
in these parameters. To conduct sensitivity analysis on $m_1$, we take the derivative of $M_1$, $M_2$, and $M_3$ with respect to parameters $\alpha, \beta, \rho, k,$ and $\sigma$.

The proof uses the following observation: for all parameters (say $\alpha$), $\frac{dM_1}{dx}$ has the opposite sign as $\frac{\partial \Pi'(M_1) - \Pi_R'(M_1)}{\partial \alpha}$. This is because $\Pi'(M_1) - \Pi_R'(M_1) = 0$ (by definition). Taking the derivative on both sides of this equation with respect to $\alpha$ then yields $\frac{\partial \Pi'(M_1) - \Pi_R'(M_1)}{\partial \alpha} + \frac{dM_1}{dx} \frac{\partial \Pi'(M_1) - \Pi_R'(M_1)}{\partial \alpha} = 0$, or $\frac{dM_1}{dx} = -\frac{\partial \Pi'(M_1) - \Pi_R'(M_1)}{\partial \alpha} \frac{\partial \Pi'(M_1)}{\partial \alpha}$. Since $\frac{\partial \Pi'(M_1) - \Pi_R'(M_1)}{\partial \alpha} > 0$ (by proof of (i)), it follows that $\frac{dM_1}{dx}$ has the opposite sign as $\frac{\partial \Pi'(M_1)}{\partial \alpha}$.

1. **Sensitivity analysis with respect to $\alpha$:** To prove $\frac{dM_1}{dx} < 0$ for $i \in \{1, 2, 3\}$, it suffices to prove $\frac{\partial \Pi'(M_1)}{\partial \alpha} > 0$. First observe that $\frac{\partial \Pi'(M_1)}{\partial \alpha} = \Pi'(M_1) = \frac{\partial \Pi(M_1)}{\partial \alpha} - \sigma \frac{1}{x} \frac{\partial \Pi(M_1)}{\partial \alpha}$. Hence it suffices to prove $\frac{\partial \Pi'(M_1)}{\partial \alpha} > 0$ and $\frac{\partial \Pi(M_1)}{\partial \alpha} > 0$. When $i = 1$, recall from the proof of (i) that $\frac{\partial \Pi}{\partial \alpha}$ is defined and $\frac{\partial \Pi}{\partial \alpha} \leq 1$, and, similarly, $\frac{\partial \Pi}{\partial \alpha} = (1 - x^2) \frac{1}{x^2}$. When $i \in \{2, 3\}$, we observe that $\frac{\partial \Pi'(M_1)}{\partial \alpha} > 0$, $\frac{\partial \Pi(M_1)}{\partial \alpha} > 0$, and $\frac{\partial \Pi(M_1)}{\partial \alpha} \leq 1$, which holds because $\frac{\partial \Pi}{\partial \alpha} \leq 1$ and, similarly, $\frac{\partial \Pi}{\partial \alpha} = (1 - x^2) \frac{1}{x^2}$. Hence it suffices to prove $\frac{\partial \Pi'(M_1)}{\partial \alpha} > 0$ and $\frac{\partial \Pi(M_1)}{\partial \alpha} > 0$.

2. **Sensitivity analysis with respect to $\beta$:** We show that $M_1, M_2$, and $M_3$ are all increasing in $\beta$. First, prove $\frac{dM_1}{dx} > 0$, it suffices to prove $\frac{\partial \Pi'(M_1)}{\partial \beta} > 0$. Substitute $\frac{\partial \Pi'(m_0)}{\partial \beta} = \frac{\partial \Pi'(m)}{\partial \beta}$ to obtain $\frac{\partial \Pi'(m_0)}{\partial \beta} > 0$. The inequality holds because $\frac{\partial \Pi}{\partial \beta} \geq 0$.

Finally, similar to the case above, to prove $\frac{dM_1}{dx} > 0$, it suffices to prove $\frac{\partial \Pi'(m_0)}{\partial \beta} > 0$, which holds because $\frac{\partial \Pi}{\partial \beta} \geq 0$.

3. **Sensitivity analysis with respect to $\rho$:** To prove $\frac{dM_1}{dx} < 0$ for $i \in \{1, 2, 3\}$, it suffices to prove $\frac{\partial \Pi'(M_1)}{\partial \rho} = \frac{\partial \Pi'(M_1)}{\partial \rho} > 0$, which holds because $\frac{\partial \Pi'(M_1)}{\partial \rho} < 0$ (by (i)).

4. **Sensitivity analysis with respect to $x$:** To prove $\frac{dM_1}{dx} < 0$ for $i \in \{1, 2, 3\}$, it suffices to prove $\frac{\partial \Pi'(M_1)}{\partial x} > 0$, which holds because $\frac{\partial \Pi'(M_1)}{\partial x} < 0$ (by (i)).

5. **Sensitivity analysis with respect to $k$:** We show that $M_i$ is increasing in $k$ for $i \in \{1, 2, 3\}$.

For $i = 1$, it suffices to prove $\frac{\partial \Pi'(M_1)}{\partial k} < 0$. We divide the proof into two cases.

If $\max \{m_1, \sqrt{2K\theta}w_0\} < M_1 < \min \{m_1, \sqrt{2K\theta}w_0\}$, then $\frac{\partial \Pi'(M_1)}{\partial k} = \frac{\partial \Pi'(M_1)}{\partial k} \frac{\partial \Pi(M_1)}{\partial k} < 0$, which holds because $\frac{\partial \Pi'(M_1)}{\partial k} < 0$ (by (i)).
\[ \frac{\partial^2 W(M_t) - \Pi''(M_t)}{\partial k} < 0, \text{ we observe that } M_t \frac{\sqrt{\pi}}{2k} < \frac{(4 - 3\lambda')y_g}{(1 - \lambda')y_g} \sqrt{\frac{\pi}{2k}} < \frac{(\lambda')^2(4 - 3\lambda')y_g}{2(1 - \lambda')(1 + x)} \text{, where the first inequality follows from (13), and the second inequality is equivalent to } \lambda' > \sqrt{\frac{2w_0}{2k + 4q/1 + x}} \text{, which holds at } m = M_1 \text{ by proof of (i) (see R2.2).} \]

By comparison, the firm’s profit under a negative correlation is lower than its profit under a positive correlation.

\[ \frac{\partial^2 W(M_t) - \Pi''(M_t)}{\partial k} < 0 \text{ by observing that } M_t \frac{\sqrt{\pi}}{2k} < \frac{\lambda'q(y - 3\lambda')}{2(1 - \lambda')(1 + x)} < \frac{\lambda'q(y - 3\lambda')}{\sqrt{2(1 - \lambda')(1 + x)}} \text{, where the first inequality holds by (13), and the second inequality holds because } \lambda' > \frac{q}{2} \text{ by proof of (i) (see R2.3).} \]

For \( i = 2 \), \( \frac{dM_2}{dk} = \frac{8y}{4 - x^2} + \frac{\sqrt{2w_0(2 - z)}(10k^2y^3 + 8kq^2y^2 + (4 - x^2)w_0 + 8(2 - x)w_0)\sqrt{2w_0}}{(x + 2)\sqrt{2k + 4q + z})} \) is positive if \( (x + 2)\sqrt{2k + 4q} > Z \), or equivalently, \( y < \frac{\sqrt{\frac{x(2 + x)}{2w_0}}}{k} \), which follows from \( M_2 < 2\sqrt{2k + 4q} \) (because \( M_2 \) increases in \( y \) and \( M_2 > 2\sqrt{2k + 4q} \) at \( y = \sqrt{\frac{x(2 + x)}{2w_0}} \)).

For \( i = 3 \), \( \frac{dM_3}{dk} = (2 + x - \sqrt{x(2 + x)}) \frac{4y + (2w_0(4 - x^2) + 8kq^2y^2)}{4 - x^2} > 0. \]

**Proof of Corollary 3.** We start by observing that without social comparison \((\alpha = 0, \beta = 0)\), the firm’s optimal profit is constant in \( \rho \in (-1, 1) \) (see the proof of Proposition 2(ii)). Next we show that, with social comparison, the firm’s profit under a negative correlation is lower than its profit under a positive correlation. This follows directly from the proof of Proposition 2(ii), where we show that \( \Pi' \) is increasing in \( \rho \in (-1, 1) \) for any \( m > 0 \). \( \Box \)

**Proof of Lemma 2.** (i) Agent \( i \in \{1, 2\} \) solves

\[ \max_{e_i, e_{ij}} u_i = (b + q)(e_i + he_{ij}) + q(e_j + he_{ij}) - \alpha b(e_j - e_i + he_{ij} - he_{ij}) \]

\[ - (\beta - \alpha)B \int_{\epsilon_{ij} - e_{ij} + he_{ij} - he_{ij}} (e_j - e_i + he_{ij} - he_{ij} + z) dG(z) - \frac{k}{2}(e_i^2 + e_j^2). \]

Taking the partial derivative with respect to \( e_i \) and \( e_{ij} \) yields the following first-order conditions:

\[ \frac{\partial E_u_1}{\partial e_i} = b + q + (\beta - \alpha)bG(e_i - e_2 + he_{21} - he_{12}) + \alpha b - ke_1 = 0, \]

\[ \frac{\partial E_u_1}{\partial e_{ij}} = qh - (\beta - \alpha)bhG(e_i - e_2 + he_{21} - he_{12}) - abh - ke_{12} = 0, \]

\[ \frac{\partial E_u_2}{\partial e_i} = b + q + (\beta - \alpha)bG(e_2 - e_1 + he_{12} - he_{21}) + ab - ke_2 = 0, \]

\[ \frac{\partial E_u_2}{\partial e_{ij}} = qh - (\beta - \alpha)bhG(e_2 - e_1 + he_{12} - he_{21}) - abh - ke_{21} = 0. \]

The second-order partial derivatives are

\[ \frac{\partial^2 E_u_1}{\partial e_i^2} = -(\beta - \alpha)bG'(e_i - e_j + he_{ij} - he_{ij}) - k < 0, \]

\[ \frac{\partial^2 E_u_1}{\partial e_{ij}^2} = -(\beta - \alpha)bG'(e_i - e_j + he_{ij} - he_{ij}) - k < 0, \]

\[ \frac{\partial^2 E_u_1}{\partial e_i \partial e_{ij}} = -(\beta - \alpha)bhG'(e_i - e_j + he_{ij} - he_{ij}) \leq 0, \]

and the Hessian is negative definite. Therefore, agent \( i \)'s maximization problem is jointly concave in his effort decisions. We find the equilibrium solution by solving the first-order conditions. Thus, if we add (16) and (18) and also add (17) and (19), we obtain

\[ (\beta - \alpha)b(1 - 2G(e_1 - e_2 + he_{21} - he_{12})) = k(e_1 - e_2), \]

\[ - (\beta - \alpha)bh^2(1 - 2G(e_1 - e_2 + he_{21} - he_{12})) = hk(e_1 - e_2). \]
Subtracting (21) from (20) now yields

\[(\beta - \alpha)b(1 + h^2)(1 - 2G(e_1 - e_2 + he_{21} - he_{12})) = k(e_1 - e_2 + he_{21} - he_{12}),\]

which has the unique solution \(e_1 - e_2 + he_{21} - he_{12} = 0\) (because the LHS of the equation is decreasing in \(e_1 - e_2 + he_{21} - he_{12}\) and the RHS is strictly increasing in \(e_1 - e_2 + he_{21} - he_{12}\). Substituting \(e_1 - e_2 + he_{21} - he_{12} = 0\) back into the first-order conditions, we obtain the unique equilibrium solution:

\[e_1 = e_2 = \frac{b + q + (\beta + \alpha)b/2}{k}, \quad e_{21} = hq - (\beta + \alpha)b/2,\]

which is feasible if and only if \(q \geq (\beta + \alpha)b/2\). If \(q < (\beta + \alpha)b/2\), then the equilibrium solution is

\[e_1 = e_2 = \frac{b + q + (\beta + \alpha)b/2}{k}, \quad e_{21} = 0.\]

(ii) The proof follows directly from (i). \(\square\)

**Proof of Proposition 3.** (i) The firm’s problem is

\[\max_{b,q,A} \Pi(b,q,A) = \max_{b,q,A} m(e^H_i(b,q) + he^{H}_i(b,q)) - (b + 2q)(\hat{e}^H_1(b,q) + he^{H}_1(b,q)) - A\]

subject to agents’ participation constraint

\[A + (b + 2q)(\hat{e}^H_1(b,q) + he^{H}_1(b,q)) - (\beta - \alpha)b\sqrt{\frac{\sigma}{\pi}} - \frac{k}{2} \hat{e}^H_1(b,q)^2 - \frac{k}{2} e^{H}_1(b,q)^2 \geq w_0\]

and the firm’s liability constraints \(b \geq 0, q \geq 0, A \geq 0\). Observe that by Lemma 2, \(\hat{e}^H_1(b,q) = h2^{-((\beta + \alpha)b/2)} > 0\) if \(q > \frac{(\beta + \alpha)b}{2}\) and \(\hat{e}^H_1(b,q) = 0\) otherwise. To find the optimal solution, we compare two possible solutions (denoted as \(\Pi^H\) and \(\Pi^{NH}\) respectively): (I) when \(q \geq \frac{(\beta + \alpha)b}{2}\), and (II) when \(0 \leq q \leq \frac{(\beta + \alpha)b}{2}\). We will show that there exists a threshold \(h_1\) such that \(\Pi^H \leq \Pi^{NH}\) if \(h \leq h_1\) and \(\Pi^H > \Pi^{NH}\) if \(h > h_1\).

**Solution (I):** Subject to \(q \geq \frac{(\beta + \alpha)b}{2}\), the Lagrangian corresponding to the firm’s optimization problem is

\[L = (m - b - 2q)(2 + (1 - h^2)x)b + 2(1 + h^2)q) - A + \lambda_1 \left(\frac{12(1 + h^2)q^2 + 4bq(4 + 2h^2 + (1 - h^2)x) + b^2(4 - 4h^2x - (1 + h^2)x^2)}{8k} + A - w_0 - by\right) + \lambda_2(q - xb)/2,\]

where \(\lambda_1\) and \(\lambda_2\) are Lagrange multipliers. The critical points satisfy the following KKT conditions: \(\frac{\partial L}{\partial b} \leq 0, b \geq 0, b \cdot \frac{\partial L}{\partial b} = 0, \frac{\partial L}{\partial b} \leq 0, A \geq 0, A \cdot \frac{\partial L}{\partial A} = 0, \lambda_1 \geq 0, \lambda_2 \geq 0, \frac{\partial L}{\partial m} = 0, \) We solve these conditions to derive five solutions, depending on the value of \(h\) and \(m\). The optimal solution is

\[
\Pi^H = \begin{cases} 
\Pi^{H, NB1} = \frac{(1 + h^2)k^2}{2k} - w_0, & \text{if } m \leq m_0^H \equiv \sqrt{\frac{2kw_0}{3(1 + h^2)}}, \\
\Pi^{H, NB2} = \frac{k^2}{2k} \frac{1}{1 + h^2} - w_0, & \text{if } m_0^H < m \leq m_1^H \equiv 4 \sqrt{\frac{2kw_0(1 + h^2)}{3(1 + h^2)(k + (1 + h^2))}}, \\
\Pi^{H, BQ} = \frac{k^2}{2k} \frac{1}{1 + h^2} - w_0, & \text{if } m_1^H < m \leq m_2^H \equiv \left(\frac{k^2}{1 + h^2} + \sqrt{\frac{2kw_0(1 + h^2)}{3(1 + h^2)(k + (1 + h^2))}}\right)^2, \\
\Pi^{H, C1} = \frac{k^2}{2k} \frac{1}{1 + h^2} - w_0, & \text{if } m_2^H \leq m \leq m_3^H \equiv \left(\frac{k^2}{1 + h^2} + \sqrt{\frac{2kw_0(1 + h^2)}{3(1 + h^2)(k + (1 + h^2))}}\right)^2, \\
\Pi^{H, C2} = \frac{k^2}{2k} \frac{1}{1 + h^2} - w_0, & \text{if } m \geq m_3^H, 
\end{cases}
\]

where \(b^{H, C1} = \frac{k^2}{1 + h^2} + \sqrt{\frac{2kw_0(1 + h^2)}{3(1 + h^2)(k + (1 + h^2))}}\). The functions \(\Pi^{H, C1}\) and \(\Pi^{H, C2}\) are the expected profits when the constraint \(q \geq xb/2\) binds, \(\Pi^{H, NB1}\) and \(\Pi^{H, NB2}\) are the expected profits when \(b \geq 0\) binds, and \(\Pi^{H, BQ}\) is the expected profit when only the PC and \(A \geq 0\) binds.
Solution (II): In this case the agents do not help each other. Therefore, similar to the proof of Proposition 1, we find five solutions depending on the value of $m$. The optimal solution is

$$
\Pi^{NH} = \begin{cases} 
\Pi^{NH,C1} = \frac{m^2}{2k} + \frac{k^2}{2(1+k)^2} \lambda - \frac{(1+x)m + my}{1+x}, & \text{if } m \leq m_0^{NH} \equiv \frac{2k + \sqrt{2k m_0(1+x)^2 + k^2 y^2}}{1+x}, \\
\Pi^{NH,C2} = \frac{(1+x)(m - \Pi^{NH,C1}(1+x))}{k}, & \text{if } m_0^{NH} < m \leq m_1^{NH} \equiv \frac{(k y + \sqrt{2k m_0 (1+x)^2 + k^2 y^2})(4k y + \sqrt{2k m_0 (1+x)^2 + k^2 y^2})}{(1+x)(2k y + \sqrt{2k m_0 (1+x)^2 + k^2 y^2})}, \\
\Pi^1, & \text{if } m_1^{NH} < m < m_2, \\
\Pi^2, & \text{if } x < 2 & m_2 \leq m \leq m_3, \\
\text{if } x < 2 & m \geq m_3,
\end{cases}
$$

where $m_2$, $m_3$, $\Pi^1$ and $\Pi^2$ are as given in the proof of Proposition 1. The functions $\Pi^{NH,C1}$ and $\Pi^{NH,C2}$ are the expected profits when the constraint $q \leq xb/2$ binds.

Since $\Pi^{NH}$ does not change in $h$, it suffices to show that $\Pi^H$ is increasing in $h$. It is straightforward to observe that $\Pi^{H,B1}$, $\Pi^{H,B2}$, $\Pi^{H,C1}$, $\Pi^{H,C2}$ are all increasing in $h$. It remains to prove $\Pi^{H,BQ}$ is increasing in $h$. For ease of exposition, we substitute $H = h^2$.

When $\Pi^H = \Pi^{H,BQ}$, the optimal solution satisfies

$$
b^H = \frac{(1 + H)(4 - 3\lambda_1)\lambda_2 - (1 - H)(1 - \lambda_1)(1 + x)m}{(1 + x)^2((1 + H^2)(1 - \lambda_1)^2 - H(2 - \lambda_1^2))} > 0,
$$

$$
q^H = \frac{m(1 + x)((1 - \lambda_1)(1 + x)(xH^2 + 2 + x) - 2H(1 + x)) - k\lambda_1 y(2H(1 - x) + 2(3 + x) - \lambda_1(4 + 2H + (1 - H)x))}{2(1 + x)^2((1 + H^2)(1 - \lambda_1)^2 - H(2 - \lambda_1^2))} > \frac{xb^H}{2},
$$

where $\lambda = \lambda_1 \in [0, 1]$ solves

$$
f_1(\lambda) = \frac{(H(2 + \lambda^2) - (1 + H^2)(1 - \lambda^2)H(1 + x)^2)}{(1 + H^2)(1 - \lambda_2^2) - H(2 - \lambda_2^2)} m^2 + \frac{2k(1 + x)y(1 + H)((1 - H + H^2)\lambda_2^2 - (1 + 2\lambda)(1 - H)^2)}{(1 + H^2)(1 - \lambda_2^2) - H(2 - \lambda_2^2)} m
$$

$$
+ \frac{k^2 y^2(-8(1 - H)^2 + (17 - 18H + 17H^2)\lambda - 12(1 + H^2)\lambda_2^2 + 3(1 + H + H^2)\lambda^3)}{(1 + H^2)(1 - \lambda_2^2) - H(2 - \lambda_2^2)} - \frac{2k y_0(1 + x)^2}{1 + H} = 0.
$$

The firm’s optimal profit is given by

$$
\Pi^{H,BQ} = \frac{k(1 + x)^2((1 + H^2)(1 - \lambda_2^2) - H(2 - \lambda_2^2)^2)}{(1 + H + \lambda_2^2)^2} - \frac{H(1 - H - \lambda_2) m(1 + x) - k(2 - \lambda_2) y}{1 + H}.
$$

From $\frac{\partial \Pi^{H,BQ}}{\partial H} = \frac{\partial \Pi^{H,BQ}}{\partial \lambda} + \partial \Pi^{H,BQ}/\partial H$, we have

$$
\frac{\partial \Pi^{H,BQ}}{\partial H} = \left( m + \frac{(1 - H)(1 - \lambda_2)(1 - \lambda_2 - H\lambda_2 - (1 - \lambda_2)(1 + x)m)}{(1 + H + \lambda_2^2)(1 - \lambda_2^2 - H(2 - \lambda_2^2))} \right) > 0.
$$

The inequality holds because $b^H > 0$ and $q^H - \frac{xb^H}{2} = \frac{m(1 + x)((1 - \lambda_2)(1 + x)(xH^2 + 2 + x) - 2H(1 + x)) - k\lambda_1 y(2H(1 - x) + 2(3 + x) - \lambda_1(4 + 2H + (1 - H)x))}{2(1 + x)^2((1 + H^2)(1 - \lambda_2^2) - H(2 - \lambda_2^2))} > 0$. This completes the proof.

(ii) The proof follows directly from the proof of (i). \qed

Proof of Proposition 4. By proof of Proposition 3, the firm’s profit without social comparison ($\alpha = 0$, $\beta = 0$) is

$$
\Pi_R = \begin{cases} 
\Pi^{R,NB} = \frac{(1 + H)m^2}{2k} - w_0, & \text{if } m < \sqrt{2k w_0 / (1 + H)}, \\
\Pi^{R,BQ} = \frac{(1 + H^2)(1 - \lambda_2^2)H(1 + H)(1 - \lambda_2) m^2}{k((1 + H^2)(1 - \lambda_2^2) - H(2 - \lambda_2^2))^2}, & \text{if } \sqrt{2k w_0 / (1 + H)} < m < (1 + H)\sqrt{2k w_0}, \\
\Pi^{R,NQ1} = \frac{2k w_0(1 + H)H(1 - \lambda_2) - k\lambda_1 y(1 + H)(1 - \lambda_2) - H(2 - \lambda_2^2))}{2(1 + H + \lambda_2^2)(1 - \lambda_2^2) - H(2 - \lambda_2^2)} > 0. & \text{if } (1 + H)\sqrt{2k w_0} < m < 2\sqrt{2k w_0}, \\
\Pi^{R,NQ2} = \frac{m^2}{2k}, & \text{if } m > 2\sqrt{2k w_0}.
\end{cases}
$$
where $\lambda = \lambda_0 \in [0, 1)$ solves

$$
\frac{(H(2 + \lambda^2) - (1 + H^2)(1 - \lambda^2))}{((1 - \lambda)^2(1 + H^2) - H(2 - \lambda^2))^2} = \frac{2kw_0}{1 + H} = 0,
$$

and satisfies

$$
b^R = -\frac{(1 + H)(1 - H)(1 - \lambda)m}{(1 + H^2)(1 - \lambda)^2 - H(2 - \lambda^2)} > 0 \quad \text{and} \quad q^R = \frac{m(1 - \lambda - H)}{(1 + H^2)(1 - \lambda)^2 - H(2 - \lambda^2)} > 0.
$$

With social comparison, the firm’s profit is $\max\{\Pi^H, \Pi^{NH}\}$. Recall from the proof of Proposition 3 that when $m < m_{1}^{NH}$, the constraint $q \leq xb/2$ binds for $\Pi^{NH}$. This implies that $\Pi^H \geq \Pi^{NH}$ in this region (because $\Pi^{NH}$ is in the feasible solution set of (I)). Similarly, when $m > m_{2}^H$, $\Pi^{NH} \geq \Pi^H$. It follows that the firm’s profit (denoted by $\Pi^*$) is

$$
\Pi^* = \begin{cases} 
\Pi^H, & \text{if } 0 < m \leq m_{1}^{NH}, \\
\max\{\Pi^H, \Pi^{NH}\}, & \text{if } m_{1}^{NH} < m < m_{2}^H, \\
\Pi^{NH}, & \text{if } m \geq m_{2}^H.
\end{cases}
$$

(i) We prove that as $m$ increases, $\Pi^*$ crosses $\Pi_R$ at most once (and from below). We do so by considering the cases of $\Pi^* = \Pi^H$ and $\Pi^* = \Pi^{NH}$ separately. We first show that $\Pi^H - \Pi_R \leq 0$ always. Next we prove that as $m$ increases in $(m_{1}^{NH}, +\infty)$, $\Pi^{NH}$ crosses $\Pi_R$ at most once (and from below).

To prove $\Pi^H - \Pi_R \leq 0$, we observe that for any $m > 0$, $\Pi^H - \Pi_R = 0$ at $y = 0$, and that $\Pi^H - \Pi_R$ decreases in $y \in [0, +\infty)$ (the proof is similar to that in the proof of Proposition 2 and hence omitted).

Next, we prove that as $m$ increases in $(m_{1}^{NH}, +\infty)$, $\Pi^{NH}$ crosses $\Pi_R$ at most once (and from below). We consider three possible regions.

**Region 1:** $\{m_{1}^{NH} < m < m_{2}^H\}$. In this region, there are three possible cases. In the first two cases, i.e., if $m \geq 2\sqrt{2kw_0}$ or if $(1 + H)\sqrt{2kw_0} \leq m < 2\sqrt{2kw_0}$, we know $\Pi^{NH}$ crosses $\Pi^R$ at most once and at below (by proof of Proposition 2(i)). It remains to prove that in the third case, i.e., if $m_{1}^{NH} < m < (1 + H)\sqrt{2kw_0}$, $\Pi^{NH} - \Pi^H = \Pi^H - \Pi^{R,BO}$ is increasing in $m$. To prove $\frac{d[\Pi^H - \Pi^{R,BO}]}{dm} > 0$, it suffices to prove (a) $\frac{\lambda_0^y}{(1 + \lambda_0^y)^2} > \sqrt{2kw_0}$ and (b) $\sqrt{\frac{2kw_0}{k(H(2-H)^2)} - \frac{\lambda_0^y}{(1 + \lambda_0^y)^2}} > 0$.

To prove (a), observe that $\frac{\lambda_0^y}{(1 + \lambda_0^y)^2} > \frac{2(1-\lambda_0^y)^2(1-\lambda_0^y)}{2(1-\lambda_0^y)^2(1-\lambda_0^y)} = \frac{2(1-\lambda_0^y)^2}{(1-\lambda_0^y)^2(1+\lambda_0^y)} > 0$, which holds because the constraint $xb^2/2 - q^2 = \frac{(3-2\lambda_0^y)^2\lambda_0^y}{(1+\lambda_0^y)^2} \geq 0$ implies $m \leq \frac{3-2\lambda_0^y}{(3-2\lambda_0^y)^2\lambda_0^y}$, which is equivalent to $m \leq \frac{3-2\lambda_0^y}{(3-2\lambda_0^y)^2\lambda_0^y}$.

To prove (b), we show $\sqrt{\frac{2kw_0}{k(H(2-H)^2)} - \frac{\lambda_0^y}{(1 + \lambda_0^y)^2}} > 0$, which holds because $b^R > 0$ and $q^R > 0$ (see (23)).

**Region 2:** $\{x < 2 \& m_2 \leq m < m_3\}$. Again, there are three possible cases. In the first two cases, i.e., if $m \geq 2\sqrt{2kw_0}$ or if $(1 + H)\sqrt{2kw_0} \leq m < 2\sqrt{2kw_0}$, we know $\frac{d[\Pi^{NH} - \Pi_R]}{dm} > 0$ by proof of Proposition 2(i). It remains to prove that in the third case, i.e., $m < (1 + H)\sqrt{2kw_0}$, $\Pi^{NH} - \Pi_R$ increases in $m$. Note that in this case,$\frac{d[\Pi^{NH} - \Pi_R]}{dm} = \frac{4kw_0 + 2\sqrt{4k^2w_0^2 + 2kw_0(1-x^2)}}{2(1-x^2)} - \frac{H(1+H)\lambda_0^m}{k(H(2-H)^2)(1-\lambda_0^m)} > 0$, where the first inequality holds because $y \geq 0$ and $x \in [0, 2)$, and the second inequality holds as shown in Region 1.
Region 3: \( \{ x < 2 \& m \geq m_3 \} \). In this region, \( \Pi_{NH} - \Pi_R = \Pi^{I.NQ2} - \Pi^{R.NQ2} = \frac{zm^2}{8k} > 0 \).

Combining Regions 1, 2, and 3, we conclude that there exists a unique threshold \( m_H \) such that \( \Pi^* - \Pi_R \leq 0 \) if \( m \leq m_H \) and \( \Pi^* - \Pi_R > 0 \) if \( m > m_H \).

(ii) The proof is similar to that of Proposition 2(ii) and hence omitted. \( \square \)

**Proof of Corollary 4.** The result follows directly from the proof of Proposition 4, where we show that \( \Pi^* > \Pi_R \) only if \( \Pi^* = \Pi^{NH} \). That is, wage transparency benefits the firm only if agents do not exert help effort under the optimal compensation scheme. \( \square \)

**Proof of Lemma 3.** The lemma follows directly from observing that

\[
\frac{\partial E_u(k)}{\partial k} = -\frac{4(b^2 + 4bq + 3q^2) - 4bq(\beta + \alpha) + b^2(\beta + \alpha)^2}{8k^2}
\]

is a quadratic, convex function of \( \beta + \alpha \), is negative at \( \beta + \alpha = 0 \) and has a positive root at \( \beta + \alpha = \frac{2b + 6q}{k} \).

Therefore, \( \frac{\partial E_u(k)}{\partial k} < 0 \) if and only if \( \beta + \alpha < \frac{2b + 6q}{k} \). \( \square \)

**Proof of Proposition 5.** The proposition holds because the LHS of equation (6) is decreasing in \( \beta + \alpha \) if \( q = 0 \) and \( b > 0 \), is constant in \( \beta + \alpha \) if \( b = 0 \), and is first increasing and then decreasing in \( \beta + \alpha \) if \( b > 0 \) and \( q > 0 \). \( \square \)

**Proof of Proposition 6.** By Proposition 2, if agents cannot engage in collaboration, then the firm benefits from social comparison if and only if \( m > m_I \).

If the agents can engage in collaboration and the collaboration constraint in (6) is not binding, then the firm benefits from social comparison if and only if \( m > m' \).

Since \( k' < k \), we have \( m' < m_I \) (see Proposition 2(ii)). Therefore, it remains only to find a sufficient condition under which the optimal solution to the firm’s unconstrained maximization problem (given effort cost \( k' \)) satisfies the collaboration constraint (6) for all \( m > m' \). This condition is

\[
\psi \leq \bar{\psi} \equiv \frac{k - k'}{8kk'}(12(q'(m'))^2 + 4(4 + \beta + \alpha)b'(m')q'(m') + (4 - (\beta + \alpha)^2)(b'(m'))^2),
\]

(24)

where \( (b'(m), q'(m)) \) are the optimal solutions given in the proof of Proposition 1 corresponding to \( \rho = 0 \), a cost of effort equal to \( k' \), and a given \( m \). To see this, first observe that (24) implies that the constraint (6) is satisfied at \( m = m' \). Second, we can confirm that \( 12(q'(m'))^2 + 4(4 + \beta + \alpha)b'(m')q'(m') + (4 - (\beta + \alpha)^2)(b'(m'))^2 \) is increasing in \( m \), so if (24) holds, then the constraint (6) is satisfied for all \( m > m' \). \( \square \)