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# Capturing Consumption Flexibility in Assortment Choice from Scanner Panel Data

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This study develops and estimates a model to empirically identify two behavioral effects (namely consumption flexibility and state dependence) that may underlie temporal and horizontal assortment choice, using scanner panel data where consumption information is unavailable. The proposed approach permits a consumer's consumption utility to be dependent on previous consumptions, thus capturing state dependence both across purchase occasions and within horizontal assortments. Moreover, consumers' purchase and consumption decisions are modeled at two distinctive and sequentially related stages, which allows for incorporating the effect of consumption flexibility. The model is estimated on scanner panel data of yogurt purchase. It is found that the two captured effects provide strong empirical support with face validity for the temporal and horizontal assortment choice patterns observed in the data. The behavioral insights derived from estimating the proposed model can also be translated into significant managerial implications.

*Key words*: assortment; choice model; consumption flexibility; marketing; multiple choice; state dependence; utility uncertainty; variety seeking
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## Introduction

Consumers' grocery shopping frequently involves purchasing multiple items from the same category. These multiple-item purchases are either homogeneous with only one variety or heterogeneous with multiple varieties. Over time, the composition of the purchased assortments could also vary in quantity or variety or both.[1] For example, a consumer might buy one Strawberry yogurt in one shopping trip, and an assortment with one Blueberry and two units of Mixed Berry during another trip. Understanding consumers' temporal and horizontal assortment choice behavior is central to many marketing strategic issues, e.g., promotion, free sample distribution, and assortment management, etc. The purpose of this paper is to provide behavioral insights into assortment choice by developing and estimating a descriptive model with scanner panel data.

There are three underlying effects that may influence scanner panel households' multiple-item purchase behavior. The first one lies in the similarity/difference among the members in a household in their relative preference over the varieties. Second, the consumption of a product might lead an individual consumer to prefer the product more or less in subsequent consumptions. This causal dependence of current utility on previous consumptions has been termed in the literature as state dependence (e.g., Erdem 1996). If a variety's utility is increased due to previous consumptions (i.e., positive state dependence or increasing marginal utility), consumers will be compelled to purchase assortments that are both temporally persistent and horizontally homogeneous. If, on the other hand, consumption utility is decreased because of previous consumptions (i.e., negative state dependence or diminishing marginal utility), consumers will be more likely to buy different varieties both across purchase occasions and within horizontal assortments.

The third incentive that may promote the simultaneous purchase of different products is the need for consumption flexibility, which may arise from the existence of uncertainty at the purchase time about future consumption utility (Kreps 1979, Walsh 1995, Guo 2006). For most shopping occasions, the purchased products are not for immediate use and future consumption utility is uncertain at the purchase time (Hauser and Wernerfelt 1990, Simonson 1990, Xie and Shugan 2001). That is, the varieties' utility may be contingency-dependent (e.g., mood,

[1] This study's unit of analysis is product/variety that is defined at the flavor level. Where no confusion arises, I will use "varied" or "heterogeneous" to describe purchases involving different varieties either temporally or horizontally, and "persistent" or "homogeneous" to capture purchases with the same variety. Moreover, I will follow the literature (e.g., Harlam and Lodish 1995, Bucklin et al. 1998) and use "assortment" to represent consumers' purchase options, which may involve single or multiple, varied or homogeneous, products.

weather) and vary across consumption occasions. This in turn implies that a less popular product may turn out to have a higher realized utility on some consumption occasions than other more popular products. As a result, consumers may have an incentive to purchase a horizontally varied assortment in order to maintain *consumption flexibility* whereby the decision about which product to consume for a particular consumption occasion is de facto postponed until the varieties' consumption utility becomes known. If homogeneous products are purchased, a commitment is effectively made to consume the same variety on future consumption occasions prior to next shopping trip. If, instead, different varieties are purchased, future consumptions can be adjusted according to the varieties' realized utility.

There is no better way to single out the within-household-heterogeneity effect than calibrating the model only on single-person households (e.g., Mayhew and Winer 1992). The identification of the other two effects is less straightforward. Both state dependence and consumption flexibility may influence the observed (horizontal) assortment choice. In particular, although positive (negative) state dependence may enhance a consumer's incentive to buy same (different) varieties simultaneously, the need for consumption flexibility may lead to the purchase of horizontally varied products. As a result, horizontally varied assortment purchase can be driven by either negative state dependence or consumption flexibility. On the other hand, even when the state dependence effect is positive, consumers may still exhibit a (convex) preference to buy different varieties together if the desire for consumption flexibility is more significant. Although the importance of both effects in consumer decision making has been documented separately in the literature (e.g., Kreps 1979, Walsh 1995, Erdem 1996), this paper is the first to empirically separate them from each other using scanner panel data where consumption information is absent. This empirical separation, as will be demonstrated in this paper, can not only improve our understanding of consumers' assortment choice behavior but also provide important managerial insights.

I develop a model on scanner data to identify the differential impact of consumption flexibility from that of state dependence on assortment choice. The proposed approach explicitly models consumer decisions at two interrelated stages. At the consumption stage that may involve multiple occasions, consumption decisions are made based on the varieties' realized utility. A variety's consumption utility is influenced by its previous consumptions, which thus allows for the state dependence effect both across purchase occasions and within horizontal assortments. Moreover, the consumption utility is dependent on some contingency that is uncertain at the purchase time. The anticipation about how this uncertainty will be resolved at the consumption stage, and how the consumption utility may evolve from one consumption occasion to the next, determines endogenously how the purchased products will be consumed and thus influences how alternative assortments are evaluated at the purchase stage.

An important feature of the proposed model is that the effect of state dependence is defined at the consumption utility level. Moreover, the proposed approach explicitly distinguishes between assortment valuation (and thus the observed purchase behavior) and the consumption utility of the underlying varieties. As a result, it permits the differences between the observed (horizontal) choice dependence among varieties and the underlying utility-level state dependence effect, as well as between the observed temporal and horizontal dependences in assortment choice. This stands in contrast to previous studies that investigate either the temporal dependence of successively purchased single-unit products or the horizontal interdependence of varieties within multiple-item assortments, but not both. First, if the focus is only on temporal loyalty/switching for single-unit shopping occasions (e.g., Erdem 1996, Fader and Hardie 1996, Ho and Chong 2003), the distinction between variety utility and assortment valuation is immaterial and the effect of consumption flexibility is necessarily absent. On the other hand, if the dependence of consumption utility across purchase occasions is ignored (e.g., Hendel 1999, Kim et al. 2002, Dube 2004), the effect of state dependence on consumption utility cannot be separated from consumption flexibility in investigating the horizontal interdependence of varieties.

More importantly, it is the specification of the state dependence effect at the consumption utility level, coupled with the observed temporal and horizontal variations in assortment purchase, that permit the identification from the effect of consumption flexibility. This is because, although both state dependence and utility uncertainty may influence horizontal assortment choice, their impacts across purchase occasions are fundamentally different. Essentially, the impact of state dependence, as operationalizd at the consumption utility level, influences consumers' assortment choice along both the temporal and the horizontal dimensions. However, the kind of utility uncertainty captured in this paper that is relevant for consumption flexibility and caused by the separation between purchase and consumption, tends to be independent across purchase occasions. Moreover, consumption flexibility matters only for multiple-unit purchase occasions but not for single-unit occasions. As a result, by capturing both temporal and horizontal variations in assortment purchase, which is feasible with scanner panel data, we are able to achieve the desired identification.[2]

I calibrate the proposed model on yogurt purchase of single-person households. The estimated parameters suggest that the varieties' utility is significantly reinforced by previous consumptions (i.e., positive state dependence), and that the consumers perceive significant uncertainty about the consumption utility of some varieties. Consumers therefore tend to buy different varieties together but similar assortments over time. The proposed model fits all in-sample and out-of-sample data significantly better than some alternative models in which these two effects are not fully captured in one way or the other. Interestingly, the alternative models underestimate, rather than exaggerate, the importance of the captured effect when the other effect is ignored. That is, the state dependence effect is understated when utility uncertainty is absent, and vice versa, suggesting that the two effects may not necessarily be competing explanations for horizontally varied assortment purchase. Moreover, one striking result is that a static model addressing only horizontal assortment choice might mislead us to infer "variety seeking" whereas consumption utility is actually self-reinforcing. Overall, these results demonstrate the importance of capturing and distinguishing the impacts of state dependence and consumption flexibility.

The insights gained from the proposed model can be translated into valuable managerial implications. To show this, I conduct policy experiments to simulate the demand impact of two counterfactual changes, and compare the managerial insights derived from the proposed model against those from the alternative models. The advantage of the proposed model is illustrated from two perspectives. First, by fully capturing the two effects characterizing assortment choice, the proposed model is able to assess the impact of some policy changes (e.g., free sample distribution, uncertainty change). Second, in comparison to the proposed model, the alternative models may lead to biased model parameters and thus misestimated demand response.

The rest of the paper proceeds as follows. I first lay out the proposed model of assortment valuation and purchase, explicitly accounting for the differential impacts of state dependence and consumption flexibility. I then discuss the estimation issues, describe the scanner panel data used for model calibration and validation, and report the empirical results. The last section concludes the paper and discusses future research.

[2] Temporal variation in assortment purchase is normally absent in survey or experiment data.

# The Model

Consider a sample of consumers, indexed by $i = 1, 2, \ldots, I$, who purchase from a set of $j = 1, 2, \ldots, J$ product varieties. On each of the shopping occasions, $t = 1, 2, \ldots, T_i$, a consumer $i$ is observed to purchase an assortment of products $d_{it} = \{d_{i1t}, d_{i2t}, \ldots, d_{iJt}\}$, where $d_{ijt}$ is a nonnegative integer representing the units bought on variety $j$. Let $n_{it} = \sum_j d_{ijt} > 0$ denote the total number of purchased units. An equivalent way to record the assortment is to document the variety for each of the $n$ units: $\bar{d}_{it} = \{\bar{d}_{i1t}, \bar{d}_{i2t}, \ldots, \bar{d}_{int}\}$, where $\bar{d}_{imt} = 1, 2, \ldots, J$, and $\bar{d}_{imt} \geq \bar{d}_{im't}$ if and only if $m \geq m'$ for all $m$, $m' = 1, 2, \ldots, n_{it}$. For instance, if $J = 3$, an assortment consisting of two units of variety $j = 1$ and one unit of variety $j = 3$ can be denoted as $d_{it} = \{2, 0, 1\}$, or equivalently $\bar{d}_{it} = \{1, 1, 3\}$. For each consumer $i$, a sequence of purchases indexed by $d_i = \{d_{i1}, d_{i2}, \ldots, d_{iTi}\}$ or $\bar{d}_i = \{\bar{d}_{i1}, \bar{d}_{i2}, \ldots, \bar{d}_{iTi}\}$ are observed. Note that the $J$ elements in $d_{it}$ capture the quantity for each variety, whereas $\bar{d}_{imt}$ represents the variety for each purchased unit. As will become clearer, the alternative notation $\bar{d}_{it}$ can facilitate illustrating, in assessing the expected assortment valuation, how the purchased units in the assortment are to be sequentially consumed and depleted on each consumption occasion.

In this section, I develop a model to examine how the consumers make the assortment purchase decision, conditional on the observed total purchase units (i.e., $n_{it}$). It is posited that a consumer's (conditional) assortment purchase decision hinges on how to allocate the $n_{it}$ units over the varieties, i.e., select a $d(n_{it}) \in D(n_{it}) \equiv \{d\colon d = \{d_1, d_2, \ldots, d_J\}$ and $n_{it} = \sum_j d_j\}$. Note that the purchase option set $D(n_{it})$ includes all possible combinations of the $J$ varieties subject to the condition that the total units of products in a considered assortment option $d$ are equal to $n_{it}$. The set of purchase options are thus exhaustive, mutually exclusive, and distinct from the union of the varieties unless $n_{it} = 1$. The purchase option set can be equivalently represented as $\bar{D}(n_{it}) \equiv \{\bar{d}\colon \bar{d} = \{\bar{d}_1, \bar{d}_2, \ldots, \bar{d}_{nit}\}$, for all $\bar{d}_m = 1, 2, \ldots, J$, $m = 1, 2, \ldots, n_{it}$, such that $\bar{d}_m \geq \bar{d}_{m'}$ if and only if $m \geq m'\}$.

The key to the proposed model is consumer valuation for each assortment in the purchase option set $D(n_{it})$. Essentially, a consumer's assortment valuation depends on the utility that can be derived from consuming the products in the purchased assortment. Without time-of-consumption information, it is reasonable to assume that the purchased products are to be consumed sequentially, and that a consumption occasion can be accordingly defined as arising when and only when the consumer desires to consume one unit of the products. As a result, on each consumption occasion, $m = 1, 2, \ldots, n_{it}$, that will arise prior to next shopping visit $t+1$, one product is selected for consumption from the set of products available on hand at that time.[3] Therefore, one can sum the consumption utilities that are obtained from the $n_{it}$ consumption occasions as an approximation for assortment valuation (gross of the impacts of marketing mix variables). However, as will be elaborated, when state dependence and uncertainty are introduced to affect consumption utility, the expected gross valuation of an assortment at the time of purchase may not be equal to the sum of the expected consumption utility of the products that constitute the assortment.

In particular, the proposed model captures a consumer's information/utility evolution and decision making at both the consumption and the purchase stages that are interdependent. First, it captures how a consumer's utility may evolve from one consumption occasion to the next, given the cumulative variety consumption and conditional on the realization of utility uncertainty. This will endogenously determine the consumer's consumption decision and hence product inventory on each consumption occasion. Second, at the time when the assortment purchase decision is made, the consumer anticipates that the future realization of utility uncertainty at the consumption stage will determine how the products in the purchased assortment are to be consumed, and recognizes that the overall assortment valuation should take into account all possible uncertainty realizations.

**Consumption Utility**

On a representative consumption occasion $m$, a consumer's utility uncertainty that was present at the purchase time is resolved. Specifically, it is postulated that the consumption utility is

$$U_{ijtm} = \varphi_{ij} + \omega_i SD_{ijtm} + \varepsilon_{ijt} + \zeta_{ijm}, \tag{1}$$

where $\varphi_{ij}$ is the variety-specific intercept, which can be interpreted as a variety's intrinsic consumption utility; $SD_{ijtm}$ indicates the cumulative consumption of variety $j$ up to consumption occasion $m$; $\varepsilon_{ijt}$ is a set of random factors that are unobservable to the researcher but observable to the consumer at both the purchase and the consumption stages; and $\zeta_{ijm}$ captures the contingent factors (e.g., mood, weather) whose values become known to the consumer only when it comes to the consumption occasion $m$. Finally, $\omega_i$ represents the impact of state dependence, with a positive estimate indicating utility reinforcement while a negative estimate suggestive of variety seeking.

[3] It is implicitly assumed that all the purchased products are to be depleted prior to next shopping visit, i.e., stockpiling is not allowed. The no-stockpiling assumption is likely to hold in the empirical application of the model that considers the perishable 6-oz. and 8-oz. yogurt varieties.

Note that $\varphi_{ij}$ and $\varepsilon_{ijt}$ are known to the consumer at the time of purchase, whereas $SD_{ijtm}$ and $\zeta_{ijm}$ are dependent on some consumption contingency that is unknown at the purchase stage. Extant choice models typically assume that the error term in the utility function is unobservable only to the researcher, but known to the consumers. The proposed model, from the consumers' perspective, explicitly decomposes the error term into the certain and uncertain parts, $\varepsilon_{ijt}$ and $\zeta_{ijm}$, respectively.

The value of $SD_{ijtm}$ could be uncertain, because the consumption sequence is unknown at the purchase time, thanks to the uncertainty on $\zeta_{ijm}$. Formally, the cumulative consumption of variety $j$ on consumption occasion $m=1$ is $SD_{ijt1} = S_{ijt}$, and for $m>1$ is given by

$$SD_{ijtm} = \lambda^{(m-1)(w_{t+1}-w_t)/n_{it}} S_{ijt} + \sum_{m'=1}^{m-1} \lambda^{(m-m')(w_{t+1}-w_t)/n_{it}} I_{i,\hat{d}_{m'},t,m'}, \tag{2}$$

where $\lambda \in (0,1)$ stands for the weekly geometric decay parameter; $w_t$ indicates the week that the shopping occasion $t$ occurs; $S_{ijt}$ captures the influence of past purchases on the consumptions following shopping occasion $t$; $I_{ijtm}$ is an indicator variable taking the value of 1 if the variety $j$ is to be consumed on occasion $m$, and 0 if otherwise; and $\hat{d}_m$ indicates the variety that is anticipated to be consumed on occasion $m$. It is important to note that, as will be specified subsequently, the consumed variety $\hat{d}_m$ needs to be determined *endogenously* in a sequential manner, conditional on both the depletion of products from the purchased assortment on the previous consumption occasions $m'=1,2,\ldots,m-1$, and also on the realization of the uncertainty shock $\zeta_{ijm}$.

The specification of $S_{ijt}$ is akin to Guadagni and Little (1983):

$$S_{ijt} = \lambda^{w_t - w_{t-1}} S_{ij,t-1} + \lambda^{(w_t-w_{t-1})/2} d_{ij,t-1}, \tag{3}$$

where $d_{ijt}$ is the purchased quantity for variety $j$ in shopping trip $t$. Because the exact time for the occurrence of the consumption occasions cannot be observed, it is assumed that the purchased units are consumed smoothly with equal time interval between shopping trips. This is reflected in the specifications for both $SD_{ijtm}$ and $S_{ijt}$, where the discounting of previous consumptions is captured on the basis of average interconsumption time.

The two components on the right-hand side of (2), $S_{ijt}$ and $I_{ijtm}$, capture the impacts of "past" and "anticipated" consumptions, respectively. Therefore, this specification allows for both temporal and horizontal utility reinforcement/satiation. Nevertheless, note that the effects of both temporal and horizontal state dependence are captured by the same parameter $\omega_i$.

This is without loss of conceptual generality, because the effect of state dependence in this paper is defined at the consumption utility level and operationalized as the dependence on *all* previous consumptions of the same variety, including both previous purchases in shopping trips $t' = 1, 2, \ldots, t-1$ and consumption occasions $m' = 1, 2, \ldots, m-1$ following the same trip $t$. As a result, the evolution of cumulative consumption, either temporally across shopping trips or horizontally within an assortment, follows the same conceptual process as captured by the construction of $SD_{ijtm}$ in (2).

Note that previous studies focusing on the impact of state dependence on temporal loyalty/switching across shopping occasions investigate only single-item choice (e.g., McAlister 1982, Lattin 1987), which is similar to a special case of the current model: $n = 1$ and hence $SD_{ijtm} = S_{ijt}$. In contrast, the proposed model examines multiple-item purchase and incorporates also the (anticipated) impact of state dependence within assortments. On the other hand, the proposed model also departs from some recent studies that examine the interdependence of varieties within horizontal assortments without allowing for temporal dependence across shopping trips (e.g., Hendel 1999, Kim et al. 2002, Dube 2004).

Consumer heterogeneity in the parameters is specified as

$$\begin{pmatrix} \varphi_i \\ \omega_i \end{pmatrix} \sim N\left\{ \begin{pmatrix} \varphi \\ \omega \end{pmatrix}, \begin{pmatrix} \sigma_\varphi^2 & 0 \\ 0 & \sigma_\omega^2 \end{pmatrix} \right\}, \tag{4}$$

where $\varphi_i$ are the $J$-dimensional variety-specific intercepts that are assumed to be independently and normally distributed with mean $\varphi$ and standard deviation $\sigma_\varphi$, and $\omega_i$ is assumed to be distributed normally with mean $\omega$ and standard deviation $\sigma_\omega$.

For simplicity, it is assumed that $\varepsilon_{ijt}$ has an i.i.d. standard normal distribution:

$$\varepsilon_{ijt} \sim N(0, 1) \quad \text{for all } i, j, t. \tag{5}$$

The uncertain error term $\zeta_{ijm}$ is assumed to have an independent normal distribution across all consumers and consumption occasions with a $J \times J$ diagonal covariance matrix structure:

$$\zeta_{im} \sim N(0, \sigma_\zeta^2) \quad \text{for all } i, m. \tag{6}$$

The zero-mean assumption for $\zeta_{ijm}$ is without loss of generality, given the variety-specific intercepts $\varphi_{ij}$. It is also important to note that here consumers are uncertain about the *realization* but not about the *distribution* of the error term $\zeta_{ijm}$, i.e., the mean and variance are both known to the consumers. As a result, consumption experience does not influence the uncertainty structure from one consumption occasion to another (i.e., nonexperience goods). One can think of the kind of uncertainty considered here as the term $\zeta_{ijm}$ being determined exogenously from a known normal distribution. This is essentially different from the kind of quality uncertainty considered in the consumer learning literature (e.g., Erdem and Keane 1996), where consumption experience could improve a consumer's knowledge about the distribution of quality.

**Assortment Valuation**

Consider a consumer $i$ who is contemplating which assortment $\bar{d}(n_{it}) \in \bar{D}(n_{it})$ to select for purchase on shopping occasion $t$. The consumer needs to determine and compare the gross values of the assortments in $\bar{D}(n_{it})$. However, the consumption utility in (1) is not known when the assortment purchase decision is made. In particular, the value of $\zeta_{ijm}$ and hence that of $U_{ijtm}$ is uncertain at the purchase time. As a result, the sequence of consuming the products in a purchased assortment is not known either. Nevertheless, when the time passes to the consumption occasion $m$, the uncertainty will be resolved naturally. As a result, the consumer can form anticipation about the consumption utility $U_{ijtm}$ and hence about the consumed product $\hat{d}_m$, conditional on each possible realization of $\zeta_{ijm}$. The consumer can then integrate over all possible realizations of $\zeta_{ijm}$ to obtain the expected gross assortment value. Formally, it is posited that the expected gross value of a representative assortment $\bar{d}(n_{it}) \in \bar{D}(n_{it})$ is[4]

$$E[V(\bar{d}(n))] = \int \sum_{m=1}^{n} \left\{ \max_{j \in \underline{d}_m} U_{jm} \right\} f(\zeta_{jm})\, d\zeta_{jm}, \tag{7}$$

where $E[\cdot]$ represents expectations; $\underline{d}_m \subseteq \bar{d}(n)$ indicates the products remaining on hand when it comes to consumption occasion $m$; and $f(\zeta_{jm})$ is the multivariate normal density function for $\zeta_{jm}$.

Let us elaborate on the proposed process by which a consumer evaluates an assortment of products in anticipation of future consumption scenarios. On each future consumption occasion $m$, the value of $\zeta_{jm}$ will be realized and revealed to the consumer. Denote one possible realization as $\zeta^l = \{\zeta_{11}^l, \zeta_{21}^l, \ldots, \zeta_{J1}^l, \ldots, \zeta_{1n}^l, \zeta_{2n}^l, \ldots, \zeta_{Jn}^l\}$. Given $\zeta^l$, the consumption utility $U_{jm}$ will become known to the consumer—because, as will be explained below, the cumulative consumption $SD_{jm}$ is also completely specified conditional on $\zeta^l$. The consumer then selects to consume, among the products $\underline{d}_m$ on hand at that time, the one with the highest realized utility. Denote the selected variety on consumption occasion $m$ as $\hat{d}_m \equiv \{j' \colon j' \in \underline{d}_m$

[4] The subscripts $i$ and $t$ are omitted here for illustration convenience.

and $U_{j',m} \geq U_{jm}$ for all $j \in \underline{d}_m\}$. The remaining products will be retained for future consumptions. When it comes to next consumption occasion $m+1$, $\underline{d}_{m+1}$ will be obtained by removing $\hat{d}_m$ from $\underline{d}_m$, i.e., $\underline{d}_{m+1} \cap \hat{d}_m = \underline{d}_m$. Moreover, given the previously consumed varieties, $\hat{d}_1, \ldots, \hat{d}_m$, the value of $SD_{j,m+1}$ and $U_{j,m+1}$ can be constructed, which in turn allows the consumer to determine $\hat{d}_{m+1}$. Note also that on the first consumption occasion $m=1$, we have $\underline{d}_1 = \bar{d}(n)$, and $SD_{j1} = S_j$. Therefore, one can exercise the aforementioned process iteratively to derive the values of $\underline{d}_m$, $U_{jm}$, and $\hat{d}_m$, for each consumption occasion $m = 1, \ldots, n$. In addition, the utility of the variety that is anticipated to be consumed on occasion $m$ is $U_{\hat{d}_m} \equiv \max_{j \in \underline{d}_m} U_{jm}$.

It is important to emphasize that the aforementioned specified consumption sequence, as characterized by the profile ($\underline{d}_m$, $U_{jm}$, $\hat{d}_m$, $U_{\hat{d}_m}$), is neither known to the consumer at the purchase time nor imposed by the researcher, but is based on the consumer's anticipation conditional on a particular uncertainty realization $\zeta^l$. The consumer anticipates that there would be different realizations of the uncertainty shock, $\zeta^l = \zeta^1, \ldots, \zeta^L$, and thus different consumption sequences. Therefore, to obtain the expected gross value of sequentially consuming an assortment of products, the consumer takes into account (i.e., integrate over) all possible uncertainty realizations and hence all possible consumption sequences. This completes the description of the proposed assortment valuation process.

### State Dependence vs. Consumption Flexibility

I now elaborate how state dependence and utility uncertainty may differentially and interactively influence assortment valuation and purchase. First, in the presence of the state dependence effect, a variety's utility can be reinforced/detracted as a result of its previous consumptions. If a variety is consumed on occasion $m$, its utility will be modified not only for each remaining consumption occasion prior to next shopping trip but also for future purchases. Therefore, *all else being equal*, the expected valuation of an assortment with homogeneous varieties is higher (lower) than the summed expected utility of the products in the assortment if the state dependence effect is positive (negative). That is, consumers tend to exhibit concave (convex) preference in assortment purchase if the underlying utility is self-reinforcing (detracting), or equivalently if the marginal utility is increasing (diminishing). In the same vein, a positive (negative) state dependence effect would drive consumers to make similar (different) purchases over time, especially for single-unit shopping trips.

Second, the separation in time between purchase and consumption may prevent consumers from knowing up-front about the exact utility of a variety on a particular consumption occasion. This, in turn, leads to uncertainty about the sequence of consumptions, i.e., on which consumption occasion a purchased variety is to be consumed. This is because, in the presence of consumption utility uncertainty, a variety with relatively higher expected utility may not always be preferred and selected for consumption among the products in the inventory, whereas a variety with relatively lower expected utility may turn out to have a higher realized utility in some situations. A consumer may then desire to pursue consumption flexibility by purchasing a set of different products, which can help increase the chance in future consumption that the preferred product is available no matter which variety turns out to be the preferred one. Therefore, consumer valuation for horizontally varied assortments can be positively influenced by utility uncertainty, because of the flexibility in adjusting consumption decisions according to the realization of utility uncertainty.

Interestingly, the presence of the consumption flexibility effect may lead to the differences between the observed (horizontal) choice dependence among varieties and the underlying utility-level state dependence effect, as well as between the observed temporal and horizontal dependences in assortment choice. This is because the observed temporal dependence is driven only by the state dependence effect, whereas the observed horizontal variety choice dependence is influenced by both the state dependence effect and the desire for consumption flexibility. As a result, for example, even when the utility-level state dependence effect is positive, consumers may prefer to buy different varieties simultaneously if the consumption flexibility effect is sufficiently significant. This reflects the distinction highlighted in this paper between variety consumption utility and assortment valuation; it is the latter that directly determines the observed assortment choice behavior.

### An Example

Consider now an example on the proposed assortment valuation process, in which utility uncertainty may lead a consumer to prefer horizontally varied assortments despite the presence of positive state dependence. Consider a consumer who decides to purchase three units and is comparing two options: a homogeneous assortment $\bar{d}^1 = \{1, 1, 1\}$, and a varied assortment $\bar{d}^2 = \{1, 1, 3\}$ that consists of two units of variety 1 and one unit of variety 3. The certain part of the consumption utility, $\widetilde{U}_j \equiv \varphi_j + \varepsilon_j$, has the following hypothetical values: $\widetilde{U}_1 = 1$ and $\widetilde{U}_3 = 0.9$. The state dependence effect is positive and specified as $\omega = 1$, $S_j = 0$, and $\lambda^{(w_{t+1}-w_t)/n} = 0.3$, i.e., a variety's utility is increased by 0.3 (0.1) if it is consumed one (two) consumption occasion ago. It is clear that, when

**Table 1** **An Example of Assortment Valuation with State Dependence and Utility Uncertainty**

| | $m=1$ | | | | $m=2$ | | | | $m=3$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\zeta_{j1}$, $\zeta_{j2}$ | $\underline{d}_m$ | $U_{jm}$ | $\hat{d}_m$ | $U_{\hat{d}_m}$ | $\underline{d}_m$ | $U_{jm}$ | $\hat{d}_m$ | $U_{\hat{d}_m}$ | $\underline{d}_m$ | $U_{jm}$ | $\hat{d}_m$ | $U_{\hat{d}_m}$ | $\sum U_{\hat{d}_m}$ |
| {1, 1}, {1, 1} | {1, 1, 3} | (2, 2, 1.9) | *1* | **2** | {1, 3} | (2.3, 1.9) | *1* | **2.3** | {3} | (0.9) | *3* | **0.9** | 5.2 |
| {1, 1}, {1, −1} | {1, 1, 3} | 2, 2, 1.9) | *1* | **2** | {1, 3} | (2.3, −0.1) | *1* | **2.3** | {3} | (0.9) | *3* | **0.9** | 5.2 |
| {1, 1}, {−1, 1} | {1, 1, 3} | (2, 2, 1.9) | *1* | **2** | {1, 3} | (0.3, 1.9) | *3* | **1.9** | {1} | (1.1) | *1* | **1.1** | 5.0 |
| {1, 1}, {−1, −1} | {1, 1, 3} | (2, 2, 1.9) | *1* | **2** | {1, 3} | (0.3, −0.1) | *1* | **0.3** | {3} | (0.9) | *3* | **0.9** | 3.2 |
| {1, −1}, {1, 1} | {1, 1, 3} | (2, 2, −0.1) | *1* | **2** | {1, 3} | (2.3, 1.9) | *1* | **2.3** | {3} | (0.9) | *3* | **0.9** | 5.2 |
| {1, −1}, {1, −1} | {1, 1, 3} | (2, 2, −0.1) | *1* | **2** | {1, 3} | (2.3, −0.1) | *1* | **2.3** | {3} | (0.9) | *3* | **0.9** | 5.2 |
| {1, −1}, {−1, 1} | {1, 1, 3} | (2, 2, −0.1) | *1* | **2** | {1, 3} | (0.3, 1.9) | *3* | **1.9** | {1} | (1.1) | *1* | **1.1** | 5.0 |
| {1, −1}, {−1, −1} | {1, 1, 3} | (2, 2, −0.1) | *1* | **2** | {1, 3} | (0.3, −0.1) | *1* | **0.3** | {3} | (0.9) | *3* | **0.9** | 3.2 |
| {−1, 1}, {1, 1} | {1, 1, 3} | (0, 0, 1.9) | *3* | **1.9** | {1, 1} | (2, 2) | *1* | **2** | {1} | (1.3) | *1* | **1.3** | 5.2 |
| {−1, 1}, {1, −1} | {1, 1, 3} | (0, 0, 1.9) | *3* | **1.9** | {1, 1} | (2, 2) | *1* | **2** | {1} | (1.3) | *1* | **1.3** | 5.2 |
| {−1, 1}, {−1, 1} | {1, 1, 3} | (0, 0, 1.9) | *3* | **1.9** | {1, 1} | (0, 0) | *1* | **0** | {1} | (1.3) | *1* | **1.3** | 3.2 |
| {−1, 1}, {−1, −1} | {1, 1, 3} | (0, 0, 1.9) | *3* | **1.9** | {1, 1} | (0, 0) | *1* | **0** | {1} | (1.3) | *1* | **1.3** | 3.2 |
| {−1, −1}, {1, 1} | {1, 1, 3} | (0, 0, −0.1) | *1* | **0** | {1, 3} | (2.3, 1.9) | *1* | **2.3** | {3} | (0.9) | *3* | **0.9** | 3.2 |
| {−1, −1}, {1, −1} | {1, 1, 3} | (0, 0 ,−0.1) | *1* | **0** | {1, 3} | (2.3, −0.1) | *1* | **2.3** | {3} | (0.9) | *3* | **0.9** | 3.2 |
| {−1, −1}, {−1, 1} | {1, 1, 3} | (0, 0, −0.1) | *1* | **0** | {1, 3} | (0.3, 1.9) | *3* | **1.9** | {1} | (1.1) | *1* | **1.1** | 3.0 |
| {−1, −1}, {−1, −1} | {1, 1, 3} | (0, 0, −0.1) | *1* | **0** | {1, 3} | (0.3, −0.1) | *1* | **0.3** | {3} | (0.9) | *3* | **0.9** | 1.2 |

*Notes.* This example computes the expected valuation for the assortment $\bar{d}^2 = \{1, 1, 3\}$, with the following hypothetical values for the certain part of the utility: $\widetilde{U}_1 = 1$ and $\widetilde{U}_3 = 0.9$. In addition, $\omega = 1$, $S_j = 0$, $\lambda^{(w_{t+1}-w_t)/n} = 0.3$, and $\zeta_{jm} \in \{1, -1\}$ denotes the uncertainty shock for the variety $j$ on consumption occasion $m$, where $\text{Prob}(\zeta_{jm} = 1) = \text{Prob}(\zeta_{jm} = -1) = 1/2$. Moreover, $\underline{d}_m$ denotes the set of products available on consumption occasion $m$; $U_{jm} = \widetilde{U}_j + \omega * SD_{jm} + \zeta_{jm}$, $m = 1, 2$, represents the conditional utility, and $U_{j3} = \widetilde{U}_j + \omega * SD_{j3} + E[\zeta_{j3}]$ the expected utility, of the variety $j \in \underline{d}_m$; and $\hat{d}_m \equiv \{j'\colon j' \in \underline{d}_m$ and $U_{j',m} \geq U_{jm}$ for all $j \in \underline{d}_m\}$ denotes the variety selected for consumption, and $U_{\hat{d}_m} \equiv \max_{j \in d_m} U_{jm}$ the utility of the consumed variety $\hat{d}_m$, on occasion $m$.

utility uncertainty is absent, the valuations for the two assortment options are $V(\bar{d}^1) = 1 + 1.3 + 1.4 = 3.7$ and $V(\bar{d}^2) = 1 + 1.3 + 0.9 = 3.2$, respectively, suggesting that the homogeneous assortment is preferred.

Consider then the case with utility uncertainty. On each consumption occasion, suppose a variety's utility can be either increased or decreased by 1, each with probability 1/2, i.e., $\zeta_{jm} \in \{1, -1\}$ and $\text{Prob}(\zeta_{jm} = 1) = \text{Prob}(\zeta_{jm} = -1) = 1/2$. The expected valuation for the homogeneous assortment remains unchanged (i.e., $E[V(\bar{d}^1)] = 3.7$). The proposed valuation process for the heterogeneous assortment $\bar{d}^2 = \{1, 1, 3\}$ is illustrated in Table 1.

The rows capture the possible realizations of the uncertainty shock, $\zeta^l = \{\zeta^l_{11}, \zeta^l_{31}, \zeta^l_{12}, \zeta^l_{32}\}$, for the first two consumption occasions, based on which the sequence of consumption profile is derived. Note that the consumption sequence is completely determined by the realizations of the uncertainty shock for the first two consumption occasions, with a total of 16 possibilities. The columns present the conditional consumption profile ($\underline{d}_m$, $U_{jm}$, $\hat{d}_m$, $U_{\hat{d}_m}$) for each consumption occasion. Starting from $\underline{d}_1 = \bar{d}^2 = \{1, 1, 3\}$, the number of products in the inventory is decreased over the consumption occasions as the product $\hat{d}_m$ with the highest realized utility $U_{jm}$ is consumed, i.e., $\underline{d}_{m+1} \cap \hat{d}_m = \underline{d}_m$. The last column adds up the anticipated utilities $U_{\hat{d}_m} \equiv \max_{j \in \underline{d}_m} U_{jm}$ of the consumed varieties. In this particular example, the variety $j = 3$ will be selected for consumption on the first or the second occasion (i.e., $\hat{d}_m = 3$, $m = 1, 2$), if and only if its realized value for the uncertainty shock turns out to be higher than that of variety $j = 1$ (i.e., $\zeta_{3m} = 1 > \zeta_{1m} = -1$). Moreover, there is only one product left on consumption occasion $m = 3$, when the expected utility is $U_{33} = 0.9$ for variety $j = 3$, and $U_{13} = 1.1$ or 1.3 for variety $j = 1$ depending on whether the other unit of variety 1 is consumed on occasion $m = 1$ or $m = 2$. Finally, the expected gross value of the heterogeneous assortment can be obtained as the average of the figures in the last column: $E[V(\bar{d}^2)] = 4.038$, which is indeed higher than the expected value for the homogeneous assortment (i.e., $E[V(\bar{d}^1)] = 3.7$).

## Estimation

### Choice Probability

The proposed model deals with the issue of mutual nonexclusiveness of multiple-variety purchase by regarding each possible assortment as an alternative purchase option. The expected gross value for a representative assortment of size $n_{it}$ is given in (7). Given that the assortment options in $\overline{D}(n_{it})$ are exhaustive and mutually exclusive, the discrete choice framework can be used to capture the conditional probability that an assortment $\bar{d}_{it}$ (or $d_{it}$) is selected for purchase:

$$P(\bar{d}_{it}) = \text{Prob}\Bigg\{E[V(\bar{d}_{it})] - \sum_{j \in \bar{d}_{it}} X_{ijt}\beta_i \geq E[V(\bar{d})] - \sum_{j \in \bar{d}} X_{ijt}\beta_i,\ \forall \bar{d} \in \overline{D}(n_{it})\Bigg\}, \quad (8)$$

where $X_{ijt}$ represents a vector of marketing mix variables (e.g., price, feature). The marketing mix variables influence assortment purchase without directly affecting the consumption utility in (1). This captures the notion that these variables are known to the consumers at the purchase time and become sunk at the consumption stage. The marketing mix response parameters are represented by $\beta_i$, which are independently and normally distributed across consumers:

$$\beta_i \sim N(\beta, \sigma_\beta^2). \tag{9}$$

The assortment choice probability in (8) is conditional on the certain random term $\varepsilon_{ijt}$ and on the random variables characterizing consumer heterogeneity, known to the consumers at the purchase time while unknown to the researcher, which can be collected as $\mu_{ijt}$. The set of parameters are the variety-specific intercept parameters ($\varphi_j$ and $\sigma_{\varphi j}$), state dependence parameters ($\omega$ and $\sigma_\omega$), weekly geometric decay parameter ($\lambda$), marketing mix response parameters ($\beta$ and $\sigma_\beta$), and uncertainty parameters ($\sigma_\zeta$). Let us summarize the parameters as $\Theta$. By integrating over the domain of the random terms $\mu_{ijt}$, one can then obtain the unconditional probability for consumer $i$'s assortment purchase behavior at time $t$:

$$P(\bar{d}_{it} \mid \Theta) = \int P(\bar{d}_{it} \mid \Theta, \mu_{ijt}) f(\mu_{ijt})\, d\mu_{ijt}, \tag{10}$$

where $f(\mu_{ijt})$ is the multivariate normal density function for $\mu_{ijt}$.

**Identification**

Generally, discrete choice models need to deal with two identification issues, namely translational and scale invariance, arising from the unchangeability of choice behavior when adding or multiplying the utility of each choice option by the same constant. In single-unit choice models where there is no distinction between the consumption utility of varieties and the purchase valuation of assortments, the number of varieties is equal to that of purchase options in the choice set. Therefore, not all variety-specific intercepts (or all entries in the random error covariance matrix) can be estimated without some appropriate parameter normalization. Nevertheless, the structure imposed in the current model has automatically achieved the necessary normalization. Note that here the number of purchase options in $\bar{D}(n_{it})$ is larger than that of alternative varieties when the assortment size $n_{it}$ is not sufficiently small. For example, when $J = 2$ and $n_{it} = 3$, there are only two variety-specific intercepts and three covariance parameters but four assortment options. Any change in these parameters necessarily modifies the consumers' relative preference over the assortment options. So the traditional identification issues are immaterial here. Nevertheless, to facilitate comparison across models, the last variety's intercept is set to zero.

The two random error terms in (1) can be identified, even though both are unobservable to the researcher. This is indebted to the temporal within-consumer variation in assortment purchase. Basically, the expected value of homogeneous assortments, where consumption flexibility is irrelevant, is not influenced by the uncertain term $\zeta$. In contrast, the certain random term $\varepsilon$ affects the valuation of both single- and multiple-unit assortments. Therefore, the entries in the covariance matrix of the uncertainty shock can be estimated without further normalization.

The proposed model is also capable of identifying the differential impacts of state dependence and utility uncertainty. If we had looked at only (static) horizontal assortment choice, either negative state dependence or utility uncertainty may drive the consumers to purchase varied assortments, and as a result no identification could be achieved. By incorporating the effect of state dependence across purchase occasions as well (i.e., $S_{ijt}$ in (2)), the proposed model can empirically decompose these two effects. This is because state dependence influences assortment valuation both temporally across and horizontally within shopping occasions through the same underlying process as in (2), whereas utility uncertainty affects only horizontal assortment valuation. Therefore, investigating the dynamic evolution of assortment purchase is not only desirable but also inevitable for identifying these two confounding explanations for horizontal variety purchase.

The following example can facilitate the understanding of the above identification discussion. Consider two varieties ($J = 2$) and a representative consumer $i$ with a sequence of five shopping occasions: $d_i = \{\{0, 1\}, \{1, 0\}, \{1, 0\}, \{2, 1\}, \{1, 2\}\}$, or equivalently $\bar{d}_i = \{\{2\}, \{1\}, \{1\}, \{1, 1, 2\}, \{1, 2, 2\}\}$. From (7) we have $E[V(\bar{d}_{it})] = \varphi_{i2} + \omega_i S_{i2t} + \varepsilon_{i2t}$ for $t = 1$, and $E[V(\bar{d}_{it})] = \varphi_{i1} + \omega_i S_{i1t} + \varepsilon_{i1t}$ for $t = 2, 3$, where the uncertain error term $\zeta_{ijm}$ is cancelled out in the expected value of the first three (single-unit) purchases. It is then obvious that the likelihood of observing the first three (single-unit) purchases is influenced by $\omega_i$ but not by $\sigma_\zeta$, whereas the choice likelihood for the other two (multiple-unit) purchase observations is jointly affected by both $\omega_i$ and $\sigma_\zeta$. Note that, however, in the models focusing only on horizontal variety choice (e.g., Hendel 1999, Kim et al. 2002, Dube 2004), $S_{ijt}$ is arbitrarily restricted to zero and hence $\omega_i$ does not influence the choice probability for the first three observations (neither does $\sigma_\zeta$). As a result, the parameters $\omega_i$ and $\sigma_\zeta$ in these models cannot be separately estimated solely from the last two multiple-unit observations.

**Estimation Procedure**

The overall log-likelihood function for the choice probability in (10) is

$$\mathrm{LL}(\Theta) = \sum_{i=1}^{I}\sum_{t=1}^{T_i} \ln P(\bar{d}_{it} \mid \Theta). \tag{11}$$

A simulated maximum likelihood method is used to approximate this log-likelihood function. The simulation involves evaluating the choice probability in (10) by numerically integrating over the random terms $\mu_{ijt}$. However, even though the normal-distribution assumption of the error terms in the consumption utility function (1) follows the Probit model, the GHK simulator is not appropriate here. This is because the purchase decision and hence the conditional choice probability in (8) are not directly based on the varieties' relative consumption utility but on the expected value of the assortments in the choice set $\bar{D}(n_{it})$. The assessment of the expected gross assortment value, as in (7), accounts for the interactive effects of state dependence and consumption flexibility. The distribution of the error term in the assortment valuation function in (7) is hence ill-defined and unlikely to be normal. To see this, note that $\varepsilon$ and $\zeta$ represent the error terms in the consumption utility but not in the expected assortment value. The effective error term of the expected assortment value is the interaction of $\varepsilon$ and $\zeta$ with other parameters and explanatory variables.

I develop a two-stage logit-smoothed accept-reject simulator to approximate the choice probability in (10). In the first stage, the proposed process of how consumers assess the expected assortment value is approximated. This approximation is conditional on each draw of $\mu_{ijt}$ from $f(\mu_{ijt})$, which in the second stage is incorporated into the logit-smoothing function to simulate the assortment choice probability (Train 2003). The details of the simulator are given in the appendix. Finally, I employ the Quasi-Newton method with inexact line search to maximize the simulated log-likelihood function, where the Hessian is numerically approximated by the BHHH algorithm.

## Data

The proposed model is applied to scanner panel data in the yogurt category provided by the A. C. Nielsen Company. In this section, I describe the scanner data used for model calibration and validation and explore the empirical horizontal and temporal patterns of assortment purchase to provide some preliminary diagnosis of the underlying drivers. The data set spans 138 weeks from January 1986 to August 1988 and was collected from two test markets: Sioux Falls, SD, and Springfield, MO. The following considerations favor the selection of yogurt over other product categories with frequent multiple-item purchase (e.g., soft drinks): (a) yogurt is perishable, making consumer inventory less desirable; (b) the one-unit-per-consumption assumption is more likely to hold given that most yogurt purchased is in the 6-oz. or 8-oz. size; (c) there is less competition from other purchase channels (e.g., vending machines, restaurants) that may reduce the proportion of yogurt consumption captured by the scanner data.

I focus on five popular flavors of the Dannon brand, whose market share is almost double that of the second largest brand Yoplait, in the dominant 6-oz. and 8-oz. sizes: Strawberry, Cherry, Blueberry, Mixed Berry, and Piña Colada. This follows Kim et al. (2002), who note that neither temporal switching between nor simultaneous purchase of different yogurt brands is often observed, i.e., consumers are brand loyal. In the current data set, less than 6% of the single-person households' yogurt purchases of the selected sizes include more than one brand. Even conditional on multiple-unit purchase, a dominant percentage of the assortments involve only one single brand, e.g., about 90% for Dannon. In addition, there are more persistent same-brand purchases over time than brand switching. For example, the purchase of a Dannon yogurt leads to buying the same brand in the next period for about 61% of the scenarios. In contrast, it has been well recognized that different yogurt flavors are often bought simultaneously (e.g., Erdem 1996). Therefore, it is parsimonious but not very restrictive to concentrate on the Dannon brand.

To minimize the possibility that the purchase of varied assortments—either temporal or horizontal—simply reflects the heterogeneous preference of different household members, the calibration sample consists of only single-person households in the Springfield market. This rules out any household-member-heterogeneity bias that may disguise the behavioral inference made at the individual consumer level.[5] Households are selected for the calibration sample with at least two purchase incidences of at least one of the five varieties. This yields 63 single-person households and 441 purchase occasions. This small sample size may limit the generalizability of the results for policy simulation but is reasonable for demonstrating the proposed model and estimation procedure.

The predictive validity of the proposed model is assessed on the single-person households in the Sioux Falls market, as well as on another two mixed samples from these two markets where the household-size restriction is removed (i.e., including both single- and multiple-person households).

[5] This would not be an issue if the objective were not to reveal the processes underlying the observed behavior, in which case the utility function could be defined at the household level (e.g., Erdem 1996).

**Table 2** Distribution of Purchase Occasions: Varieties and Units

| Varieties/ units | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10+ | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 201 | 78 | 17 | 10 | 4 | 5 | 0 | 2 | 0 | 0 | 317 |
| 2 | 0 | 46 | 13 | 16 | 4 | 1 | 0 | 1 | 0 | 0 | 81 |
| 3 | 0 | 0 | 8 | 2 | 8 | 11 | 3 | 0 | 0 | 0 | 32 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 7 | 2 | 1 | 11 |
| Total | 201 | 124 | 38 | 29 | 16 | 17 | 3 | 10 | 2 | 1 | 441 |

**Table 3** Frequency of Variety Pairs in Horizontal Multiple-Unit Assortments

| | Strawberry | Cherry | Blueberry | Mixed Berry | Piña Colada | Total |
|---|---|---|---|---|---|---|
| Strawberry | 0.13 | 0.27 | 0.27 | 0.08 | 0.24 | 142 |
| Cherry | 0.27 | 0.12 | 0.34 | 0.08 | 0.19 | 142 |
| Blueberry | 0.30 | 0.36 | 0.14 | 0.05 | 0.14 | 132 |
| Mixed Berry | 0.14 | 0.14 | 0.09 | 0.53 | 0.10 | 79 |
| Piña Colada | 0.32 | 0.25 | 0.18 | 0.07 | 0.18 | 107 |

The validation samples differ from the calibration sample in household size or market environment or both, thus constituting more stringent predictive validity assessment than alternative random sampling approach (e.g., the calibration and validation samples consisting of randomly assigned single-person households from both markets). The number of households is 34 for the Sioux Falls single-person sample, and 423 and 316 for the Springfield and the Sioux Falls mixed samples, respectively. In the Springfield validation sample, there are 3,246 purchase occasions. The Sioux Falls single and mixed samples include 160 and 1,888 purchase occasions, respectively.

Table 2 breaks down the purchase occasions by the number of varieties and units for the calibration sample. As can be seen, 54.42% of the occasions involve multiple-unit purchase. The proportion of multiple-variety purchase is also quite substantial (i.e., 28.12%), which is increased to 51.67% if conditional on multiple-unit assortment. The validation samples also exhibit a considerable proportion of multiple-unit and/or multiple-variety purchase occasions. In the Springfield validation sample, multiple-unit and multiple-variety assortments constitute 59.49% and 33.46% of the purchase occasions, respectively. The figures in the Sioux Falls market are 41.25% and 23.13% for the single-person sample and 48.41% and 27.17% for the mixed sample.

Table 3 presents the frequency of variety pairs purchased simultaneously in multiple-unit assortments for the calibration sample. The number of purchase incidences involving each variety is reported in the last column.[6] The diagonal entries represent homogeneous purchase of the same variety, whereas the off-diagonal figures capture the frequency of various variety pair combinations. The numbers within each row reveal the distribution of a particular variety's homogeneous versus varied purchase incidence with each of the other varieties. Table 3 indicates that there is substantial difference across the varieties in terms of the frequency of horizontally varied purchase. For example, Mixed Berry has a much lower frequency of being bought together with other varieties.

The varieties' temporal transition pattern across purchase occasions is presented in Table 4. One incidence of temporal transition from variety $j$ to $j'$ is counted, whenever variety $j$ is purchased on occasion $t$ and $j'$ purchased on $t+1$. The diagonal entries in Table 4 reflect the extent of temporal persistence, whereas the off-diagonal figures document the frequency that different varieties are purchased sequentially across consecutive shopping trips. These transition statistics indicate that there is positive state dependence in driving the consumers' temporal assortment purchase.

Comparing the figures in Table 3 with the corresponding entries in Table 4 reveals some preliminary insights into the relative importance of the forces underlying each variety's purchase, which are to be further confirmed in next section through empirically estimating the proposed model. In particular, Strawberry, Cherry, Blueberry, and Piña Colada are less frequently bought in homogeneous assortments than in conjunction with other varieties. These horizontally varied choices are more likely to be driven by utility uncertainty than by negative state dependence, because consumers exhibit a relatively high level of temporal persistence in purchasing these varieties. However, there is a higher degree of both horizontal homogeneity and temporal persistence in the purchase of Mixed Berry, indicating that utility uncertainty is unlikely to be a significant concern for this

[6] For example, for an assortment including Strawberry, Cherry, and Blueberry, one purchase incidence is recorded for each of the three pairs involved: (Strawberry, Cherry), (Strawberry, Blueberry), and (Cherry, Blueberry). For multiple-unit purchase of a single variety, one purchase incidence is documented for that variety.

**Table 4** Frequency of Temporal Variety Transitions

| | Time $t+1$ | | | | | |
|---|---|---|---|---|---|---|
| Time $t$ | Strawberry | Cherry | Blueberry | Mixed Berry | Piña Colada | Total |
| Strawberry | 0.30 | 0.23 | 0.19 | 0.08 | 0.21 | 141 |
| Cherry | 0.27 | 0.32 | 0.22 | 0.07 | 0.12 | 132 |
| Blueberry | 0.23 | 0.28 | 0.30 | 0.08 | 0.13 | 120 |
| Mixed Berry | 0.15 | 0.16 | 0.10 | 0.52 | 0.08 | 62 |
| Piña Colada | 0.21 | 0.17 | 0.16 | 0.07 | 0.39 | 103 |

variety. Similar patterns are also found in the validation samples.

This discussion further illustrates the intuition on how the observed temporal variation in assortment choice can help identify the impact of consumption flexibility from state dependence. If the value of a parameter in $\sigma_\zeta$ were increased, the purchase incidence of horizontally varied assortments would rise (Table 3), whereas the temporal transition pattern involving that variety (Table 4) would remain basically uninfluenced. In contrast, if it were state dependence that led to the change in the observed horizontal assortment choice, we should expect to see a similar change in the temporal transition pattern.

Finally, the marketing mix variables $X_{ijt}$ include price and feature. No variety was displayed in the stores in the empirical samples. For each consumer $i$, variety $j$, and shopping occasion $t$, the average price and feature (across the SKUs with the same flavor for the dominant 6-oz. and 8-oz. sizes) in the same week/store for this purchase occasion are constructed. The variables' correlation across varieties is high but not perfect (see Table A.1 in the appendix). Therefore, the marketing mix variables are retained in the empirical analysis for two considerations: (1) It is an empirical issue whether the correlation is too high to yield statistically significant coefficient estimates, and (2) It can help illustrate how marketing mix variables can be incorporated if the proposed model is applied to other product categories with frequent multiple-item purchase and low correlation in marketing mix (e.g., soft drinks).

## Results

This section presents the results from model estimation and validation. To reveal the differential effects of state dependence and utility uncertainty on assortment choice, the full model is compared against several alternative models. First, I am interested in the consequence of ignoring utility uncertainty. To this end, the standard deviations of utility uncertainty are arbitrarily set to zero: $\sigma_\zeta = 0$, which leads to Model 1. Given that utility uncertainty may promote the purchase of horizontally varied assortments, this restriction is expected to underestimate (overestimate) the utility reinforcement (variety seeking) effect if utility uncertainty is actually significant. Second, this bias is expected to be exaggerated if the state dependence effect across purchase occasions is further foreclosed (Model 2): $S_{ijt} = 0$ and $\sigma_\zeta = 0$.[7] This is conceptually similar to Kim et al. (2002), who model horizontal assortment purchase only. I intend to show that models focusing only on the horizontal property but not the temporal evolution of assortment purchase may generate qualitatively misleading interpretation of consumer behavior. Third, I also estimate a nested model where the state dependence effect is completely removed by setting $\omega_i = 0$ while utility uncertainty is allowed. Model 3 will underestimate (overestimate) utility uncertainty if consumer utility is actually reinforced (detracted) by previous consumptions.

I start with assessing the in-sample and out-of-sample comparative fit statistics of the full and the alternative models, which are presented in Table 5. The parameter estimates on the calibration sample are then discussed and reported in Table 6. Next, I present the results from a numerical simulation, where a series of estimations are performed on data set generated using the full model with a prespecified set of parameters. The main objective of the simulation is to demonstrate the accuracy of the proposed estimation procedure in recovering the model parameters and the robustness of the results from the calibration sample. Finally, it is shown by conducting policy analysis that the conceptual insights obtained from estimating the proposed model can yield valuable managerial implications.

### Model Fit

The model fit statistics (log-likelihood, AIC, and BIC) for the calibration and the validation samples are reported in Table 5. The main result is that the full model fits significantly better than the alternative models in all samples in terms of all fit criteria. Note first that the full model performs significantly better than Models 1 and 2 at the 0.01 level on all fit statistics and in all samples. The null hypothesis is then rejected that the utility uncertainty parameters $\sigma_\zeta$ are zero. This suggests that accounting for utility uncertainty improves both in-sample and out-of-sample fit. To assess the significance of incorporating the state dependence effect, we can compare the full model with Model 3. All fit criteria favor the full model over Model 3 in all samples, indicating that the null hypothesis is not supported that state dependence is not significantly different from zero. These results demonstrate the importance of accounting for both

[7] To ensure fair comparison with the full model, the diagonal covariance matrix of the certain error term $\varepsilon_{ijt}$ is also estimated for Models 1 and 2. Moreover, variants of Models 1 and 2 are assessed with the estimation of the full covariance matrix for $\varepsilon_{ijt}$. The model variants, as reported in Table 5, do not yield significant improvement in any goodness-of-fit criteria for either the calibration or the validation samples, and in some cases can even lead to worse performance with the adjustment for the number of parameters (i.e., AIC, BIC) or in terms of out-of-sample log-likelihood value. In addition, parameter estimates under the more general model variants are fairly similar to those reported in Table 6. Therefore, Models 1 and 2 are specified with only the diagonal covariance matrix for $\varepsilon_{ijt}$, and their variants with full covariance structure are omitted for discussion in the paper.

**Table 5** **Goodness of Fit Statistics on Scanner Data**

| | Full model | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|
| Calibration sample | | | | |
| −LL | 1,075.08 | 1,110.76 (1,108.25) | 1,120.78 (1,116.15) | 1,185.56 |
| AIC | 1,095.08 | 1,129.76 (1,133.25) | 1,139.78 (1,141.15) | 1,202.56 |
| BIC | 1,135.97 | 1,168.61 (1,184.36) | 1,178.63 (1,192.26) | 1,237.32 |
| Validation sample (−LL) | | | | |
| Sioux Falls single | 329.73 | 359.20 (360.54) | 371.19 (365.36) | 374.76 |
| Springfield mixed | 7,738.11 | 7,828.43 (7,832.57) | 8,261.62 (8,263.56) | 8,287.42 |
| Sioux Falls mixed | 4,103.42 | 4,114.79 (4,128.59) | 4,468.22 (4,474.36) | 4,428.01 |

*Note.* Numbers in the parentheses are the model fit statistics under alternative specifications of Models 1 and 2, where the full covariance parameters for the certain error term $\varepsilon_{ijt}$ are estimated.

utility uncertainty and state dependence in explaining assortment choice behavior.

I then evaluate the improvement in model fit of accounting for the state dependence effect across purchase occasions. Recall that Model 1 allows cumulative consumption from past purchase occasions to influence current assortment valuation, whereas Model 2 does not (i.e., $S_{ijt} = 0$). Comparing these two models' fit statistics leads to Model 2 being rejected in favor of Model 1 in all samples. This shows that it is

**Table 6** **Parameter Estimates on Scanner Data**

| Parameters | Full model | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|
| Strawberry intercept | | | | |
| $\varphi_1$ | 0.987 (0.109) | 0.973 (0.161) | 0.801 (0.198) | 1.047 (0.090) |
| $\sigma_{\varphi 1}$ | 0.337 (0.521) | 0.171 (0.591) | 0.104 (0.569) | 0.155 (0.128) |
| Cherry intercept | | | | |
| $\varphi_2$ | −0.125 (0.163) | 0.301 (0.176) | 0.149 (0.237) | −0.064 (0.148) |
| $\sigma_{\varphi 2}$ | 0.279 (0.312) | 0.049 (0.508) | 0.600 (0.541) | 0.263 (0.535) |
| Blueberry intercept | | | | |
| $\varphi_3$ | 0.087 (0.156) | 0.418 (0.195) | 0.544 (0.178) | 0.180 (0.134) |
| $\sigma_{\varphi 3}$ | 0.475 (0.116) | 0.041 (0.551) | 0.154 (0.507) | 0.288 (0.393) |
| Mixed Berry intercept | | | | |
| $\varphi_4$ | 0.604 (0.122) | 0.734 (0.204) | −0.683 (0.558) | 0.560 (0.102) |
| $\sigma_{\varphi 4}$ | 0.069 (0.640) | 0.150 (0.429) | 10.296 (0.842) | 0.436 (0.517) |
| State dependence | | | | |
| $\omega$ | 0.859 (0.124) | 0.500 (0.121) | −0.301 (0.075) | |
| $\sigma_{\omega}$ | 0.519 (0.114) | 0.455 (0.241) | 0.658 (0.585) | |
| Weekly decay factor | | | | |
| $\lambda$ | 0.829 (0.027) | 0.917 (0.027) | 0.583 (0.623) | |
| Price | | | | |
| $\beta_P$ | 1.292 (2.966) | 0.577 (2.230) | 0.865 (2.148) | −0.257 (1.965) |
| $\sigma_{\beta P}$ | 1.033 (1.000) | 0.119 (10.244) | 0.059 (1.000) | 0.874 (1.000) |
| Feature | | | | |
| $\beta_F$ | −0.515 (0.986) | −0.625 (1.059) | −0.527 (1.278) | −0.781 (1.268) |
| $\sigma_{\beta F}$ | 0.053 (1.000) | 0.052 (1.000) | 1.021 (4.959) | 0.725 (4.245) |
| Certain error | | | | |
| $\sigma_{\varepsilon 1}$: Strawberry | | 0.436 (0.346) | 0.661 (0.505) | |
| $\sigma_{\varepsilon 2}$: Cherry | | 0.431 (0.194) | 0.132 (0.343) | |
| $\sigma_{\varepsilon 3}$: Blueberry | | 0.649 (0.413) | 0.336 (0.540) | |
| $\sigma_{\varepsilon 4}$: Mixed Berry | | 0.473 (0.560) | 2.639 (0.752) | |
| Uncertainty error | | | | |
| $\sigma_{\zeta 1}$: Strawberry | 1.575 (0.276) | | | 0.546 (0.292) |
| $\sigma_{\zeta 2}$: Cherry | 2.429 (0.575) | | | 0.831 (0.571) |
| $\sigma_{\zeta 3}$: Blueberry | 2.168 (0.817) | | | 1.000 (0.269) |
| $\sigma_{\zeta 4}$: Mixed Berry | 0.051 (0.902) | | | 0.166 (0.242) |
| $\sigma_{\zeta 5}$: Piña Colada | 1.521 (0.692) | | | 0.586 (0.307) |

*Notes.* Numbers not in the parentheses are the estimated parameters; numbers in the parentheses are the standard errors of the corresponding estimates. The following normalizations are made: $\varphi_5 = 0.000$, $\sigma_{\varphi 5} = 0.000$, and $\sigma_{\varepsilon 5} = 1.000$. $\sigma_{\varepsilon}$ is restricted to one in the full model and Model 3, and other unestimated parameters in Models 1–3 are restricted to zero.

important to account for the temporal impact of state dependence from past purchase. Another interesting comparison is regarding the relative importance of utility uncertainty versus state dependence in driving horizontally varied assortment purchase. Note that it is not fair to compare Model 1 directly with Model 3 for this purpose, because the former also accounts for temporal variation in assortment choice whereas the latter does not. We can instead compare Model 2 with Model 3, where either state dependence or utility uncertainty is specified as the underlying driver for horizontal variety choice, respectively. Model 2 performs better than Model 3 on in-sample log-likelihood value, whereas their performance becomes closer on the ground of AIC and BIC because Model 2 uses more parameters. Moreover, in terms of prediction, Model 2 continues to perform better except for the Sioux Falls validation sample.

It is worthwhile to note that here the parameter estimates obtained from a single-person sample are used to assess the predictive performance of the models for both single- and multiple-person households. Predictions for multiple-member households can be potentially difficult, because household member heterogeneity is a potential explanation for varied choice. Moreover, two of the validation samples are from another city (i.e., Sioux Falls) where the market environment is supposedly different. Nevertheless, despite the difference in household size or market environment or both, all three validation samples provide consistent support for the model fit results obtained from the calibration sample. The extent to which the full model performs better than the alternative models in the validation samples is comparable to the calibration sample. These results constitute strong support for the importance of both state dependence and utility uncertainty in driving the observed assortment purchase behavior, for both single- and multiple-person households.

**Parameter Estimates**

The model parameters estimated on the calibration sample are reported in Table 6. I start the discussion with the estimates from the full model. The means of the variety-specific intercepts (i.e., $\varphi$) are not significantly different from zero except for Strawberry and Mixed Berry. This indicates that consumers have a significant intrinsic preference for these two varieties, whereas the other varieties do not differ significantly from each other in terms of their intrinsic ability to satisfy the consumers' yogurt consumption need. The mean price response coefficient is negative but not significant. The standard deviation of the price response parameter is statistically insignificant either. This can be attributed to the relatively high empirical correlation of the prices across varieties (see Table A.1). Similarly, the feature response parameters are statistically insignificant.

The mean state dependence parameter $\omega$ is statistically significant and positive, with statistically significant standard deviation $\sigma_\omega$ as well. Because $\sigma_\omega$ is small (0.519) relative to $\omega$(0.859), most consumers seem to have positive yet heterogeneous state dependence parameter $\omega_i$. This suggests that, across purchase occasions, a consumer tends to purchase the same variety/assortment. Note that comparing the diagonal with the corresponding off-diagonal figures in Table 4 provides face validity for this inferred temporal choice pattern. In the same vein, the positive state dependence parameter also implies that, *all else being equal*, assortments with homogeneous items are valued more than those with different varieties. In terms of horizontal choice behavior, consumers are hence expected to buy more homogeneous assortments than varied ones. However, this is supported in Table 3 only for Mixed Berry but not for the other varieties.

The presence of utility uncertainty offers a reconciliation for this otherwise paradoxical result. The uncertainty parameter estimates $\sigma_\zeta$, as presented in Table 6, are significantly different from zero except for Mixed Berry. This suggests that consumers at the purchase time are pretty sure of their future utility for Mixed Berry, whereas the consumption utility of the other varieties is dependent on future contingency. We can then use this result to explain the observed pattern of horizontal assortment choice presented in Table 3. Despite the desire to consume the same variety across future consumption occasions (i.e., positive state dependence), buying different varieties together can increase the expected value of the purchased assortment because future consumption choices will become more flexible. However, purchasing Mixed Berry with other varieties is less likely to enhance consumption flexibility, given that the estimated uncertainty parameter is small and insignificant for Mixed Berry.

Next, I compare the parameter estimates between the full and the alternative models, which provides additional support for the behavioral processes inferred from the full model. Note that the alternative models do not sufficiently specify the two underlying processes for assortment choice, and may therefore generate bias in the parameter estimates governing these processes. Interestingly, ignoring an underlying process may not necessarily overestimate the importance of the other captured process.[8] In

[8] The alternative models here turn out to underestimate the effect that is being captured. This is because positive state dependence and utility uncertainty lead to opposite purchase behavior. If instead the consumers desired to seek variety, ignoring the state dependence effect would overestimate utility uncertainty, and vice versa.

particular, the state dependence impact in Model 1, where utility uncertainty is ignored, is underestimated in comparison to the full model. This is not surprising because positive state dependence is at odds with the horizontally varied assortment purchases as documented in Table 3, which are arbitrarily prevented from being explained by consumption flexibility. Moreover, if a model captured only horizontal variety choice while cutting both the state dependence effect across purchase occasions and the consumption flexibility effect, a qualitatively different estimate on state dependence would be obtained: $\omega$ becomes significantly negative in Model 2. This misleading result is ensured in light of the frequency of observing different varieties in horizontal assortment purchase (Table 3). Similarly, ignoring state dependence does not necessarily exaggerate the importance of utility uncertainty: In Model 3, the statistical significance of the utility uncertainty parameters becomes dramatically lower except for Blueberry, and the magnitude of the uncertainty parameter estimates is underestimated except for Mixed Berry. This suggests that, without adequately specifying the state dependence effect, utility uncertainty alone is not sufficient to restore the observed assortment purchase behavior.

**Numerical Simulation**

To provide additional evidence for the above results, a simulation exercise is run on synthetically generated data set using the full model with a prespecified set of parameters. The data set is created by simulating the assortment purchase behavior of the consumers in the calibration sample, assuming that the full model is the "true" model and using the same explanatory variable values. The true parameters, as shown in the second column of Table 7, represent the interesting case, inferred from the above calibration results, that the consumers' utility is increasing for a previously consumed variety and that there is uncertainty at the purchase time about future consumption utility.

The exercise of data set generation and model estimation is repeated many times. The average of the estimated parameters and goodness-of-fit statistics for the full and the alternative models are reported in columns 3–6 of Table 7. The results highlight the following. First, the full model estimates recover the true parameters quite accurately. The point estimates in the third column generally correspond to what they should be with reasonable confidence. Second, incorporating the impact of state dependence or utility uncertainty or both, improves the performance in explaining the simulated assortment choice. This can be seen from comparing the average model fit statistics of the full model against the other three models, and Model 1 against Model 2 as well.

Third, examining the average parameter estimates in Models 1–3 further demonstrates the importance of fully specifying the effects of both state dependence and consumption flexibility. The average estimate on the mean state dependence effect in Model 1 is significantly lower than the true value (i.e., 0.580 versus 0.800), confirming that arbitrarily shutting down utility uncertainty underestimates the state dependence effect. This bias can be even stronger if the state dependence effect across purchase occasions is further foreclosed (Model 2). Note also that the magnitude of the mean state dependence estimate in Model 2 is quite small in comparison to the standard deviation (i.e., $\omega = 0.156$ and $\sigma_\omega = 0.750$). This implies that, if we only captured the impact of state dependence on horizontal assortment choice, we could obtain a negative state dependence estimate for a substantial proportion of consumers, which may mislead us to infer that these consumers are seeking varied consumption although they actually desire to consume the same variety over time. On the other hand, we might misestimate the impact of utility uncertainty if the positive effect of state dependence were not correctly teased out: $\sigma_\zeta$ are significantly underestimated in Model 3 wherein the state dependence effect is arbitrarily absent.

Overall, these results suggest that the full model is capable of recovering the underlying effects driving the simulated choice. The importance of accounting for both state dependence and utility uncertainty also receives additional support.

**Policy Analysis**

The value of the proposed model lies not only in the correct recovery of the underlying behavioral processes but also in the policy implications one can derive. In this section, I conduct some policy simulations to demonstrate the managerial implications that are in agreement with the behavioral inference obtained from the full model. To this end, the extent to which each variety's aggregate demand responds to two counterfactual changes is assessed.

I start with simulating each consumer $i$'s demand for each variety $j$ on shopping occasion $t$ by integrating over the predicted choice probabilities of the assortments involving that variety. Simulated percentage changes in weekly demand in response to various policy experiments are then computed by aggregating the predicted individual demand. The average of the simulated percentage changes across weeks, weighted by the weekly share, constitutes the long-term demand response. Table 8 reports the average own percentage changes in weekly demand in response to hypothetical variety-specific free

**Table 7 Average Parameter Estimates and Model Fit Statistics on Simulated Data**

| Parameters | True | Full model | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|---|
| Strawberry intercept | | | | | |
| $\varphi_1$ | 0.100 | 0.098 (0.221) | 0.316 (0.221) | 0.043 (0.428) | 0.091 (0.178) |
| $\sigma_{\varphi 1}$ | 0.500 | 0.498 (0.341) | 0.269 (0.264) | 0.505 (0.454) | 0.534 (0.287) |
| Cherry intercept | | | | | |
| $\varphi_2$ | 0.100 | 0.113 (0.143) | 0.339 (0.195) | 0.117 (0.268) | 0.076 (0.160) |
| $\sigma_{\varphi 2}$ | 0.500 | 0.421 (0.237) | 0.253 (0.221) | 0.545 (0.449) | 0.492 (0.219) |
| Blueberry intercept | | | | | |
| $\varphi_3$ | 0.100 | 0.109 (0.204) | 0.463 (0.212) | 0.410 (0.291) | 0.102 (0.124) |
| $\sigma_{\varphi 3}$ | 0.500 | 0.526 (0.293) | 0.187 (0.112) | 0.380 (0.331) | 0.528 (0.278) |
| Mixed Berry intercept | | | | | |
| $\varphi_4$ | 0.100 | 0.116 (0.145) | 0.256 (0.194) | −0.219 (0.434) | 0.039 (0.150) |
| $\sigma_{\varphi 4}$ | 0.500 | 0.496 (0.290) | 0.256 (0.244) | 0.898 (0.587) | 0.645 (0.273) |
| State dependence | | | | | |
| $\omega$ | 0.800 | 0.923 (0.161) | 0.580 (0.147) | 0.156 (0.523) | |
| $\sigma_\omega$ | 0.500 | 0.517 (0.118) | 0.345 (0.180) | 0.750 (0.722) | |
| Weekly decay factor | | | | | |
| $\lambda$ | 0.800 | 0.806 (0.042) | 0.833 (0.068) | 0.696 (0.366) | |
| Price | | | | | |
| $\beta_P$ | −1.500 | −1.566 (1.621) | −1.524 (1.540) | −2.078 (1.951) | −1.422 (1.398) |
| $\sigma_{\beta P}$ | 1.000 | 1.225 (1.082) | 1.063 (1.378) | 2.104 (2.438) | 1.299 (1.374) |
| Feature | | | | | |
| $\beta_F$ | 0.500 | 0.906 (1.615) | 0.840 (1.876) | 1.445 (3.325) | 0.687 (0.912) |
| $\sigma_{\beta F}$ | 1.000 | 1.586 (1.966) | 1.378 (2.199) | 2.172 (4.339) | 1.158 (0.999) |
| Certain error | | | | | |
| $\sigma_{\varepsilon 1}$: Strawberry | 1.000 | | 0.437 (0.280) | 1.001 (0.510) | |
| $\sigma_{\varepsilon 2}$: Cherry | 1.000 | | 0.446 (0.237) | 1.090 (0.392) | |
| $\sigma_{\varepsilon 3}$: Blueberry | 1.000 | | 0.317 (0.228) | 0.632 (0.336) | |
| $\sigma_{\varepsilon 4}$: Mixed Berry | 1.000 | | 0.671 (0.281) | 1.061 (0.755) | |
| Uncertainty error | | | | | |
| $\sigma_{\zeta 1}$: Strawberry | 1.500 | 1.442 (0.580) | | | 0.410 (0.220) |
| $\sigma_{\zeta 2}$: Cherry | 1.500 | 1.581 (0.444) | | | 0.377 (0.208) |
| $\sigma_{\zeta 3}$: Blueberry | 2.500 | 2.614 (0.360) | | | 0.961 (0.226) |
| $\sigma_{\zeta 4}$: Mixed Berry | 0.500 | 0.648 (0.541) | | | 0.172 (0.124) |
| $\sigma_{\zeta 5}$: Piña Colada | 1.500 | 1.761 (0.467) | | | 0.410 (0.229) |
| −LL | | 841.25 | 861.99 | 1,024.41 | 1,053.71 |
| AIC | | 861.25 | 880.99 | 1,043.41 | 1,070.71 |
| BIC | | 902.14 | 919.84 | 1,082.26 | 1,105.47 |

*Notes.* Numbers not in the parentheses are the average parameter estimates; numbers in the parentheses are the average standard errors of the corresponding estimates. The following normalizations are made: $\varphi_5 = 0.000$, $\sigma_{\varphi 5} = 0.000$, and $\sigma_{\varepsilon 5} = 1.000$. $\sigma_\varepsilon$ is restricted to one in the full model and Model 3, and other unestimated parameters in Models 1–3 are restricted to zero.

sample distribution and utility uncertainty increase, respectively. It is assumed that one unit of a variety is distributed for free and consumed by consumers shopping in the second week. In the case of utility uncertainty, the impact on aggregate demand of increasing a variety' uncertainty estimate (i.e., $\sigma_\zeta$) by 100% is assessed.

The results based on the full model suggest that the positive state dependence effect can yield considerable demand increase if free samples are distributed. The sales return to free sample distribution can be as high as 21% (i.e., Mixed Berry). Nevertheless, there is substantial heterogeneity across the varieties in terms of the effectiveness of free sample distribution. The demand change for Strawberry is significantly lower than for other varieties. This can be potentially explained by the difference in the varieties' intercept estimates ($\varphi$). Recall that consumers have a significantly higher intrinsic preference for Strawberry over other varieties, which can be relatively difficult to be further escalated through free sampling.

One key advantage of the proposed model is that it can be used to assess the impact of changing consumers' uncertainty perception, which in practice can be influenced by commercials, (online) information provision, product life cycle, etc. The proposed model provides a quantifiable means to measuring the magnitude and the demand impact of utility uncertainty. Table 8 reveals that the varieties differ considerably in the extent of aggregate demand change in response

**Table 8 Counterfactual Percentage Change in Own Aggregate Demand on Scanner Data**

| Counterfactuals | Full model | Model 1 | Model 3 |
|---|---|---|---|
| Free sample distribution | | | |
| Strawberry | 1.184 | 1.303 | |
| Cherry | 6.471 | 39.973 | |
| Blueberry | 10.071 | 35.041 | |
| Mixed Berry | 21.484 | 30.769 | |
| Piña Colada | 8.826 | 11.978 | |
| Increased uncertainty | | | |
| Strawberry | 0.606 | | 2.438 |
| Cherry | 33.325 | | 14.110 |
| Blueberry | 24.383 | | 14.691 |
| Mixed Berry | 0.001 | | 0.652 |
| Piña Colada | 19.547 | | 9.840 |

*Note*. The figures reflect percentage changes in own long-term demand in response to two counterfactuals, one unit variety-specific free sample distribution and a 100% increase in the variety-specific uncertainty parameter ($\sigma_\xi$), respectively.

to a 100% increase in the utility uncertainty parameter. Note first that increasing uncertainty can substantially improve a variety's sales by as much as 33% (i.e., Cherry). Nevertheless, the percentage change in a variety's aggregate demand may not necessarily be proportional to the importance of that variety's estimated uncertainty parameter. The demand response is significantly lower not only for the variety with the lowest uncertainty estimate (i.e., Mixed Berry) but also for the variety with significantly higher intercept estimate (i.e., Strawberry). Therefore, it seems that increasing uncertainty can improve aggregate demand only for those varieties with both relatively high uncertainty estimate and relatively low intrinsic preference. It is the purchase of these varieties that is most likely to be driven by the consumers' desire to maintain consumption flexibility.

Table 8 also reports the results of policy experiments performed on the alternative models. Note first that the assessment of counterfactual demand response cannot be run on some of the alternative models. For example, Models 1 and 2 cannot evaluate the demand impact of utility uncertainty change. In addition, the long-term demand implication of free sample distribution is missing in Models 2 and 3. Recall also that in Model 2 the state dependence effect on (horizontal) assortment choice is estimated to be significantly negative ($-0.301$). Should we rely on this estimate for promotion planning, we might mistakenly conclude that free samples are detrimental to sales. Second, the alternative models seem to prescribe biased demand implications in comparison to the full model. Interestingly, however, the bias in the demand change in response to free sampling is upward for Model 1, where the state dependence parameter $\omega$ is underestimated (Table 7). This could be because in the long run the presence of the consumption flexibility effect in the full model dilutes the positive impact of free samples on demand. On the other hand, Model 3 underestimates not only the utility uncertainty parameters but also their impacts on the counterfactual aggregate demand responses. These results suggest that the alternative models lack or misestimate the counterfactual demand implications that can be otherwise inferred from the full model.

## Conclusion

Previous studies have investigated multiple-item purchase without explicitly decomposing the underlying rationales, namely state dependence and consumption flexibility. To fill this gap, this paper proposes a model that is capable of identifying these two effects even with scanner panel data where consumption information is missing. A two-stage simulation technique is used to estimate the model on scanner data of yogurt purchase, which suggests that consumption utility is self-reinforcing (i.e., positive state dependence) and uncertain for some varieties as well. These two recovered effects provide strong empirical support with sound face validity for the observed temporal and horizontal patterns of assortment choice. The importance of capturing and distinguishing these two effects is demonstrated. Moreover, it is illustrated that the insights inferred from the proposed model can be translated into substantial managerial implications.

There are some caveats in the current analysis. Note that assortment choice is conditional on the observed purchase quantity that is further assumed to be equal to the number of consumption occasions. The empirical analysis has carefully chosen a context in which this assumption is likely to hold, i.e., yogurt. For example, the perishability of yogurt increases the likelihood that the purchased products are depleted prior to next shopping occasion, and the one-unit-per-consumption assumption is quite reasonable for the popular small-sized yogurts. Nevertheless, future research can investigate the consumers' decision on purchase quantity, where temporal price change can be an important explanatory variable and assortment choice and purchase quantity can be interdependent. Consumer inventory also remains a potential topic for future investigation. On the one hand, utility uncertainty may affect how stockpiling decisions are made. Whereas on the other hand, the stockpiled products may influence the choice of assortment that is to be added into the current inventory. Moreover, the model can be extended to allow for the depletion of multiple products per consumption occasion.

Although this paper demonstrates that the absence of consumption information may not necessarily prevent us from making behavioral inference from purchase data, it is no doubt that consumption data can complement purchase information and enable us to derive additional insights. The underlying processes captured in this paper, and many others (e.g., stockpiling), can be recovered with increasing efficiency if consumption data is available. With consumption incidence data (i.e., when a variety is consumed), for example, one can estimate a joint model of purchase quantity and assortment choice. Moreover, some interesting issues, such as preference time-inconsistency across purchase and consumption, can be investigated with consumption information.

Other avenues for future research are numerous. Alternative operationalizations to account for state dependence can be utilized (e.g., McAlister 1982, Lattin 1987, Trivedi et al. 1994, Erdem 1996, Fader and Hardie 1996, Ho and Chong 2003). The current model can also be extended to incorporate forward-looking consumption decision into the assortment valuation process. In the current setup, consumers are presumed to simply consume the best available product on each consumption occasion. This assumption could underestimate the relative valuation of horizontally varied assortments, if indeed consumers take into account the impact of current consumption decision on future consumption/purchase (Walsh 1995). Moreover, it would be interesting to investigate alternative sources of utility uncertainty. Another useful extension would be to parameterize the utility uncertainty term. Identifying the drivers of utility uncertainty (e.g., consumer characteristics, commercials, etc.) can be pursued in future research. Overall, I hope that this research inspires more interest in the study of utility uncertainty and consumption flexibility and, more generally, in making behavioral inference using field data with observed behavior.

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## Appendix
The two-stage logit-smoothed accept-reject simulator is adapted from Train (2003, Chap. 5) and can be implemented using the following steps to approximate the choice probability in (10):

*Step* 1A. Make a draw for the $J$-dimensional vector of random term $\mu_{ijt}$ from the multivariate normal density $f(\mu_{ijt})$, and label it as $\mu_{it}^g = \{\mu_{i1t}^g, \ldots, \mu_{iJt}^g\}$.

*Step* 1B. Make a draw for the $J$-dimensional vector of errors $\zeta_{jm}$ from an independent multivariate normal density with zero mean and standard deviation $\sigma_\zeta$. Repeat this for each of the $m = 1, 2, \ldots, n_{it}$ consumption occasions. Label the draw as $\zeta_i^{gl} = \{\zeta_{i11}^{gl}, \zeta_{i21}^{gl}, \ldots, \zeta_{iJ1}^{gl}, \ldots, \zeta_{i1,n_{it}}^{gl}, \zeta_{i2,n_{it}}^{gl}, \ldots, \zeta_{iJ,n_{it}}^{gl}\}$.

*Step* 1C. Using the draws obtained in Steps 1A and 1B, calculate the consumption utility $U_{ijtm}^{gl}$ as in (1). Plug this into (7) and calculate $V^{gl}(\bar{d}(n_{it})) = \sum_{m=1}^{n_{it}} \max_{j\in \underline{d}_m} U_{ijtm}^{gl}$ for all $\bar{d}(n_{it}) \in \bar{D}(n_{it})$.

*Step* 1D. Repeat Steps 1B and 1C for $L$ times. Label the repetitions as $l = 1, 2, \ldots, L$.

*Step* 1E. The simulated expected gross value for an assortment $\bar{d}(n_{it}) \in \bar{D}(n_{it})$, conditional on the draw $\mu_{it}^g$ in Step 1A, is given by $\hat{E}^g[V(\bar{d}(n_{it}))] = 1/L \sum_{l=1}^{L} V^{gl}(\bar{d}(n_{it}))$.

*Step* 2A. Plug the simulated expected gross value obtained in Step 1E, $\hat{E}^g[V(\bar{d}(n_{it}))]$, into the logit formula. That is, calculate

$$\hat{P}^g(\bar{d}(n_{it})) = \frac{\exp\left(\left(\hat{E}^g[V(\bar{d}(n_{it}))] - \sum_{j\in \bar{d}(n_{it})} X_{ijt}\beta_i\right)/\gamma\right)}{\sum_{\bar{d}\in \bar{D}(n_{it})} \exp\left(\left(\hat{E}^g[V(\bar{d})] - \sum_{j\in \bar{d}} X_{ijt}\beta_i\right)/\gamma\right)},$$

where $\gamma > 0$ is a scale parameter that is to be specified by the researcher.

*Step* 2B. Repeat Steps 1A–2A for $G$ times. Label the repetitions as $g = 1, 2, \ldots, G$.

*Step* 2C. The simulated unconditional probability in (10) for a purchase observation $\bar{d}_{it}$ made by consumer $i$ at time $t$ is then given by $\tilde{P}(\bar{d}_{it}) = (1/G) \sum_{g=1}^{G} \hat{P}^g(\bar{d}_{it})$.

In simulating the expected gross assortment value (i.e., Steps 1B–1E), a set of 50 random draws are generated for $\zeta_{ijm}$ for each $i$, $j$, and $m$, i.e., $L = 50$. These steps are run conditional on each draw $\mu_{it}^g$ from $f(\mu_{ijt})$. In simulating the assortment choice probability (i.e., Steps 2A–2C), the draw $\mu_{it}^g$ and the Steps 1B–1E are repeated for 50 times, i.e., $G = 50$.

**Table A.1 Distribution and Correlation of the Varieties' Prices**

| | | Correlation | | | | |
|---|---|---|---|---|---|---|
| | Mean | Strawberry | Cherry | Blueberry | Mixed Berry | Piña Colada |
| Strawberry | 61.068 | 8.383 | 0.877 | 0.876 | 0.835 | 0.874 |
| Cherry | 61.389 | | 9.480 | 0.829 | 0.790 | 0.816 |
| Blueberry | 61.463 | | | 9.663 | 0.806 | 0.823 |
| Mixed Berry | 62.018 | | | | 8.120 | 0.823 |
| Piña Colada | 60.980 | | | | | 8.480 |

*Notes.* The second column and the diagonal figures, measured in cents, are the means and standard deviations of the varieties' prices for the calibration sample, respectively. The other off-diagonal entries are the price correlation coefficients for each pair of the varieties.

In summary, this simulator consists of two stages. In the first stage (i.e., Steps 1A–1E), the expected gross assortment value in (7) is simulated. In the second stage (i.e., Steps 2A–2C), the simulated expected gross assortment value obtained in the first stage, $\widehat{E}^g[V(\bar{d}(n_{it}))]$, is used to simulate the assortment choice probability in (10). The logit-smoothed accept-reject simulator in the second stage is akin to Train (2003, Chap. 5). The use of the logit-smoothing function in Step 2A has some nice properties in comparison to the alternative simulator that uses a discrete accept/reject indicator function. First, it can avoid zero values for the simulated choice probability. Second, the simulated choice probability is a smooth and twice-differentiable function of the parameters.

In the empirical results reported in the main text, the scale parameter in the logit-smoothing function in Step 2A is set as $\gamma = 1$. To assess the sensitivity of the results, the proposed and the alternative models in the paper are reestimated, conditional on different specifications for the scale parameter, i.e., $\gamma = 0.2, 0.4, 0.6, 0.8$. For each of the alternative specifications, no significant qualitative difference is detected from the specification adopted in the paper (i.e., $\gamma = 1$) except that the parameter values are scaled down accordingly in similar extent.

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