XI CHEN, RALF VAN DER LANS, and TUAN Q. PHAN*

Seeding influential social network members is crucial for the success of a viral marketing campaign and product diffusion. In line with the assumption that connections between customers in social networks are binary (either present or absent), previous research has generally recommended seeding network members who are well-connected. However, the importance of connections between customers varies substantially depending on the relationship's characteristics, such as its type (i.e., friend, colleague, or acquaintance), duration, and interaction intensity. This research introduces a new Bayesian methodology to identify influential network members and takes into account the relative influence of different relationship characteristics on product diffusion. Two applications of the proposed methodology—the launch of a microfinance program across 43 Indian villages and information propagation in a large online social network—demonstrate the importance of weighting connections in social networks. Compared with traditional seeding strategies, the proposed methodology recommends substantially different sets of seeds that increased the reach by up to 10% in the first empirical application and up to 92% in the second.

Keywords: product diffusion, social network analysis, seeding strategy, Bayesian method, multigraph network

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Uncovering the Importance of Relationship Characteristics in Social Networks: Implications for Seeding Strategies

A person's decision to adopt a new product or service often depends on the behaviors of friends, colleagues, and acquaintances. Consequently, the structure of a social network and the position of influencers in a network are important drivers of the spread of information and the diffusion of products (Van den Bulte and Wuyts 2007). To capitalize on these social effects, companies such as Philips, HP, and Microsoft have adopted seeding strategies that target influential customers in social networks to launch new products (Libai, Muller, and Peres 2013). However, because of the complexity of social networks, determining which customers are most influential is a nontrivial question that has generated much theoretical interest (Godes and Mayzlin 2009; Goldenberg et al. 2006; Iyengar, Van den Bulte, and Valente 2011). Recently, as a result of the increasing availability of social network data, this question has received a growing amount of attention from both practitioners and academics. These efforts have led to a more detailed understanding of how networks stimulate diffusion and which people possess influential positions (Goldenberg et al. 2009; Hinz et al. 2011; Hu and Van den Bulte 2014; Iyengar, Van den Bulte, and Valente 2011; Katona, Zubcsek, and Sarvary 2011; Tucker 2008; Yoganarasimhan 2012). For instance, Goldenberg et al. (2009) and Yoganarasimhan (2012) demonstrate that a person's number of connections has a positive

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effect on the diffusion process. Tucker (2008) finds that, in addition to the number of connections, the effect of betweenness (how many times a person is on the shortest path between two members of the social network) on adoption may be even stronger. In line with this notion, previous research has recommended using network metrics to pinpoint influential customers in a social network (Hinz et al. 2011; Iyengar, Van den Bulte, and Valente 2011).

Although previous studies have compared the effects of many different network measures on diffusion, they have generally assumed a binary network structure with connections that are either absent or present (0 vs. 1, respectively). This is a strong assumption, given ample evidence in marketing and sociology that consumers are connected through relationships with different characteristics of varying importance (Ansari, Koenigsberg, and Stahl 2011; Brown and Reingen 1987; De Bruyn and Lilien 2008; Granovetter 1973; Trusov, Bodapati, and Bucklin 2010). Furthermore, relationships often vary in importance depending on the type of product in the diffusion process (Schulze, Schöler, and Skiera 2014). Consequently, researchers have not been able to observe the importance of relationship characteristics in social networks, and shedding light on this importance is a primary issue in marketing and sociology (Aral and Walker 2014). Dover, Goldenberg, and Shapira (2012) recognize the limitation of assuming 0–1 connections between customers in diffusion processes and have called for future research incorporating information on the strength of connections. The present study follows up on this call and introduces a novel approach to identify influential customers in a social network. Our proposed multinetwork methodology uncovers (1) the relative importance of different characteristics of relationships in a social network and (2) which people in the subsequent weighted social network should be seeded to generate the highest impact on the diffusion of products.

We demonstrate the effectiveness of our multinetwork methodology in two empirical applications. The first application involves the diffusion of a microfinance program rollout in 43 Indian villages. The second application focuses on information propagation in a large online social network consisting of more than 42,000 users. In both applications, we found that taking into account the importance of relationship characteristics is crucial to identify influential network members. Moreover, our proposed methodology was able to increase the reach of seeding strategies by up to 10% in the first empirical application and up to 92% in the second.

The rest of the article is organized as follows. We first discuss social networks and the strength of connections as a function of relationship characteristics. Next, we introduce our multinetwork methodology, which we illustrate in two empirical applications. We conclude with a discussion of our findings, implications, and directions for further research.

CONNECTIONS IN SOCIAL NETWORKS

Strength of Connections

Granovetter (1973) argues that connections in networks vary in strength and that this has major implications for the diffusion of information. Although Granovetter categorized connections between people as either strong or weak, he recognized that the underlying strength of a connection is a continuous variable. Granovetter proposes that the continuous variable for the strength of a connection is a function of duration, emotional intensity, intimacy, and exchange of services between people. The strength of a connection thus depends on not only the characteristics of the relationship but also the type of information or service exchanged. Several empirical studies in marketing support these observations. For instance, in a study among adolescents, Moschis and Moore (1979) find that friend relationships are most important for the adoption of products in which peer acceptance plays a role (e.g., sunglasses), whereas parent relationships are essential for products with a higher perceived risk in terms of price and performance (e.g., hair dryer). Similarly, among sorority members at a university, Reingen et al. (1984) find that social influence on brand choice heavily depends on the type of product and characteristics of the relationship (e.g., roommate, friend, neighbor, study partner). In online settings, De Bruyn and Lilien (2008) find that people are more likely to open e-mails from senders with whom they have a stronger relationship, such as friends and family. Aral and Walker (2014), in a large-scale field experiment on Facebook, find that the recency of a relationship is a strong predictor for friends to adopt a recommended application. In contrast, Godes and Mayzlin (2009) discover that word of mouth through weak relationships (acquaintances) has stronger impact on sales than that through strong relationships (friends and relatives). Schulze, Schöler and Skiera (2014) demonstrate that these effects are moderated by the type of product and find that apps for hedonic products are shared more effectively by strong relationships (friends) than weak relationships (strangers).

Despite the evidence that strengths of connections between people vary systematically, previous research has generally treated the social network as given and has a priori determined the strength of connections between people (e.g., Banerjee et al. 2013; Goldenberg et al. 2009; Hinz et al. 2011; Iyengar, Van den Bulte, and Valente 2011; Katona, Zubcsek, and Sarvary 2011; Tucker 2008; Yoganarasimhan 2012). Specifically, previous research has used the following approaches to deal with different relationship characteristics. First, many studies ignore the differential influence of relationship characteristics and design one binary network (e.g., Banerjee et al. 2013; Hinz et al. 2011; Katona, Zubcsek, and Sarvary 2011; Tucker 2008; Yoganarasimhan 2012). Second, other studies recognize the limitations of one binary network and relate multiple binary networks, each corresponding to a different relationship characteristic, to diffusion (e.g., Aral and Walker 2012; Hu and Van den Bulte 2014). Third, some studies determine the importance of different relationship characteristics a priori and subsequently integrate them to obtain a weighted network (e.g., Newman 2001; Rothenberg et al. 1995). In a weighted network, a connection between people receives a positive value corresponding to its strength (see Van den Bulte and Wuyts 2007). Finally, other studies use a combination of the aforementioned methods to test the robustness of the results against these different approaches (e.g., Iyengar, Van den Bulte, and Valente 2011; Van den Bulte and Lilien 2001). All approaches determine the importance of relationship characteristics a priori, even though their importance varies. Such treatment could lead to misspecified networks, which in turn could lead to biased estimates of social influence and a suboptimal selection of seeds, as discussed by both sociologists and econometricians (Leenders 2002; LeSage and Pace 2014; Páez, Scott, and Volz 2008). Next, we discuss how weighted networks may capture diffusion in complex social networks.

Social Relationships in Weighted Networks

A natural way to capture the strength of connections between people is to represent the strength in a weighted network. In a weighted network, each connection between two people receives a continuous value representing the connection's strength (Van den Bulte and Wuyts 2007). High (low) positive values correspond to strong (weak) connections between people. Although weighted networks are more difficult to analyze, several researchers have developed network measures that extend their dichotomous network counterparts to weighted networks (e.g., Bonacich and Lloyd 2001; Newman 2004; Opsahl, Agneessens, and Skvoretz 2010).

The analysis of weighted networks has received much attention in the literature, especially for neural networks, transportation networks, and food webs in biology (e.g., Luczkovich et al. 2003; Opsahl et al. 2008; Watts and Strogatz 1998). In such networks, assigning weights to connections is relatively straightforward because strengths between connections are observed. For instance, in a food web, weights are characterized by carbon flows between species, whereas in transportation networks, travel time or the number of vehicles commuting between locations can be used to evaluate the weight of a connection. Assigning weights to connections in social networks, however, is more difficult. Strengths of connections are not observed and depend on the characteristics of underlying relationships and the information exchanged. Some prior research has attempted to observe strengths of connections, however. For instance, to study the role of social network structure on disease transmission, Rothenberg et al. (1995) weighted connections on the basis of their riskiness, with weights of .5 for injectable drug use, .3 for sexual contacts, and .1 for noninjectable drug use. Similarly, Newman (2001) weighted the connections in a social network of coauthors with the inverse of the number of authors on a paper. Recently, in a marketing context, Ansari, Koenigsberg, and Stahl (2011) assigned downloads between artists as the weight for connections between artists in an online social network. In a study on the prescription behavior of physicians, Iyengar, Van den Bulte, and Valente (2011) used the number of different types of relationships (discussion or referral) between physicians as the weight in the "total" network.

An important commonality of these empirical applications is that weights are assigned a priori. Some researchers have recognized this problem and compare different weights on the basis of statistical fit (Leenders 2002; Páez, Scott, and Volz 2008) or robustness of results (Iyengar, Van den Bulte, and Valente 2011; Van den Bulte and Lilien 2001). A disadvantage of these approaches is that they are only able to compare a limited number of possible predetermined weights. In reality, this number is infinite because the weight of a connection is a continuous variable. As we have discussed, the weight of a connection depends not only on the characteristics of the relationships it consists of but also on the type of information exchanged. Thus, ideally, connection weights should be inferred from the actual diffusion taking place on the network, rather than assigned a priori by the researcher. To the best of our knowledge, such a methodology does not exist. The only research in marketing that estimates weights in social networks is the study by Trusov, Bodapati, and Bucklin (2010). In an online social network, they estimated how susceptible a person was toward social influence and which connections influenced a person's login behavior. However, their model was developed specifically to infer influence and susceptibility from repeated login decisions in an online social network and focused on egocentric networks only. Thus, their approach cannot be applied to seeding decisions in diffusion processes in which consumers make only one adoption decision. In the next section, we introduce a new approach that infers weights of connections on the basis of relationship characteristics and the actual diffusion process.

THE MULTINETWORK APPROACH FOR SEEDING DECISIONS

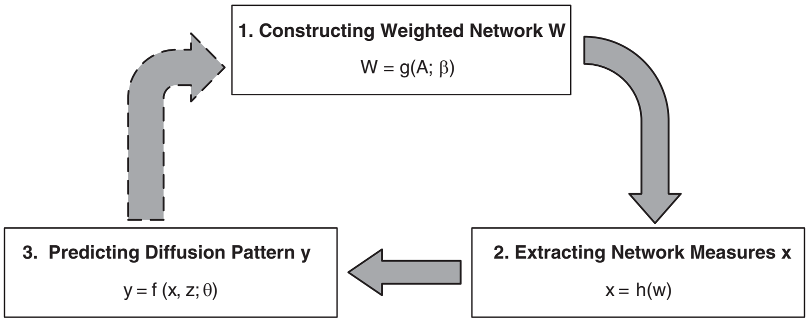
Consider the diffusion of a product or service on a social network consisting of N people who are connected through K different networks, each representing a different relationship characteristic. For instance, a network could represent only friend relationships, while another network could represent the duration of relationships. A network $k \in K$ is represented by an $N \times N$ adjacency matrix A_k with elements a_{ij}^k representing the relationship characteristic k between sender i and receiver j. These elements could be dichotomous (0 vs. 1), indicating whether a relationship characteristic is absent or present, or continuous, representing a numeric property of a relationship k, such as its duration. Furthermore, we permit $a_{ij}^k \neq a_{ji}^k$ to allow both directed and undirected relationship characteristics. For instance, $a_{ij}^k = 1$ and $a_{ji}^k = 0$ in a network of advisors and students in which individual i is an advisor, individual j is a student, and k represents the advisor–advisee relationship.

To initiate the diffusion process, we assume that S people on the social network are seeded by the company, with S being a subset of N. For each seed $s \in S$, we count the number of people y_s (which may include seed s itself) who adopt a product as a result of an information cascade initiated by seed s. Thus, y_s represents the reach of seed s, and $\sum_{s=1}^{S} y_s$ represents the total number of adopters in the diffusion process. Note that the definition of y_s could be more general and could include a time index t to indicate the number of adopters in period t due to seed s. Moreover, the number of seeds s may also change over time to reflect situations in which a company seeds additional people at different moments in time (Van der Lans et al. 2010). The goal of our multinetwork methodology is to construct a weighted social network that optimally links the weighted network measures of seed s to the observed reach y_s. To do so, our approach consists of three components. The first component creates the weighted network, which is a function of multiple networks representing different relationship characteristics. The second component computes network measures of each seed s from the constructed weighted network. Finally, the third component relates these derived network measures, in combination with possible other control variables, to the obtained reach of each seed. Figure 1 summarizes the three components of our multinetwork procedure and shows how they are related schematically. In the following subsections, we discuss the specification of the three model components, our Bayesian estimation strategy, and model identification.

Constructing the Weighted Social Network

Given the K adjacency matrices A_k, we define the weighted network construction function $g(\cdot)$, which generates $N \times N$ weighted adjacency matrix W with elements w_{ij} as follows:

Figure 1
CONCEPTUALIZATION OF MULTINETWORK APPROACH



Notes: In Component 1, W is a weighted adjacency matrix and is specified as a function of different characteristics of relationships, summarized in A and parameter vector β. In Component 2, weighted network measures x (e.g., weighted degree centrality, eigenvector centrality) are derived using function $h(\cdot)$ of the weighted network W. Finally, in Component 3, the reach of seeds y is a function of network measures x, control variables z, and parameter vector θ. The dashed feedback arrow from Component 3 to Component 1 represents the feedback loop in the Bayesian iterative estimation procedure.

$$w_{ij} = \begin{cases} 0 & \text{if } a_{ij}^k = 0, \forall k = 1, ..., K \\ g(a_{ij}; \beta) & \text{otherwise} \end{cases} \tag{1}$$

In Equation 1, $w_{ij} \in [0, \infty)$ represents the weight of the connection between individuals i and j given the diffusion process. Vector $a_{ij} = [a_{ij}^1, a_{ij}^2, ..., a_{ij}^K]$ represents the set of all K relationship characteristics between i and j, and β is a vector of parameters. Function $g(\cdot)$ is allowed to represent any (non)linear function, as long as the outcome is nonnegative. In this research, similar to Ansari, Koenigsberg, and Stahl (2011), we choose an exponential function for $g(\cdot)$, such that $g(a_{ij}; \beta) = \exp(\beta_0 + \sum_{k=1}^{K} \beta_k a_{ij}^k)$ is nonnegative.[1] If none of the K relationship characteristics is present between individuals i and j (i.e., $a_{ij}^k = 0, \forall k = 1, ..., K$), we set the corresponding weight w_{ij} equal to zero.

Extracting Network Measures

Given the weighted social network W, the researcher needs to determine for each seed s which network measures x_s are important for diffusion. As we have discussed, previous research in marketing has proposed several network measures that are important for diffusion, such as degree centrality and betweenness centrality. Although these measures are all derived from unweighted networks, previous research has developed corresponding measures for weighted networks (e.g., Bonacich and Lloyd 2001; Newman 2004; Opsahl, Agneessens, and Skvoretz 2010). To derive network measures x_s from weighted social network W, we introduce the deterministic function $h(\cdot)$ defined as follows:

$$x_s = h(W). \tag{2}$$

In Equation 2, x_s could contain J network measures that are expected to explain the diffusion process y_s. Examples of such network measures are weighted degree centrality, eigenvector centrality, and weighted betweenness centrality.

Relating Network Measures to Diffusion

In the final component of our proposed methodology, we relate network measures x, in addition to possible control variables z, to the S × 1 vector y summarizing the reach of the diffusion process. As discussed previously, outcome variable y_s may represent for each seed s = 1, ..., S and time period t = 1, ..., T the number of adopters reached by s. In that case, y equals an S × T matrix with element y_{st} representing the reach of seed s at time t. Note that the outcome variable can be adapted to incorporate diffusions of multiple products or diffusions on multiple social networks.

To identify how the weighted network measures of seed s predict the number of adopters, we propose a diffusion equation $f(\cdot)$ that relates both control variables z and network measures x to reach y:

$$y = f(x, z; \theta). \tag{3}$$

In Equation 3, θ represents model parameters that relate weighted network measures x and control variables z to diffusion. Note that Equation 3 is flexible and may represent any (non)linear relationship between reach y, network measures x, and control variables z. For instance, in our first empirical application we chose a linear specification for $f(\cdot)$, and in our second we used a generalized linear model with Poisson link function.

Model Estimation and Identification

Combining Equations 1 and 3 results in the following model likelihood:

$$y = f\{h[g(A; \beta)], z; \theta\}, \tag{4}$$

which is highly nonlinear as a result of the possible nonlinearity of $g(\cdot)$ and $h(\cdot)$. However, as explained previously and illustrated in Figure 1, the model parameters β and θ can be naturally decomposed in separate blocks. Conditional on β, estimation of θ is relatively straightforward depending only on the diffusion Equation 3. The Bayesian estimation framework, which estimates model parameters iteratively conditional on the value of other parameters, is therefore a natural tool to estimate the model parameters. Conditional on β, one can use standard Bayesian estimation procedures, such as the Gibbs sampler, to draw θ. Furthermore, conditional on θ, a Metropolis–Hastings step can be used to draw β because the posterior distribution usually has an unfamiliar form. In

[1]In the robustness checks in our empirical applications, we also tested different specifications of $g(\cdot)$.

summary, our proposed multinetwork methodology for diffusion processes is flexible and optimally integrates multiple networks of relationship characteristics into a weighted network. Our approach enables us to identify which seeds are most influential, given their weighted network position, and other possible control variables.

To apply the Bayesian estimation procedure, we need to ensure that the model is identified. Identification of θ is similar to a standard model without estimation of β, and identification thus depends on Equation 3 given β. However, identification of β is less straightforward because network measures computed by $h(\cdot)$ in Equation 2 are often relative measures, such that $h(W) = h(\alpha W)$, with $\alpha > 0$. To control for this scaling issue, we need to fix one of the elements of parameter vector β for identification. For instance, if $g(a_{ij}; \beta) = \exp(\beta_0 + \sum_{k=1}^{K} \beta_k a_{ij}^k)$, it is sufficient to set the constant to zero ($\beta_0 = 0$).

Multinetwork Approach for Seeding Decisions

Seeding decisions are executed at the start of the diffusion process. To be able to apply the multinetwork approach for optimal seed selection, marketers need to know the parameter estimates β. There are three ways to obtain these estimates at the launch of a campaign. First, it is possible to estimate the parameters on a small set of seeds and use this information to optimally select the remaining set of seeds. This approach is popular in viral marketing campaigns, in which companies test a few seeding strategies on a smaller set of customers before starting the actual launch of the campaign (Van der Lans et al. 2010). Our second empirical application adopts a holdout seeding procedure following this approach. Second, if a company has information on the diffusion process of comparable products on the same social network, it is possible to use this information to determine the importance of relationship characteristics. Such an approach is similar to the "guessing by analogy" approach that is popular to predict sales before the product launch (Jiang, Bass, and Bass 2006; Lilien, Rao, and Kalish 1981). Finally, marketers could also obtain information about the importance of relationship characteristics by sequentially launching the products in different, but comparable, social networks. Such a sequential market rollout strategy is common practice because it reduces the risk of a new product launch (Bronnenberg and Mela 2004; Kalish, Mahajan, and Muller 1995). The microfinance company in our first empirical application applies this strategy.

EMPIRICAL APPLICATION I: SEEDING A MICROFINANCE DIFFUSION PROGRAM IN INDIAN VILLAGES

We applied our multinetwork methodology to a data set depicting the diffusion of a program launched by a microfinance institution called Bharatha Swamukti Samsthe (BSS) across 43 Indian villages (Banerjee et al. 2013; Jackson, Rodriguez-Barraquer, and Tan 2012). This institution is located in Bangalore (southwest India) and provides microcredit to households in small villages in southern India. To promote its microfinance program, BSS marketing strategy depended on word of mouth and seeded an initial group of leaders in each village, such as teachers, shopkeepers, priests, and social workers. These seeds were stimulated to spread the microfinance program among their contacts so that, subsequently, these contacts could in turn influence their own contacts. Because seeds were not selected on the basis of their position in the network, this situation is a suitable setting to study the effect of network positions of seeds on total diffusion. Banerjee et al. (2013) investigated this research question and collected social network data in 43 Indian villages as well as data on the final reach of the diffusion of the program. Using aggregate diffusion patterns and average network positions of seeds in each village, they found that adoption was higher for villages that were seeded with leaders who had, on average, a higher eigenvector centrality than degree centrality. However, similar to other research, Banerjee et al. (2013) assumed a binary network in which all connections between households had the same strength.

The goal of this empirical application is threefold. First, we want to test whether different relationship characteristics affect the reach of the diffusion process differently. Second, we want to examine whether degree or eigenvector centrality captures social influence better if we take into account the importance of relationship characteristics. Third, using a holdout sample of villages, we want to benchmark optimal seeding strategies on the basis of our approach to traditional seeding strategies based on binary networks. Next, we describe the data set, the definitions of Equations 1–3, the estimation results, and implications for seeding strategies.

Data Description

To study the effects of the network position of seeds on the total reach of the diffusion process, Banerjee et al. (2013) collected relationship characteristics between households in 75 villages six months before BSS launched its program. Because these villages are relatively isolated, with a median distance of 46 km between them, there were no connections between households from different villages, which resulted in 75 independent social networks, each corresponding to a village. Because of operational difficulties, BSS finally launched the microfinance program in 43 of these villages. For each of these 43 villages, BSS provided diffusion data, which described whether each household adopted the microfinance program.

Table 1 provides descriptive statistics of the social networks that we used to validate our methodology. On average, the 43 villages contained 223.2 (SD = 56.2) households, of which an average of 26.9 (SD = 9.2) were classified as seeds (in total, BSS seeded 1,157 households). Diffusion, as measured by the percentage of households that participated in the program, varied substantially across villages (average percentage: 18.5%, SD = 8.4%). We used information about the social relationships between households and their characteristics using surveys, which is a common procedure in marketing as well as sociology to construct social networks (Van den Bulte and Wuyts 2007). In total, we found 12 types of relationships between households that revealed considerable overlap due to their similarity. To identify the underlying relationship dimensions, we used categorical factor analysis on the tetrachoric correlation matrix between the 12 types of relationships (Parry and McArdle 1991). Using Kaiser's measure of sampling accuracy (MSA), we classified two types of relationships as independent (i.e., "are related to" MSA = .28 and "go to temple with" MSA = .13). The remaining ten types of relationships loaded on two factors with eigenvalues larger than one. From these results, we categorized the 12 measured relationship characteristics in four underlying dimensions: (1) economic relationships ("borrow money from," "lend money to," "borrow

Table 1
MICROFINANCE DIFFUSION: DESCRIPTIVES OF SOCIAL NETWORKS

	All Households	*Seeds*	*Nonseeds*
Network size	223.2 (56.2)	26.9 (9.2)	196.3 (50.2)
Total Network			
Degree centrality	1.27 (1.00)	1.75 (1.21)	1.21 (.95)
Eigenvector centrality	.58 (.53)	.84 (.65)	.55 (.50)
Economic Relations			
Degree centrality	1.21 (1.00)	1.67 (1.17)	1.15 (.96)
Eigenvector centrality	.50 (.57)	.72 (.70)	.46 (.54)
Social Relations			
Degree centrality	1.25 (1.00)	1.74 (1.23)	1.18 (.95)
Eigenvector centrality	.55 (.54)	.81 (.69)	.52 (.51)
Religious Relations			
Degree centrality	.46 (1.00)	.62 (1.19)	.44 (.97)
Eigenvector centrality	.07 (.36)	.12 (.50)	.06 (.34)
Family Relations			
Degree centrality	1.12 (1.00)	1.34 (1.10)	1.09 (.99)
Eigenvector centrality	.25 (.61)	.29 (.66)	.24 (.61)

Notes: Values are means, with standard deviations in parentheses. Degree centrality is normalized for comparison across networks.

kerosene or rice from," and "lend kerosene or rice to"), (2) social relationships ("give advice to," "help with a decision," "obtain medical advice from," "engage socially with," "invite to one's home," and "visit in another's home"), (3) religious relationships ("go to temple with"), and (4) family relationships ("are related to"). We thus obtained for each village four networks corresponding to the four types of relationships between households in a village. The elements of these adjacency matrices were either 0 (no relationship) or 1 (a relationship is present). We coded a relationship as present between two households if a household mentioned the other household in one of the survey questions. This is similar to Banerjee et al. (2013), who constructed only one adjacency matrix (we call it the "total network" in Table 1) with a relationship present if any of the 12 types of relationships existed. Table 1 presents for each adjacency matrix summary statistics of the two network measures that Banerjee et al. (2013) used in their study (i.e., degree centrality and eigenvector centrality). Next, we describe how we constructed for each seed s in each village v its obtained reach y_{vs}, weighted network measures x_{vs}, and control variables z_{vs}.

The reach of a seed s in village v (y_{vs}). We observed for each household in village v whether this household adopted the microfinance program. However, because we did not observe word-of-mouth communication in the network, we did not observe through which seed(s) a household was informed about the microfinance program. We assigned an adoption to seed s in village v if this seed had the shortest path to the adopted household in the unweighted (binary) social network of village v. If there were m seeds nearest to an adopter, we distributed this adopter equally among these seeds (i.e., we increased the reach of these seeds by 1/m). This approach is similar to Van den Bulte and Lilien (2001) and Iyengar, Van den Bulte, and Valente (2011), who attributed adoptions of drugs by physicians equally across their adopted neighbors.[2] Table 2 reports summary statistics of the seeds' reach in our database. On average, each seed generated 1.45 adoptions (SD = 1.60).

Network measures (x_{vs}). For each seed s in village v, we computed two network measures: weighted degree centrality and eigenvector centrality. Weighted degree centrality of seed s corresponds to the weighted sum of connections of that seed. Weighted eigenvector centrality takes into account not only the weighted sum of connections of a seed but also the centrality of these connections, with central neighbors contributing more to the centrality of a seed s (Bonacich and Lloyd 2001). According to Borgatti (2005), degree and eigenvector centrality are network measures at two extremes that are ideally suited to capture influence. Degree centrality assumes an underlying transmission process wherein only direct connections are involved. In contrast, eigenvector centrality assumes an underlying transmission process that involves unrestricted walks (i.e., each network member may influence their neighbors, and neighbors may subsequently influence their neighbors, etc.). It is an empirical question which of these two network measures best describes the underlying diffusion process. The formulas for these weighted centrality measures are specified as follows:

$$(5) \qquad x_{vs1} = \sum_{j=1}^{N_v} w_{vsj} \text{ (weighted degree centrality), and}$$

$$(6) \qquad x_{vs2} = [x_{v2}]^{(s)} \text{ (eigenvector centrality),}$$

where x_{v2} is the solution to the following system of equations:

$$W_v x_{v2} = \lambda_{max}^{W_v} x_{v2}.$$

In Equation 5, w_{vsj} denotes the sjth entry of weighted adjacency matrix W_v of village v and N_v represents the number of households in village v. In Equation 6, operator $[\bullet]^{(s)}$ denotes element s of a vector, and $\lambda_{max}^{W_v}$ is the maximum eigenvalue of adjacency matrix W_v. Table 2 reports summary statistics of a seed's (unweighted) degree and eigenvector centrality using binary social networks. The high correlation between degree and eigenvector centrality and the reach of a seed (.58 and .53, respectively) serves as preliminary evidence that the position of a seed in the network affects its reach. We also find that degree and eigenvector centrality measures are highly correlated (r = .93), which prevents us from including them simultaneously in Equation 3. In addition to these two network measures, Table 2 also reports summary statistics of the following control variables.

Percentage of seeds. We constructed this variable by dividing the number of seeds in a village by the total number of households (network size) in that village. To reduce possible collinearity, we mean-centered this variable. We used this variable to control for saturation effects because seeds in villages that contained many other seeds may generate fewer adopters.

Household characteristics. We constructed these variables from various survey questions concerning different aspects of households. We first constructed four dummy variables to summarize roof materials of different households. If roof material was thatch, then the variable Roof (Thatch) = 1, and 0 otherwise. The other three roof types were represented similarly: Roof (Tile), Roof (Stone), and Roof (Sheet). We constructed the No. of Rooms variable from the survey question, "How many rooms do you have in your house?" We also mean-centered this variable to control for possible collinearity. The Electricity variable is a dummy variable with a

[2]We also tried different rules to construct seeds' reach and found consistent estimation results (see the robustness checks in Web Appendices A.1 and A.2).

Table 2

MICROFINANCE DIFFUSION: DESCRIPTIVES STATISTICS OF ALL VARIABLES

					Correlation Matrix											
Variables	*M*	*SD*	*Max*	*Min*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Dependent Variable																
1. Reach of a seed (y)	1.45	1.60	13.26	.00	1.00											
Network Measures																
2. Degree centrality	1.75	1.21	9.40	.00	.58	1.00										
3. Eigenvector centrality	.84	.65	4.63	.00	.53	.93	1.00									
Control Variables																
4. Percentage of seeds	.00	.03	.07	−.06	−.22	−.02	−.01	1.00								
5. Roof (thatch)	.28	.45	1.00	.00	.05	.00	−.01	−.01	1.00							
6. Roof (tile)	.31	.46	1.00	.00	−.01	.01	.01	.09	−.42	1.00						
7. Roof (stone)	.20	.40	1.00	.00	−.08	−.09	−.09	−.02	−.32	−.33	1.00					
8. Roof (sheet)	.16	.37	1.00	.00	.04	.12	.11	−.07	−.28	−.29	−.21	1.00				
9. Number of rooms	.00	1.58	15.25	−2.75	.09	.26	.26	−.02	−.12	−.03	−.05	.29	1.00			
10. Electricity	.73	.45	1.00	.00	−.04	.11	.12	−.07	−.13	−.01	.01	.19	.26	1.00		
11. Latrine	.39	.49	1.00	.00	.05	.15	.14	−.03	−.19	.01	−.03	.29	.33	.32	1.00	
12. House	.93	.26	1.00	.00	.01	.04	.03	−.02	.07	.03	−.09	−.02	.06	.04	.05	1.00

value of 1 if a household privately owned electricity and 0 otherwise. Similarly, we coded the variables Latrine and House, which equal 1 if a household privately owned a latrine or a house, respectively, and 0 otherwise.

Model Specification and Estimation

To apply our multinetwork methodology, we used Equations 5 and 6 to extract the weighted network measures x (i.e., step 2 of the multinetwork approach, see Equation 2). To construct the weighted networks W (step 1 of the multinetwork approach, see Equation 1), we used the following specification:

$$(7) \qquad w_{vij} = \begin{cases} 0 & \text{if } a^k_{vij} = 0, \forall k = 1, \ldots, 4 \\ \exp\left(\sum_{k=1}^{4} \beta_k a^k_{vij}\right) & \text{otherwise.} \end{cases}$$

In Equation 7, a^k_{vij} corresponds to relationship characteristic K between households i and j in village v and represents the importance of relationship characteristic k. Finally, we used the following equation for $f(\cdot)$ in Equation 3 to relate network measures x and control variables z to the reach of seed s in village v:

$$(8) \qquad y_{vs} = \theta_0 + \theta_1' z_{vs} + \theta_2 x_{vsm} + \varepsilon_{vs}, \text{ with } \varepsilon_{vs} \sim N(0, \sigma^2).$$

Equation 8 regresses control variables z_{vs} and network measure x_{vsm} on the reach y_{vs} of seed s in village v, with $m \in \{1, 2\}$ corresponding to weighted degree and eigenvector centrality, respectively. We assumed ε_{vs} follows independent normal distributions with mean zero and variance σ^2.

For model estimation, we assumed diffuse priors. To estimate the parameters in Equation 8, we used standard Gibbs steps in our Bayesian framework (see, e.g., Rossi, Allenby, and McCulloch 2005). To estimate β in Equation 7, we used a Metropolis–Hastings step because the posterior distribution is nonstandard. We estimated our model using a total of 20,000 iterations, with a burn-in period of 10,000 iterations, long before the Markov chain converged. Appendix A details our estimation procedure. Application of this procedure to synthetic data shows that our model recovered all parameters well. To determine model fit, we computed the log-marginal density (LMD) using the method proposed by Chib and Jeliazkov (2001).

Estimation Results

Table 3 presents the estimation results of four models, two centrality measures (degree vs. eigenvector centrality) × two approaches (traditional vs. multinetwork approach). In the traditional approach, we followed previous research and defined a binary adjacency matrix W in Equation 7, with $w_{ij} = 1$ if $a^k_{ij} = 1$ for any k = 1–4 and $w_{ij} = 0$ otherwise. For both approaches, Model 1 uses degree centrality (x_{vs1}) as network measure to explain the reach of a seed, while Model 2 uses eigenvector centrality (x_{vs2}). As we expected, for each of the four models, network centrality measures positively relate to the reach of a seed. The traditional approach suggests that eigenvector centrality is a stronger explanatory network measure for the reach of a seed than degree centrality (LMD Model 2: −1,642 vs. −1,647 for Model 1).[3] In contrast, our multinetwork approach shows that weighted degree centrality fits better than eigenvector centrality (LMD Model 1: −1,627 vs. −1,641 for Model 2). Moreover, these LMD measures also demonstrate that taking into account the importance of relationship characteristics improves model fit.

To interpret the importance of relationship characteristics, we focus on Model 1 of the multinetwork approach because this model best fits the data. Parameter estimates of the different types of relationships suggest that social relationships are the most important driver of the adoption of the microfinance program (β = .24; 97.9% of posterior draws are positive). Notably, economic relationships (β = −.23; 98.3% of posterior draws are negative) tend to be the least important for the adoption of the microfinance program. This result is in line with previous research showing that information received from stronger (social) relationships is more influential than that from weaker (economic) relationships (Brown and Reingen 1987; De Bruyn and Lilien 2008).

Finally, the parameter estimates of the control variables are stable across all four models (see Table 3). As we expected, the

[3]Note that Banerjee et al. (2013) estimate the model at the aggregate diffusion level within each village, and thus they do not take into account heterogeneity across seeds.

Table 3
MICROFINANCE DIFFUSION: ESTIMATION RESULTS

Variables	*Traditional Approach*				*Multinetwork Approach*			
	Model 1		*Model 2*		*Model 1*		*Model 2*	
Construction of Weighted Network								
Economic					**−.23**	(−.38, −.05)	**.30**	(.18, .44)
Social					**.24**	(.085, .385)	**.36**	(.24, .47)
Religious					.48	(−.02, .93)	.20	(−.01, .43)
Family					−.10	(−.20, .02)	.04	(−.05, .21)
Diffusion Equation								
Constant	**.50**	(.07, .92)	**.67**	(.25, 1.11)	**.52**	(.094, .95)	**.63**	(.20, 1.07)
Control Variables								
Percentage of seeds	**−11.69**	(−14.21, −9.77)	**−12.10**	(−14.61, −9.35)	**−11.74**	(−14.10, −9.19)	**−12.83**	(−15.38, −10.35)
Roof_1 (thatch)	−.06	(−.39, .28)	.01	(−.35, .37)	−.048	(−.40, .29)	−.05	(−.40, .31)
Roof_2 (tile)	−.14	(−.48, .19)	−.08	(−.43, .28)	−.15	(−.49, .18)	−.15	(−.50, .20)
Roof_3 (stone)	−.24	(−.58, .12)	−.18	(−.53, .17)	−.24	(−.58, .11)	−.23	(−.58, .12)
Roof_4 (sheet)	−.22	(−.59, .15)	−.14	(−.52, .25)	−.23	(−.59, .15)	−.21	(−.59, .17)
Number of rooms	−.04	(−.10, .01)	−.03	(−.09, .02)	−.04	(−.09, .01)	−.03	(−.08, .03)
Electricity	**−.36**	(−.54, −.19)	**−.40**	(−.58, −.21)	**−.35**	(−.53, −.18)	**−.35**	(−.53, −.17)
Latrine	.03	(−.14, .19)	.06	(−.11, .23)	.03	(−.14, .20)	.07	(−.01, .23)
House	−.05	(−.33, .21)	.04	(−.26, .33)	−.08	(−.35, .21)	−.03	(−.32, .25)
Network Centrality Measures								
Degree	**.81**	(.74, .86)			**.82**	(.75, .88)		
Eigenvector			**1.36**	(1.24, 1.48)			**1.07**	(.94, 1.19)
Variance of error	**1.56**	(1.44, 1.71)	**1.69**	(1.56, 1.84)	**1.55**	(1.43, 1.68)	**1.61**	(1.48, 1.75)
LMD	−1,647.19		−1,642.28		−1,627.13		−1,640.61	

Notes: 95% posterior intervals are reported in parentheses. Values in boldface are significant.

percentage of households that is seeded in a village has a negative effect on the total reach of a seed (θ = −11.74; all posterior draws are negative). In addition to the percentage of seeded households, the only other significant control variable is whether a household privately owns electricity (θ = −.35; all posterior draws are negative).

In summary, our estimation results demonstrate that (1) different relationship characteristics indeed have varied influence on the diffusion process, and (2) ignoring the importance of different relationship characteristics may lead researchers to a different conclusion of which network measure better captures social influence. Next, we demonstrate how the multinetwork approach results in better seeding strategies than the traditional approach.

Implications for Seed Selection

To validate whether the multinetwork approach is a valuable tool for seeding decisions, we performed holdout seeding practices using the following procedure. First, we randomly split the data into four disjoint subsets of villages with approximately 10–11 villages in one subsample. Then, we reestimated the models on each subsample and used the remaining villages as a holdout sample to determine forecasting accuracy and seeding performance. We computed in- and out-of-sample fit statistics, performed a seeding analysis using each of the four subsamples, and averaged the results. Table 4 presents the results of the in- and out-of-sample fit statistics. Corroborating the results using LMD (see Table 3), the proposed multinetwork approach systematically outperforms the traditional approach on all fit statistics.

To illustrate whether the multinetwork approach is a valuable tool for reseeding practices, we estimated our model on four subsamples of 290 seeds and ranked the remaining 867 seeds in holdout samples on the basis of their predicted reach. Table 5 presents the average actual reach across the four holdout subsamples of different reseeding strategies. Subsequently, we computed the actual reach drawing on the traditional as well as the multinetwork approach if BSS decided to seed the best n out of 867 seeds in the holdout sample, with n = 1, 2, ..., 867. Table 5 presents the results and shows that the multinetwork approach obtains a higher reach for both network measures. The multinetwork approach does especially well if a relatively small proportion of seeds is selected, which is important because the goal of a seeding strategy is to select a relatively small number of customers for targeting. For instance, if BSS selects n = 100 seeds in the holdout sample, the multinetwork approach with degree centrality (Model 1) obtains a reach that is 10.14% higher than its traditional counterpart, while the model with eigenvector centrality increases reach by 10.68% (Model 2).

Robustness Checks

To ensure that the estimates of the importance of different relationships and the superior holdout seeding performance of the multinetwork approach are robust against different specifications and alternative benchmark models, we performed six robustness checks (see Web Appendices A.1–A.6). First, we tested an alternative rule to assign adopting households to seeds. Instead of equally dividing an adopter among multiple seeds with shortest distance, we assigned the household to the seed with the largest number of shortest paths to that household.[4] Web Appendix A presents the results of this

[4]If this rule again resulted in multiple seeds, we looked at the number of paths with length equal to the shortest path plus one and repeated the process until each household was assigned to one unique seed.

Table 4
MICROFINANCE DIFFUSION: IN- AND OUT-OF-SAMPLE FIT STATISTICS

	In-Sample				*Out-of-Sample*			
	Traditional Approach		*Multinetwork Approach*		*Traditional Approach*		*Multinetwork Approach*	
Criterion	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*
MAD	.99	.95	.87	.89	1.01	.97	.88	.90
MAPE	.68	.65	.60	.61	.69	.66	.60	.61
RMSE	1.36	1.31	1.24	1.26	1.39	1.33	1.26	1.28
R-square	.39	.41	.54	.51	.38	.40	.53	.51

assignment rule. Similar to our previous findings, the multinetwork approach fits the data better than the traditional approach based on LMD. Moreover, we found that weighted degree centrality better explains the reach of a seed and shows that social relationships are the most important, while economic relationships are the least important, corroborating our previous findings. Both in- and out-of-sample fit statistics also confirm the relative performance of different models as presented in Web Appendix A. Finally, in a holdout sample seeding strategy, the multinetwork approach obtains a higher actual reach than the traditional approach (see Web Appendix A). One may also argue that the assignment of seeds should be based on the weighted network instead of the original binary network. To account for this, we performed another robustness check that constructs seeds' reach at each iteration of the Markov chain Monte Carlo (MCMC) sampler on the basis of the draw of the importance of relationship characteristics θ. We then computed the shortest (weighted) path between a seed and an adopter and assigned the adopter to the nearest seed(s). If, in rare occasions, multiple nearest seeds exist, we distributed the adopter evenly among them. Web Appendix A documents this robustness check. Note that the holdout seeding practice is different because seeds' reach is not fixed, but computed in line with the importance of relationship characteristics. Despite these differences, our results are stable, and the multinetwork approach outperforms the traditional approach (see Web Appendix A).

Second, we also tested whether there were possible interaction effects between different relationships characteristics (see Web Appendix A). Due to multicollinearity issues, we were only able to include three interaction terms. Our estimation results reveal that none of the interactions were significant and that adding these interactions did not substantially change the estimation results or the holdout seeding performance.

Third, we further estimated a model in which we used a different functional form for the weighted network construction function $g(\cdot)$ (see Equations 1 and 7). Although we selected the exponential function drawing on previous research, this function only assigns strictly positive weights to connections if a relationship exists. To allow for connections with zero weights if a relationship exists, we applied a stochastic network construction function that follows a binomial distribution based on Trusov, Bodapati, and Bucklin (2010). Web Appendix A presents the estimation results. The LMD indicates that weighted degree centrality fit the data much better than eigenvector centrality and that the exponential function fit better overall. For weighted degree centrality, the holdout seeding performance using the stochastic network construction function still outperformed the traditional approach, though it does slightly worse than the multinetwork approach with the exponential function. Consistent with LMD and other in- and out-of-sample fit statistics (see Web Appendix A), eigenvector centrality did not perform well in the holdout seeding procedure.

Finally, previous research has generally ignored relationship characteristics and assumed a binary network. However, some researchers estimated models with multiple network measures corresponding to each relationship characteristic (e.g., Aral and Walker 2012; Hu and Van den Bulte 2014) or

Table 5
MICROFINANCE DIFFUSION: RESEEDING PERFORMANCE

	Traditional Approach		*Multinetwork Approach*		*Percentage Improvement*	
Number of Selected Seeds	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*
50 (5.77%)	182.42	177.44	200.75	203.75	10.04%	14.83%
100 (11.53%)	343.84	319.17	378.72	353.25	10.14%	10.68%
150 (17.30%)	469.41	450.12	501.46	473.56	6.82%	5.12%
200 (23.07%)	576.95	556.89	598.55	576.96	3.74%	3.60%
250 (28.84%)	665.91	642.41	690.41	680.57	3.68%	5.94%
300 (34.60%)	756.29	717.80	780.66	768.93	3.22%	7.12%
400 (46.14%)	892.59	872.47	921.34	919.38	3.22%	5.38%
500 (57.67%)	1,015.82	979.51	1,041.66	1,020.73	2.54%	4.21%
600 (69.20%)	1,117.19	1,099.47	1,128.52	1,122.53	1.01%	2.10%
700 (80.74%)	1,189.04	1,172.83	1,198.49	1,188.22	.80%	1.31%
800 (92.27%)	1,237.21	1,231.61	1,242.11	1,232.23	.40%	.05%
867 (100.00%)	1,258.24	1,258.24	1,258.24	1,258.24	.00%	.00%

assigned weights a priori to relationship characteristics (e.g., Newman 2001; Rothenberg et al. 1995). Web Appendices A.5 and A.6 compare our multinetwork approach with, respectively, the procedure using multiple network measures and a priori assigning equal weights to different relationship characteristics. Again, the LMD and other in- and out-of-sample fit statistics favored the multinetwork approach in terms of model fit. More importantly, the holdout seeding performance demonstrates that estimating the importance of different relationship characteristics leads to better seeding decisions in terms of actual obtained reach.

This microfinance diffusion application illustrates that the multinetwork approach not only improved in- and out-of-sample model fit but also resulted in better reseeding strategies. Although robustness checks show that our results are robust for several assumptions, a limitation of this data is that we did not observe actual cascades and therefore needed to make assumptions to construct the reach of seeds. In addition, the sizes of the social networks in microfinance diffusion are relatively small (223.2 households in a village, on average). Our second empirical application addresses these concerns and involves a large online social network in which we observe information propagation among network members. In this application, we aim to further test the robustness of our proposed method in a very different empirical setting (online vs. offline, large vs. small networks, and microfinance product adoption vs. information dissemination) and showcase the generalizability of our results.

EMPIRICAL APPLICATION II: INFORMATION PROPAGATION IN AN ONLINE SOCIAL NETWORK

Data Description

Our data set contains anonymized detailed records of information transmission in a large online social network platform. The social network consists of 42,858 enrolled undergraduate students of a major university in the United States. The detailed data set contains background information for each student (age, gender, and date of joining the online social network) and detailed information about each student's online behavior, including one-to-one messaging, public posting, and commenting. For each message, we observed the sender, receiver, time of the message, and message content.

To study the effect of a seed's position on the spread of information, we focused on information cascades that were generated during and after the 2010 Super Bowl. We selected the Super Bowl because many brands launched new ads during this event that were not announced in advance. Moreover, these ads tended to be of high quality and were, therefore, often mentioned in social media. The Super Bowl thus provides an external shock to the diffusion cascades of ads posted on social media, which enables us to validate our method. Triggered by this event, 1,620 seed students initiated messages about advertisements used during the Super Bowl (see Table 6). Subsequently, we observed many instances in which friends of these posters forwarded or posted messages about the same advertisements to their friends. For each student who initiated a message about a Super Bowl advertisement, we were able to identify the subsequent actual cascades (who influenced whom on what and when). Therefore, we were able to observe actual reach of seeds. Note that each cascade involved a message about only one advertisement, but different cascades could involve different ads. This is similar to previous research investigating the effects of network structure on the spread of different YouTube videos (Yoganarasimhan 2012). The cascades involving Super Bowl advertisements reflect the complexities observed in other large-scale social network studies (Hinz et al. 2011), such that there are many short diffusion paths and much fewer long paths (i.e., the average reach of a seed is 1.66, but the highest reach of a seed is 22; average cascade length is 1.12, but the longest is 4, equal to the diameter[5] of the network; for details, see Table 7).

To uncover the importance of different relationship characteristics in the social network, we used two sources of information. First, we used the number of messages exchanged between students in the two month period before the Super Bowl. Second, we obtained the duration of each dyadic link in the network during the Super Bowl. Whereas the former reflects the effect of interaction frequency, the latter captures possible recency effects. As Aral and Walker (2014) show, frequency and recency are strong predictors of tie strength. This setting thus enables us to validate whether our methodology is in line with previous research by uncovering a positive weight for the number of messages exchanged and a negative weight for relationship duration (implying a positive recency effect). Next, we discuss how we constructed the variables to apply the multinetwork approach to predict the reach of different seeds in the social network.

The reach of a seed (y_s). The reach of a seed is constructed by counting the number of users involved in the cascades initiated by the corresponding seed.[6] Table 7 reports summary statistics for the reach of seeds in our data set. On average, each seed generated 1.66 adoptions or messages (SD = 2.07). In addition, we observed that many seeds influenced only one adopter (approximately 65% in our data) and thus generated cascades of length one. However, some seeds generated a very high reach of up to 22 messages posted and cascades of length four, which equals the diameter of this network.

Network measures (x_s). Similar to the previous application, we used weighted degree and eigenvector centrality to predict the reach of seeds (see Equations 5 and 6). Table 7 provides initial support that seeds with higher degree and eigenvector centralities obtain a greater reach (correlation coefficient = .12 for both measures). We again find a high correlation between these two centrality measures (correlation coefficient = .80), which prevents us from including both in one regression.

Control variables (z_s). For each seed, we included age, gender, and how long the seed was a member of the social platform at the moment of the Super Bowl. Age and membership duration were standardized across all students in the network and gender was dummy coded (1 = male, 0 = female). In addition, because we expect that early messages about the Super Bowl are more influential than later messages, we also included a timing variable that indicated the elapsed time (log of seconds) between the start of the event and sending time of

[5]The diameter of a network refers to the longest of all the calculated shortest paths between two people in the network.

[6]In only 5.64% of the cascades, it was not possible to uniquely assign a user to a seed, because a user received messages about the same ad from multiple sources that were initiated by different seeds. In these cases, we assigned the user to the seed that initiated the earliest message. We also estimated our model by assigning the user to the seed that initiated the latest message, which resulted in almost identical results because the two dependent variables are highly correlated (correlation coefficient = .984).

Table 6
INFORMATION PROPAGATION: DESCRIPTIVES OF SOCIAL NETWORKS

	All Households	*Seeds*	*Nonseeds*
Network size	42,852	1,620	41,232
Total Network (Binary)			
Degree centrality	1.05 (1.00)	1.67 (1.16)	1.03 (.99)
Eigenvector centrality	.30 (.54)	.53 (.69)	.30 (.53)
Number of Messages Exchanged (Weighted)			
Degree centrality	.16 (1.00)	.76 (1.77)	.14 (.95)
Eigenvector centrality	.0009 (.06)	.002 (.11)	.0008 (.05)
Relationship Duration (Weighted)			
Degree centrality	1.05 (1.00)	1.66 (1.16)	1.03 (.99)
Eigenvector centrality	.30 (.54)	.52 (.69)	.30 (.53)

Note: Values are means, with standard deviations in parentheses. Degree centrality is normalized for comparison across networks.

the message. Table 7 presents descriptive statistics of these variables. It indicates that younger students were more likely to initiate discussions about Super Bowl ads than older members (average standardized age = −.49), but correlations with reach are weak. In addition, seeding times are shown to be negatively correlated with seeds' reach (correlation coefficient = −.14), indicating that early seeds may be more influential.

Model Specification and Estimation

To apply our multinetwork methodology, we used degree and eigenvector centrality to extract the weighted network measures x, similar to Empirical Application I (Equations 5 and 6). To construct the weighted networks W (step 1 of the multinetwork approach; see Equation 1), we used the following specification:

$$w_{ij} = \begin{cases} 0 & \text{if } a^k_{vij} = 0, \forall k = 1, 2 \\ \exp\left(\sum_{k=1}^{2} \beta_k a^k_{ij}\right) & \text{otherwise.} \end{cases} \tag{9}$$

In Equation 9, a^k_{ij} corresponds to the number of messages exchanged (k = 1) or relationship duration (k = 2) between users i and j, and β_k represents the importance. Finally, because the reach y_s is a count variable, we used a generalized linear model with Poisson link function to specify the diffusion equation $f(\cdot)$ (Equation 3). The expectation of the Poisson distribution is as follows:

$$E(y_s|x_s, z_s) = \exp(\theta_0 + \theta_1' z_s + \theta_2 x_{sm}). \tag{10}$$

Equation 10 regresses control variables z_s and network measure x_{sm} on the reach y_s of seed s, with $m \in \{1, 2\}$ corresponding to weighted degree and eigenvector centrality, respectively. We estimated our model using a total of 20,000 iterations, with a burn-in period of 10,000 iterations, long before the Markov chain converged (for the details of MCMC sampler, see Appendix B).

Estimation Results

Table 8 presents the estimation results of four models, two centrality measures (degree vs. eigenvector centrality) × two approaches (traditional vs. multinetwork approach), with the traditional approach defined as in Empirical Application I. Similar to the first empirical application, the multinetwork approach outperforms the traditional approach for both models based on LMD (Model 1: −2,498 vs. −2,623; Model 2: −2,535 vs. −2,624, respectively for the multinetwork and traditional approach). Notably, similar to the first empirical application, the multinetwork approach suggests that weighted degree centrality is a better predictor of the reach of a seed than eigenvector centrality (LMDs: −2,498 vs. −2,535). This is consistent with the descriptive results that most cascades are short. Because both models produce similar results and Model 1 provides better fit, we focus on Model 1 to interpret the estimation results and the importance of different relationship characteristics.

Table 7
INFORMATION PROPAGATION: DESCRIPTIVES OF ALL VARIABLES

					Correlation Matrix						
Variables	*M*	*SD*	*Min*	*Max*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Dependent Variable											
1. Reach of seeds (y)	1.66	2.07	1.00	22.00	1.00						
Network Measures											
2. Degree centrality	1.67	1.16	.01	7.68	.12	1.00					
3. Eigenvector centrality	.53	.69	.00	5.24	.12	.80	1.00				
Control Variables											
4. Age	−.49	1.23	−2.06	2.94	−.04	−.07	−.01	1.00			
5. Gender	.32	.47	.00	1.00	.00	−.12	−.12	.16	1.00		
6. Membership duration	.01	.19	−.79	.39	.03	.11	.14	.54	−.02	1.00	
7. Seeding time	1.71	1.00	.00	3.85	−.14	−.08	−.09	.05	.06	−.02	1.00

Table 8
INFORMATION PROPAGATION: ESTIMATION RESULTS

	Traditional Approach		Multinetwork Approach	
Variables	*Model 1*	*Model 2*	*Model 1*	*Model 2*
Construction of Weighted Network				
Number of messages exchanged			**4.76** (1.91, 6.25)	**1.46** (1.37, 1.56)
Relationship duration			**−.44** (−.54, −.34)	**−.23** (−.33, −.14)
Diffusion Equation				
Intercept	**.90** (.85, .94)	**.93** (.89, .97)	**.85** (.80, .89)	**.91** (.86, .95)
Age	−.02 (−.03, .00)	−.02 (−.04, .00)	**−.02** (−.04, −.00)	−.02 (−.03, .00)
Gender	.00 (−.04, .04)	.00 (−.04, .05)	.00 (−.03, .04)	.00 (−.04, .04)
Membership duration	.10 (−.02, .22)	.11 (−.01, .23)	.06 (−.05, .17)	.08 (−.03, .20)
Seeding time	**−.06** (−.08, −.04)	**−.06** (−.08, −.04)	**−.07** (−.08, −.05)	**−.06** (−.08, −.04)
Network Measures				
Degree	**.04** (.03, .06)		**.06** (.05, .07)	
Eigenvector		**.07** (.04, .09)		**.10** (.08, .12)
LMD	−2,623.46	−2,623.97	−2,497.71	−2,535.25

Note: 95% posterior intervals are reported in parentheses. Values in boldface are significant.

As we expected, the number of messages exchanged is positively related to social influence in the network ($\beta = 4.67$; all posterior draws positive). Moreover, longer relationships are associated with weaker social influence ($\beta = -.44$; all posterior draws negative), confirming recent findings of frequency and recency effects on tie strength (Aral and Walker 2014). These results not only demonstrate that the importance of relationship characteristics vary in the social network but also validate the uncovered importance of relationship characteristics by the proposed methodology.[7] Finally, the parameter estimates of control variables are stable across the four models, with significant effects for age and seeding time (age: $\theta = -.02$, 97.7% posterior draws negative; seeding time: $\theta = -.07$, all posterior draws negative).

Implications for Seed Selection

Similar to the previous application, we assessed the holdout reseeding performance of our approach and compared it with the traditional benchmark models. To do so, we randomly divided the seeds into four disjoint subsamples of 405 seeds that we used as estimation sample and the remaining 1,215 seeds as holdout sample. Table 9 presents the prediction accuracy for both in- and out-of-sample fit statistics, averaged across the four samples. Corroborating the model selection criterion on the basis of LMD, models of the multinetwork approach outperform their traditional counterparts by all criteria (mean absolute deviance [MAD], mean absolute percentage error [MAPE], root mean square error [RMSE], and R-square), in both the estimation and the holdout sample. Moreover, the results also validate the notion that weighted degree centrality is a better predictor of the reach of a seed than eigenvector centrality.

Table 10 presents the results of the reseeding strategies based on the four different models. The multinetwork approach again outperformed the traditional approach and did particularly well when a relatively small number of seeds were selected. Compared with the previous empirical application, the improvements were much larger. If 50 seeds were selected (4.12% of the holdout sample), the multinetwork approach obtained reaches that were 91.91% and 31.29% higher than the traditional approach for degree and eigenvector centrality, respectively.

Robustness Checks

Similar to the first empirical application, we performed two robustness checks to ensure the stability of our results (see Web Appendices C.1–C.2). In Web Appendix C, we compare the proposed multinetwork approach with the use of multiple network measures as well as with a priori weighted networks. Both robustness checks demonstrate the superiority of the proposed multinetwork approach over these alternatives (for details, see Web Appendix C).

DISCUSSION

Connections between consumers in social networks vary in strength, depending on the characteristics of relationships and the information exchanged. Although different relationship characteristics may have different impacts on diffusion processes, previous research has generally ignored this information and treated all relationship characteristics as equal. In this research, we developed a new methodology that uncovers the importance of relationship characteristics on the basis of the observed diffusion process of a product, service, or message. Our approach results in a model that can be decomposed into three components that are conditional on each other. The Bayesian estimation procedure, consisting of a Gibbs sampler nested with a Metropolis–Hastings step, is therefore a natural tool to estimate the model parameters. Our Bayesian approach is efficient and can easily be applied to large-scale social networks, as we demonstrated in our second empirical application. The major computational challenge for large-scale social networks may be the computation of network measures (step 2 in Figure 1), such as eigenvector centrality that requires calculating the inverse of large matrices. In extreme situations in which tens of millions of users are involved, it may be useful to approximate these network measures (Brandes and Pich 2007; Brin and Page 1998).

In two empirical applications that varied substantially in terms of network size (small vs. large), context (online vs. offline),

[7]In Web Appendix B, we further validate the multinetwork approach using the number of messages exchanged and relationship duration.

Table 9
INFORMATION PROPAGATION: IN- AND OUT-OF-SAMPLE FIT STATISTICS

	In-Sample				*Out-of-Sample*			
	Traditional Approach		*Multinetwork Approach*		*Traditional Approach*		*Multinetwork Approach*	
Criterion	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*
MAD	.815	.817	.752	.788	.842	.844	.771	.810
MAPE	.383	.384	.358	.377	.388	.388	.361	.381
RMSE	2.009	2.009	1.742	1.791	2.161	2.162	1.785	1.836
R-square	.026	.030	.090	.074	.023	.023	.082	.063

Notes: R-square is McFadden's pseudo R-square.

and diffusion process (a microfinance product vs. an online message about an advertisement), the proposed multinetwork approach demonstrates that the importance of relationship characteristics substantially vary. In both applications, recognizing these differences not only resulted in a better statistical fit but also led to better seeding strategies. In the first application, the multinetwork approach proposed seeding strategies that increased the number of actual adoptions of a microfinance program by up to 10%. In the second empirical application, our approach was able to increase the actual reach in a holdout sample by up to 92% compared with a benchmark that ignored the importance of different relationship characteristics. Notably, in both empirical applications we found that weighted degree centrality was a better criterion for seed selection than eigenvector centrality. However, as we have explained, degree and eigenvector centrality capture two extremes of transmission processes (Borgatti 2005). It is possible that a measure in between these two extremes better captures the diffusion process. Banerjee et al. (2013) proposes such a measure: diffusion centrality. In Web Appendix D, we compare different versions of this measure with degree and eigenvector centrality using the multinetwork approach. Interestingly, degree centrality remains optimal in both empirical applications, although diffusion centrality obtained similar performance in the second empirical application. This finding is also in line with Hinz et al. (2011), who report that targeting seeds with high degree centrality results in a higher reach than high global betweenness centrality.

It would be useful for future research to investigate whether specific campaign characteristics, such as the type of product or service and characteristics of consumers in the network, are able to explain the importance of relationship characteristics. Such an analysis would enable marketers to predict the importance of relationship characteristics in a social network before the start of the diffusion process. It would also be worthwhile for researchers to allow for multiple weighted social networks. In our application, we constructed only one weighted social network. However, it is possible that in other applications, diffusion is better explained by two or more weighted social networks. For instance, previous research in organizational networks of employees has distinguished between formal and informal relationships (Soda and Zaheer 2012; Tucker 2008). As argued by Soda and Zaheer (2012), formal relationships (consisting of workflows and organizational structures) transmit different types of information, compared with informal relationships (consisting of social relationships such as friendships). The former are more likely to transmit approvals and task-related information, whereas the latter are more likely to transmit advice and affect. These types of information could have a different impact on the adoption decisions of employees, such as the adoption of a new technology. In addition, it would be useful to investigate the importance of different relationship characteristics on diffusion measures other than reach, such as the speed of diffusion or repeated purchases.

Future studies could also extend our methodology in several ways. First, in our first application, we found through categorical factor analysis that different types of relationships loaded on the same underlying relationship dimensions. Further research could extend our methodology to model such similarities directly. Second, in our model, the strength of

Table 10
INFORMATION PROPAGATION: HOLDOUT SEEDING PERFORMANCE

Number of Selected Seeds	*Traditional Approach*		*Multinetwork Approach*		*Percentage of Improvement*	
	Model 1 (Degree)	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*	*Model 1 (Degree)*	*Model 2 (Eigenvector)*
50 (4.12%)	111.25	126.25	213.50	165.75	91.91%	31.29%
100 (8.23%)	256.00	255.00	323.50	294.25	26.37%	15.39%
150 (12.35%)	384.50	402.00	434.00	425.25	12.87%	5.78%
200 (16.46%)	480.50	493.50	520.00	511.50	8.22%	3.65%
250 (20.58%)	571.00	569.00	615.50	609.75	7.79%	7.16%
450 (37.04%)	956.75	948.50	963.00	983.00	.65%	3.64%
650 (53.50%)	1,260.25	1,255.25	1,296.50	1,282.75	2.88%	2.19%
850 (69.96%)	1,545.75	1,532.00	1,572.00	1,572.75	1.70%	2.66%
1,050 (86.42%)	1,822.00	1,810.50	1,827.00	1,812.75	.27%	.12%
1,215 (100.00%)	2,014.50	2,014.50	2,014.50	2,014.50	.00%	.00%

connections between consumers in a social network only varied with the characteristics of relationships. Future studies could incorporate additional heterogeneity through a hierarchical Bayesian approach that allows strengths to be individual or dyad specific. This would require a data set containing diffusion processes with multiple adoption observations for people. Third, we assumed all strengths between connections of consumers in the social network to be positive. It would be worthwhile to study whether some characteristics of relationships are negative and, thus, inhibit the diffusion process. For instance, Chandrashekaran, Grewal, and Mehta (2010) showed that nonrepeat buyers of shareware inhibited the diffusion process through spreading negative word of mouth, while repeat buyers facilitated diffusion through positive word of mouth. Finally, the weighted network construction function in Equation 1 assumes that two customers are not connected if there is no observed relationship between those customers. In reality, such customers may still influence each other, and it would be beneficial for further research to extend our approach to allow for such interactions. A possible direction would be to incorporate network formation models in the network construction function (Braun and Bonfrer 2011).

In conclusion, in this study we proposed a methodology that derives the strengths of connections as a function of relationship characteristics. We demonstrate our approach to be flexible and suitable for any diffusion process in which social network data are available. We believe that our methodology may be a valuable tool for managers to optimize seeding strategies to facilitate the diffusion of their products and services.

APPENDIX A: MCMC SAMPLER FOR MICROFINANCE DIFFUSION

Priors

We used the following diffuse priors for the model parameters:

$$\theta \sim N(0, 100\, I_{M\times M}), \beta \sim N(0, 100\, I_{4\times 4}), \text{and } \sigma^2 \sim IG\left(\frac{r_0}{2}, \frac{s_0}{2}\right),$$

where I indicates identity matrices, and IG refers to the inverse gamma distribution. In our application, we set $r_0 = s_0 = 2$. This leads to the MCMC procedure described next.

MCMC Procedure

In our MCMC sampler, we draw sequentially from the following distributions:

1. Metropolis–Hasting steps to draw β_k, k = 1, ..., 4. Given the current value of β_k, draw a new value $\beta_k^{new} = \beta_k + \rho_k \varphi$, with $\varphi \sim N(0, 1)$. ρ_k is dynamically tuned to ensure an appropriate level of acceptance rate (between 25% and 45%; see Browne and Draper 2000). The new proposed value is accepted with the following probability:

$$P_{accept} = \min\left\{1, \frac{\phi(\beta_k^{new}|0, 100)L\{y|z, h[g(A; [\beta_k^{new}, \beta_{-k}])], \theta, \sigma^2\}}{\phi(\beta_k|0, 100)L\{y|z, h[g(A; \beta)], \theta, \sigma^2\}}\right\},$$

where $\phi(\cdot|0, 100)$ represents the probability density function of a normal distribution with mean 0 and variance 100. Furthermore, $L(\cdot)$ is the likelihood function as obtained by combining Equations 5–8.

2. $\theta \sim N[Q(\sigma^{-2}X'y), Q]$, with $X = [1\ z\ x]$ and $Q = (\sigma^{-2}X'X + .01\, I_{M\times M})^{-1}$.
3. $\sigma^2 \sim IG\left(\frac{r_n}{2}, \frac{s_n}{2}\right)$, with $r_n = r_0 + NS$ and $s_n = s_0 + (y - X\theta)'(y - X\theta)$, where NS denotes the number of seeds.

APPENDIX B: MCMC SAMPLER FOR ONLINE INFORMATION PROPAGATION

Priors

We used the following diffuse priors for the model parameters:

$$\theta \sim N(0, 100\, I_{M\times M}) \text{ and } \beta \sim N(0, 100\, I_{2\times 2}),$$

where I indicates identity matrices. This leads to the following MCMC procedure.

MCMC Procedure

In our MCMC sampler, we draw sequentially from the following distributions:

1. See step 1 of the MCMC sampler in the microfinance diffusion application (Appendix A).
2. To draw the parameters in Poisson regression, we use an independent Metropolis–Hastings step, where the proposal distribution is assumed to be a multivariate normal distribution centered at the maximum likelihood estimates of the Poisson regression, with variance–covariance matrix set to the asymptotic covariance matrix (i.e., approximated by inverse of Hessian H of the log-likelihood). The new proposal value θ^{new} is accepted with the following probability:

$$P_{accept}(\theta^{new}) = \min\left\{1, \frac{\phi(\theta^{new}|0, 100I_M)L\{y|z, h[g(A; \beta)], \theta^{new}\}}{\phi(\theta|0, 100I_M)L\{y|z, h[g(A; \beta)], \theta\}} \times \frac{\phi(\theta|\theta^{MLE}, H^{-1})}{\phi(\theta^{new}|\theta^{MLE}, H^{-1})}\right\}.$$

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