Consumer Stockpiling and Price Competition in Differentiated Markets

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In many storable-goods markets, firms are often aware that consumers may strategically adjust purchase timing in response to expected price dynamics. For example, in periods when prices are low, consumers stockpile for future consumption. This paper investigates the dynamic impact of consumer stockpiling on competing firms' strategic pricing decisions in differentiated markets. The necessity of equilibrium consumer storage for storable products is re-examined. It is shown that preference heterogeneity generates differential consumer stockpiling propensity, thereby intensifying future price competition. As a result, consumer storage may not necessarily arise as an equilibrium outcome. Economic forces are also investigated that may mitigate the competition-intensifying *effect of consumer inventories and that, hence, may lead to equilibrium consumer storage.*

1. Introduction

When offering price promotions, firms are often aware that consumers may adjust their purchase timing in response to dynamic price changes. Specifically, in periods when prices are low, consumers may accelerate their purchases of a product that is expected to be consumed in the future when prices go up. As a result, consumer stockpiling could create intertemporal demand shifts from the future to the current period. This temporary demand expansion is distinct from sales increases generated by brand switching. In differentiated markets, intertemporal demand

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shifts may influence price competition in subsequent periods. Therefore, to ascertain the strategic rationale underlying competing firms' inventory-inducing promotion efforts, it is important to investigate the dynamic effects of consumer storage.

Consumer stockpiling could potentially generate incentives for competing firms to offer price promotions, in order to compete away potential future demand of the competitors. To investigate this possibility we consider price competition through time in a two-period model among differentiated firms facing heterogeneous consumers. Consumers in each period desire to consume exactly one unit of any product and do not change preferences over time, but have the possibility of buying more than one unit in one period for storage and future consumption. Both firms and consumers are forward-looking, and consumers take into account their relative product preferences and expected arbitrage opportunities from price dynamics. Firms are fully aware of how consumer stockpiling behavior responds to price changes, and balance the immediate benefits from expanding temporary demand against the consequent dynamic effects of consumer storage on future price competition.

We identify the effect that consumers exhibit a differential propensity for stockpiling depending on their relative preferences for the different products. In particular, those consumers with a relatively higher preference for a product are more likely to stockpile that product and are therefore out of the market in later periods, leading to a lower degree of differentiation and more intense price competition in future periods. This then reduces the incentives of the firms in the initial period to lower prices such that consumers stockpile. In the stylized model we consider, under fixed consumer preferences through time, this effect ends up dominating, and the firms do not cut prices enough in the first period to lead consumers to stockpile. Note that the key to this no-storage equilibrium is the positive relationship between product preference and stockpiling propensity.

To highlight the role of the strategic effect of consumer storage in sustaining the no-storage equilibrium, we also investigate two potential factors which may counter this effect and as a result may recover consumer stockpiling as an equilibrium. If consumers change preferences from period to period, the positive relationship between consumer preference and stockpiling propensity may not be significantly extended into the future period. One can then obtain that the change of preferences drives up future market differentiation and preference heterogeneity, counteracting the competition-intensifying effect of consumer inventory and thus resulting in equilibrium consumer stockpiling. Similarly, if the firms discount future payoffs sufficiently more than the consumers do, the competition-intensifying effect of consumer inventory will be a less important concern for the firms and positive equilibrium storage can also be obtained.

Two recent papers are closely related to the results presented here: Hong et al. (2002) and Anton and Das Varma (2005). Hong et al. (2002) analyze the effect of consumer heterogeneity in price searching and storage ability on price dynamics. In their model, the only set of stockpiling consumers are "shoppers" (akin to Varian, 1980) who always seek to buy at the lowest price. The dynamic effect of storage is hence to mitigate future price competition when the market is comprised only of "loyal" consumers. In contrast, this paper shows that when no restriction is imposed on consumers' stockpiling capabilities, the consumers who are more likely to stockpile a product are those who have relatively higher preferences, that is, the "captives" ("loyals") in Hong et al. (2002). Therefore, future price competition is intensified as a result of stockpiling by the "loyal" consumers.

Anton and Das Varma (2005) study quantity competition for a homogeneous good with vertical heterogeneous consumer preferences. In that case, as the product is homogeneous, there is no decrease in product differentiation if consumers stockpile the product. However, as assumed in that paper, if the consumers that stockpile are the ones that value the product most, more stocks lead to lower prices in the next period which may decrease the incentives for the firms to cut prices in the initial period to induce stockpiling. But as demand increases with lower prices, this incentive not to cut prices in the first period ends up not being too large, and Anton and Das Varma find that if the discount factor is sufficiently large the unique equilibrium involves storage. In contrast, this paper considers differentiated goods (instead of a homogeneous good), price competition (instead of quantity competition), and complete market coverage in equilibrium, and finds that for fixed consumer preferences the equilibrium involves no storage. The assumptions of product differentiation and price competition (although not necessarily the assumption of complete market coverage) can be seen as interesting conditions to investigate.

Note that product differentiation with consumer heterogeneity is essential for the argument above that stockpiling leads to more intense future competition by reducing the level of differentiation in the market. An important contribution of this paper with respect to Anton and Das Varma can be seen as identifying the effect that under differentiation stockpiling leads to greater future price competition because of differential stockpiling propensity across consumers. In the homogeneous-good case of Anton and Das Varma, price competition would lead to a no-storage equilibrium as the differentiation case with fixed preferences. However, price competition under differentiation and partially changing preferences leads to an equilibrium with storage. Anton and Das Varma also show that a monopolist would not choose to offer quantities such that there is stockpiling in equilibrium. This could be seen as potentially related to the no-storage equilibrium under differentiation (market power) in this paper. However, the mechanism by which one gets no storage is quite different in the two settings. In the monopoly case, the market is not fully covered, and the no-storage result comes from the firm fully internalizing that sales today will crowd out sales tomorrow. In the differentiation case considered in this paper, the market is fully covered, and the no-storage result comes from the firms realizing that stockpiling will lead to intense price competition in the future.

In other related work, Salop and Stigltz (1982) consider consumer stockpiling under monopolistic competition. Bell et al. (2002) present an extension of Salop and Stiglitz when consumers may accelerate consuming stockpiled products. Jeuland and Narasimhan (1985) consider the use of stockpiling as a way for firms to price discriminate between different types of consumers. Lal et al. (1996) investigate the strategic effects of stockpiling by a retailer in a distribution channel. Gupta (1988) investigates empirically the role of stockpiling in consumers' response to price promotions.[1] For other empirical investigations of consumer stockpiling in response to price promotions, see also Blattberg and Neslin (1990), Erdem et al. (2003), and Hendel and Nevo (2006).

Other related literatures are the ones on competition with durable goods (e.g., Bucovetsky and Chilton, 1986; Bulow, 1986; Carlton and Gertner, 1989), and on competition with the possibility of forward contracts (e.g., Allaz and Vila, 1993). Under competition with durable goods, a firm, when selling, substitutes for possible renting in the next period. However, there the no arbitrage condition requires the durable sale price to be equal to the discounted value of the rental prices, while here the no arbitrage condition requires the current nondurable sale price to be equal to the discounted future price. Under competition with the possibility of forward contracts the no arbitrage condition is that the forward price be the same as the expected future spot price, which is also different from the no arbitrage condition under the possibility considered here of consumer stockpiling.

The remainder of this paper is organized as follows. The next section lays out the base model. Section 3 analyzes the dynamic effects

1. For empirical evidence on price promotions see, for example, Villas-Boas (1995) and Zhao (2006).

of consumer inventory on subsequent market differentiation and price competition, and establishes the characteristics of storage demand. In Section 4, the strategic concerns underlying pricing decisions are identified and the no-storage equilibrium is established. The roles of preference uncertainty and different consumer versus firm patience in balancing the pricing incentives are explored in Section 5, where the characteristics of storage equilibria are also analyzed. The last section concludes the paper and discusses directions for future research.

2. The Base Model

Consider a duopoly market in which two firms, $i \in \{A, B\}$, produce nondurable products A and B, respectively, at zero marginal costs. The game consists of two periods, indexed by $n = 1, 2$. In each of these two periods, a consumer desires to consume one unit of either product. No additional utility can be accomplished from consuming more than one unit of either product in any period.

The market consists of a continuum of consumers, the mass of which is normalized to 1. Consumer preferences are heterogeneous and captured by the location $x \in [0, 1]$ (Hotelling, 1929). The cumulative preference distribution function $F(x)$ is assumed to be smooth, continuous, and twice differentiable everywhere on [0, 1] with strictly positive density $f(x)$, where $F(0) = 0$ and $F(1) = 1$. $F(x)$ is assumed to satisfy the monotonic hazard rate (MHR) property, that is, the inverse hazard rate $H(x) \equiv \frac{[1 - F(x)]}{f(x)}$ is strictly decreasing in x. Moreover, the density function $f(x)$ is assumed to be symmetric around 1/2, which implies that $h(x) \equiv \frac{F(x)}{f(x)}$ is strictly increasing in x. In the presentation of the equilibrium strategies in Section 4 we focus on uniform preferences, that is, $F(x) = x$ and $f(x) = 1$, for all $x \in [0, 1]$.

Product A's and B's location on the Hotelling line are fixed at 0 and 1, respectively.[2] The variable x could represent a consumer's most preferred product attribute level, hence measuring the consumer's relative preference for product B over A. In particular, a consumer located at x derives gross utility $u_A(x) = v - xt$ from consuming one unit of product A, and $u_B(x) = v - (1 - x)t$ from product B. The parameter v is the stand-alone value of consuming either of the products compared to no consumption, and $t > 0$ can be viewed as measuring the per-unit disutility of consuming a mismatched product. If a consumer does not consume anything, then $u(x) = 0$.

2. We also analyzed arbitrary product locations on [0, 1] under uniform preferences, and obtain similar results.

The stand-alone value v is assumed to be sufficiently large such that in any period each consumer desires to consume at least one product. Moreover, the products are assumed to be storable such that consumers could purchase more than one unit in period one and stockpile for future consumption. Note that the first-period consumption and stockpiling decisions are completely separated. In determining whether and which product to stockpile, a consumer takes into account the products' current and expected future prices, as well as their relative preferences. Consumers' stockpiling decisions, arising from this trade-off, could potentially lead to intertemporal demand shifts and serially correlated equilibrium prices. In the base model, it is assumed that consumers do not change preferences over time. This assumption is relaxed in Section 5.1 in order to examine how changing preferences may influence firms' equilibrium prices through mitigating the dynamic effects of consumer stockpiling on future price competition.

In each period, the firms simultaneously make pricing decisions P_{in}, $i = A, B$, $n = 1, 2$. In period one, the firms cannot commit to the prices to be charged in period two. After observing the first-period prices, consumers make their purchase/stockpiling decisions. Each consumer acts noncooperatively and simultaneously with respect to whether or not to build up an inventory and which product to stockpile. Consumers who stockpile exit the market and make no further purchases in period two.[3] The consumers and firms are assumed to be risk-neutral, and maximizing discounted expected wealth-equivalent payoffs using a common discount factor λ, with $0 \leq \lambda \leq 1$. The assumption of common discount factor will be relaxed in Section 5.2 to investigate the differential impact of consumer versus firm patience.

As a benchmark, let us consider the nonstockpiling case when advance purchases are not feasible, potentially because inventory costs are prohibitive. The consumers and firms hence care only about the current-period payoffs, resulting in the static model replicating itself in two identical periods. Without loss of generality, we can then restrict attention to a model with a single period. Let P_i be the price charged by firm i, $i = A, B$. The market demand is characterized by the cut-off point $x(P_A, P_B) = \frac{t - P_A + P_B}{2t}$, where consumers at $x \leq x(P_A, P_B)$ buy product A, and consumers at $x \geq x(P_A, P_B)$ purchase product B. The firms' objective functions are thus given by $\pi_A(P_A, P_B) = P_A \cdot F(x(P_A, P_B))$ and $\pi_B(P_A, P_B) = P_B \cdot [1 - F(x(P_A, P_B))]$, respectively.

3. A consumer who stockpiles may enter the market again in period two if the entry cost is sufficiently low and the benefits of finding a better consumption are sufficiently high. In the current context, as will be shown, the consumers who are induced to stockpile a product tend to have relatively higher preferences for the product than those who are not, and, therefore, are in fact out of the market in period two.

Taking the first-order derivative of the profit functions with respect to P_A and P_B, respectively, yields $P_A = 2h(x(P_A, P_B))t$, and $P_B = 2H(x(P_A, P_B))t$. It can be verified that the second-order conditions are given by $-1 - h'(\cdot)$ and $-1 + H'(\cdot)$, which are both negative as implied by the MHR and symmetry conditions. The solution to the first-order conditions is thus sufficient, which gives rise to the best response functions $P_A(P_B)$ and $P_B(P_A)$, respectively. Moreover, the slopes of the best response functions satisfy $\frac{dP_A(P_B)}{dP_B} = \frac{h'(\cdot)}{1+h'(\cdot)} < 1$, and $\frac{dP_B(P_A)}{dP_A} = \frac{-H'(\cdot)}{1-H'(\cdot)} < 1$. As a result, the best response curves cross each other exactly once, the intersection of which yields the following unique equilibrium prices and profits: $P_A^* = P_B^* = 2h(1/2)t$, and $\pi_A^* = \pi_B^* = h(1/2)t$. Under a uniform preference distribution, these reduce to t and $t/2$, respectively.

3. Consumer Stockpiling and Market Differentiation

In this section, we investigate the dynamic interaction between consumer stockpiling and market differentiation. We start by showing that preference heterogeneity in differentiated markets creates differential consumer propensity to stockpile. In the current context, this implies that consumers who have relatively higher preferences for a firm are more likely to stockpile that firm's product. We then examine the implications of heterogeneous consumer stockpiling for price competition in the subsequent period. It is shown that as consumers with stronger preferences stockpile and are out of the market in the next period, preference heterogeneity and market differentiation in the future period are driven down. The analysis highlights the role of consumer stockpiling in intensifying future price competition. Lastly, rational stockpiling behaviors are characterized while taking into account the carrying-over impact of consumer inventory. We also demonstrate that consumer response to price can be stronger for storage demand than for brand switching.

3.1 Differential Consumer Stockpiling Propensity

The consumers' first-period stockpiling decisions involve choosing among three alternative options. A consumer located at x can choose to stockpile either product A or B, with expected net payoff $E[\mu_A^S(x)] = \lambda(\nu - xt) - P_{A1}$ or $E[\mu_B^S(x)] = \lambda(\nu - (1-x)t) - P_{B1}$, respectively. Alternatively, the consumer may choose to wait and purchase in the second period with an expected net utility:

$$E[\mu^{NS}(x)] = \max\left\{\lambda\left(\nu - xt - P_{A2}^E\right), \lambda\left(\nu - (1-x)t - P_{B2}^E\right)\right\}, \quad (1)$$

where P^E_{i2} is the expected second-period price for product i, $i = A, B$. It is easy to check that the relative desirability of stockpiling product A over the nonstockpiling option, $E[\mu^S_A(x)] - E[\mu^{NS}(x)]$, is (weakly) decreasing in x. Moreover, it is obvious that the preference for stockpiling product A over B, $E[\mu^S_A(x)] - E[\mu^S_B(x)]$, is decreasing in x. Therefore, if a consumer at $\tilde{x}$ chooses to stockpile product A, all the consumers at $x < \tilde{x}$ would prefer to follow suit. Similarly, it can be shown that if a consumer at $\tilde{x}$ chooses to stockpile product B, then all the consumers at $x > \tilde{x}$ also choose to do so. This implies a positive relationship between product preference and stockpiling propensity, which is consistent with the empirical evidence that loyal consumers are more price sensitive than nonloyal consumers are in quantity decisions (e.g., Krishnamurthi and Raj, 1991). The intuition in the current study is that consumers tend to commit to consuming the product for which they have relatively higher preferences, but indifferent consumers are less willing to make a consumption commitment. This is different from the assumption in Hong et al. (2002) that "shoppers may hold inventories" whereas "captives" may not (which could be potentially possible because of different inventory costs).

This differential stockpiling propensity due to preference heterogeneity allows us to denote the (possibly empty) set of consumers who stockpile product A or B as $[0, x_A]$ or $[1 - x_B, 1]$, respectively. The consumers without any inventory are in the interval $[x_A, 1 - x_B]$, which constitutes the effective market in period two. Note that x_i, $i = A, B$, represents the amount of temporary demand increase in period one for firm i, which is borrowed from period two. The first-period consumer storage for each firm and the effective second-period market are depicted in Figure 1.

3.2 The Dynamic Effects of Consumer Stockpiling

The heterogeneity in consumer stockpiling propensity can introduce strategic implications for the firms' second-period pricing decisions,

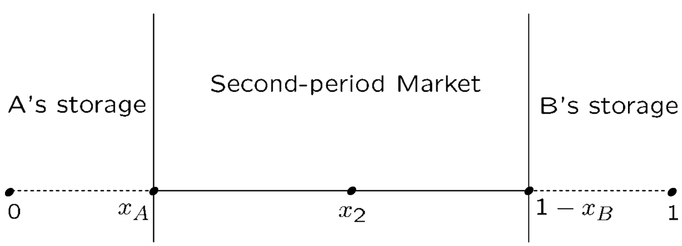

FIGURE 1. DIFFERENTIAL STOCKPILING PROPENSITY AND THE SECOND-PERIOD MARKET

which are absent in undifferentiated markets. When the products are not differentiated, the demand-shifting effect of stockpiling exerts equal influence on the competing firms in the subsequent period. In other words, consumer inventories affect future competition only through rescaling the total market demand faced by all firms. This is the case under either monopolistic (Salop and Stiglitz, 1982) or quantity competition with homogeneous goods (Anton and Das Varma, 2005). In the current context, however, what matters for price competition is not only *how much* demand is shifted, but also *who* stockpiles *which* product. Consumer stockpiling not only reduces the total size of the second-period demand, but also endogenously decreases future market differentiation since the consumers remaining in the market have more homogeneous preferences than those in the first period.

To analyze the dynamic effects of consumer stockpiling, note first that a firm might in equilibrium have zero demand in the second period. As consumer inventory for a product increases, the number of "loyal" consumers for that product in the second period is driven down. In competing for the remaining consumers who have weaker preferences for its product, the firm has to sufficiently undercut its competitor. When price competition becomes sufficiently intense due to decreased market differentiation, the firm may end up with zero demand even if it prices at marginal cost. Formally, let $x_2(P_{A2}, P_{B2}) = \frac{t - P_{A2} + P_{B2}}{2t}$ be the indifferent consumer type in the second period, which depending on the prices charged may fall on the boundary of the market range $[x_A, 1 - x_B]$. The firms' profit functions are then $\pi_{A2} = P_{A2} \cdot [F(\bar{x}(P_{A2}, P_{B2})) - F(x_A)]$ and $\pi_{B2} = P_{B2} \cdot [F(1 - x_B) - F(\bar{x}(P_{A2}, P_{B2}))]$, respectively, where $\bar{x}(P_{A2}, P_{B2}) = \min\{\max\{x_A, x_2(P_{A2}, P_{B2})\}, 1 - x_B\}$. To characterize the equilibrium, let us define

$$X \equiv \big\{(x_a, x_b) \in [0, 1]^2 : 1 - F(x_a) - F(x_b) + f(\dot{x})(1/2 - \dot{x}) > 0, \quad \text{where} \quad \dot{x} = \max\{x_a, x_b\}\big\}.$$

Lemma 1: *If $(x_A, x_B) \in X$, there is a unique interior equilibrium in the second period where both firms have positive sales: $\bar{x}^* = x_2^* \in (x_A, 1 - x_B)$. If $(x_A, x_B) \notin X$, the second-period equilibrium is on the boundary of $[x_A, 1 - x_B]$, where the firm with more first-period storage charges the marginal cost and has zero sales, and the other firm sells to the whole second-period market.*

This lemma suggests that the characteristics of the second-period equilibrium depend on the dispersion between consumer inventories x_A and x_B. If the consumer inventories are sufficiently close (within X), both firms in equilibrium have positive sales. If firm A has sufficiently more consumer inventory than firm B does, for instance, in equilibrium firm A has zero sales even if it charges the marginal cost whereas firm

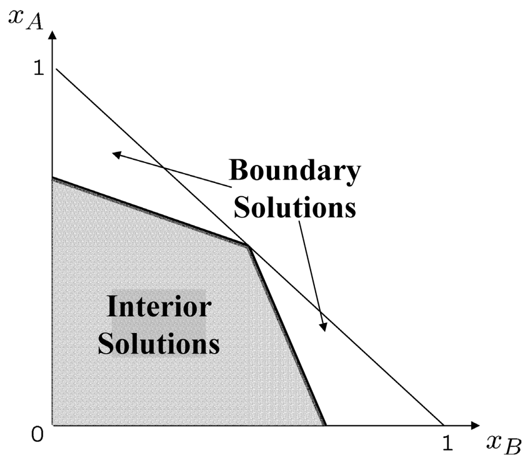

FIGURE 2. THE CHARACTERISTICS OF EQUILIBRIUM SOLUTIONS IN THE SECOND PERIOD

B captures the whole second-period market. The interior-solution set X defines positive values (x_a, x_b) that are relatively close to each other in magnitude. The set X for uniform preferences is represented by the shadowed area in Figure 2.

Proposition 1: *In an interior second-period equilibrium: (i) The firm with less first-period storage has a higher equilibrium price, market share, and profit than the firm with more first-period storage. (ii) As a firm's first-period storage increases, both firms' second-period prices drop. A firm's price drops more with its own storage increase than its rival's price does.*

This proposition highlights the role of consumer inventory in intensifying future price competition. As consumer storage increases, the firms subsequently compete more intensely and the equilibrium prices hence decrease. Intuitively, firm A's pricing decision, for example, reflects the trade-off between competing for the marginal consumer at x_2^* and extracting inframarginal rents from consumers on $[x_A, x_2^*]$. Given the differential storage propensity, the marginal impact of a price reduction on x_2^* is independent of consumer inventory x_A, whereas the extraction loss in inframarginal rents is lower as x_A increases. Therefore, increasing consumer inventory makes a firm more aggressive in pricing, which in turn drives down the rival firm's price. Proposition 1 also suggests that the own-effect of consumer storage is stronger than the cross-effect: a firm's second-period price responds more aggressively to its own consumer inventory than its competitor's does. This is because, for instance, x_A does not influence firm B's inframarginal consumers on $[x_2^*, 1 - x_B]$.

This *competition-intensifying* effect is absent in homogeneous markets (e.g., Salop and Stiglitz, 1982). The opposite result obtained by Hong et al. (2002) that consumer inventory softens price competition, can be attributed to their premise that only "shoppers" can stockpile, which does not occur in equilibrium if equal consumer stockpiling abilities are allowed. Note also that the asymmetric inventory effect on future pricing does not exist in quantity competition where the firms share the same set of inframarginal consumers (Anton and Das Varma, 2005).

Under uniform preferences, one can readily verify that the second-period equilibrium is interior if and only if $4x_A + 2x_B < 3$ and $2x_A + 4x_B < 3$. In an interior equilibrium, the indifferent consumer is at $x_2^*(x_A, x_B) = \frac{1}{2} + \frac{x_A - x_B}{3}$. We also have $P_{A2}^*(x_A, x_B) = (1 - \frac{4x_A + 2x_B}{3})t$ and $P_{B2}^*(x_A, x_B) = (1 - \frac{2x_A + 4x_B}{3})t$. It is then obvious that $P_{A2}^*(x_A, x_B) \geq P_{B2}^*(x_A, x_B)$ if and only if $x_A \leq x_B$. Moreover, $\frac{dP_{A2}^*}{dx_A} = \frac{dP_{B2}^*}{dx_B} = -\frac{4}{3}t$ and $\frac{dP_{A2}^*}{dx_B} = \frac{dP_{B2}^*}{dx_A} = -\frac{2}{3}t$.

3.3 Storage Demand

We now investigate the characteristics of the first-period storage demand x_A and x_B, which result from aggregating rational consumer stockpiling decisions. As discussed in Section 3.1, a consumer's stockpiling decision involves the trade-off between the current benefits of a low price and the expected future payoff of postponing an immediate purchase. In particular, those consumers located close to zero on the Hotelling line compare $E[\mu_A^S(x)]$ with $E[\mu^{NS}(x)]$, and those close to one compare $E[\mu_B^S(x)]$ with $E[\mu^{NS}(x)]$. In evaluating the nonstockpiling option $E[\mu^{NS}(x)]$, consumers take into account the impact of storage demand on the second-period market equilibrium. The following lemma can facilitate characterizing the consumers' evaluation of $E[\mu^{NS}(x)]$:

Lemma 2: *In equilibrium, the firms' second-period profits are strictly positive:* $x_2^* \in (x_A, 1 - x_B)$.

This lemma suggests that a firm never charges a sufficiently low first-period price such that its storage sales are high enough to drive its second-period profit down to zero. In other words, the firms can expect that the second-period equilibrium is interior. This implies that the expected net utility of no inventory for a consumer at the stockpiling margin (x_A or $1 - x_B$) can be simplified: $E[\mu^{NS}(x_A)] = \lambda(\nu - x_A t - P_{A2}^E)$, and $E[\mu^{NS}(1 - x_B)] = \lambda(\nu - x_B t - P_{B2}^E)$.

Let us then examine how the storage demand x_A and x_B are determined. To this end, we can define a firm's second-period equilibrium price as a function of the inventories accumulated: $P^*_{i2}(x_A, x_B)$, $i = A, B$. Note that rational expectations stipulates that $P^E_{i2} = P^*_{i2}(x_A, x_B)$, $i = A, B$. From Proposition 1, we know that P^*_{i2} is smooth, differentiable, and strictly decreasing in both x_A and x_B. Therefore, the inverses of the single-value functions $P^*_{A2}(x_A, 0)$ and $P^*_{B2}(0, x_B)$ exist, which can be defined as $P^{*-1}_{A2}(\cdot)$ and $P^{*-1}_{B2}(\cdot)$, respectively. Let us also denote the (symmetric) second-period equilibrium prices when $x_A = x_B = 0$ as $P^* = P^*_{A2}(0, 0) = P^*_{B2}(0, 0) = 2h(1/2)t$. We can then characterize the firms' storage demand as:

$$(x_A, x_B) = \begin{cases} (0, 0), & \text{if } P_{A1} \geq \lambda P^* \text{ and } P_{B1} \geq \lambda P^*; \\ \left(P^{*-1}_{A2}(P_{A1}/\lambda), 0\right), & \text{if } P_{A1} < \lambda P^* \text{ and} \\ & \quad P_{B1} \geq \lambda P^*_{B2}\left(P^{*-1}_{A2}(P_{A1}/\lambda), 0\right); \\ \left(0, P^{*-1}_{B2}(P_{B1}/\lambda)\right), & \text{if } P_{A1} \geq \lambda P^*_{A2}\left(0, P^{*-1}_{B2}(P_{B1}/\lambda)\right) \text{ and} \\ & \quad P_{B1} < \lambda P^*; \\ (\underline{x}_A, \underline{x}_B), & \text{otherwise,} \end{cases}$$

where $(\underline{x}_A, \underline{x}_B)$ solve simultaneously:

$$P_{A1} = \lambda P^*_{A2}(x_A, x_B) \tag{2}$$

$$P_{B1} = \lambda P^*_{B2}(x_A, x_B). \tag{3}$$

In order to have a better understanding of the storage demand, note first that if the first-period prices are sufficiently high, storage demand would be zero. Let us take firm A as an example. If $P_{A1} \geq \lambda P^*$, we must have $x_A = 0$, where $E[\mu^S_A(0)] = \lambda\nu - P_{A1} \leq \lambda(\nu - P^*) \leq E[\mu^{NS}(0)]$. This is because: (1) If the consumer at $x = 0$ deviated, from Proposition 1 firm A's expected second-period price would fall below P^*, making stockpiling product A even less attractive; (2) If the consumer at $x = 0$ does not favor the stockpiling option, neither do consumers at $x > 0$.

A firm could still have zero storage demand if its price is not sufficiently low but its rival's is. This represents the two asymmetric cases in which only one firm has positive storage demand. For example, if $P_{A1} < \lambda P^*$ and $P_{B1} \geq \lambda P^*_{B2}(P^{*-1}_{A2}(P_{A1}/\lambda), 0)$, then $x_A = P^{*-1}_{A2}(P_{A1}/\lambda)$ and $x_B = 0$. To see this, note that given P_{A1} and $x_B = 0$, the consumer at $x_A = P^{*-1}_{A2}(P_{A1}/\lambda)$ is indifferent between stockpiling product A and not stockpiling: $E[\mu^S_A(x_A)] = \lambda(\nu - x_A t) - P_{A1} = \lambda(\nu - x_A t) - \lambda P^*_{A2}(P^{*-1}_{A2}(P_{A1}/\lambda), 0) = \lambda(\nu - x_A t) - \lambda P^*_{A2}(x_A, 0) = E[\mu^{NS}(x_A)]$. However, the consumer at $1 - x_B = 1$ prefers not to stockpile given

P_{A1}, P_{B1} and $x_A = P_{A2}^{*-1}(P_{A1}/\lambda)$, since $E[\mu_B^S(1 - x_B)] = \lambda\nu - P_{B1} \leq \lambda\nu - \lambda P_{B2}^*(P_{A2}^{*-1}(P_{A1}/\lambda), 0) = \lambda\nu - \lambda P_{B2}^*(x_A, 0) = E[\mu^{NS}(1 - x_B)]$. This result is due to the cross-effect of consumer storage on second-period firm pricing, as identified in Proposition 1: given $x_A > 0$, consumers anticipate that in the second period firm B's price will be decreasing with x_A, which makes stockpiling product B increasingly unattractive.

When both P_{A1} and P_{B1} are sufficiently low, both firms have some consumers who carry their product as inventories. In this case, as the number of consumers who stockpile a product increases, the expected future prices for both products decrease and so does the intertemporal consumer gain from stockpiling either of the products. As long as the gain is positive, the number of stockpiling consumers continues to grow. The gain from stockpiling a product would be brought to zero when its storage demand increases to the point where its discounted future equilibrium price is equal to its current price. It is at this point that the marginal consumer at x_A (or $1 - x_B$) is indifferent between stockpiling product A (or B) and not stockpiling.

Proposition 2: *The storage demand $x_A(P_{A1}, P_{B1})$ and $x_B(P_{A1}, P_{B1})$ are continuous in both P_{A1} and P_{B1}, and have the following features: (i) The own-price sensitivity of storage demand is nonpositive and the cross-price sensitivity is nonnegative. (ii) The own-price sensitivity of storage demand is stronger than the cross-price sensitivity. (iii) The own- and cross-price sensitivity of storage demand decrease with consumer patience.*

This proposition summarizes the characteristics of storage demand. It suggests that lowering prices can allow a firm to induce more consumers to purchase its product in advance for future consumption. This is captured by the *own-price* effect of storage demand, through which the static market sales can be expanded by shifting future demand toward the current period. Moreover, there exists a *cross-price* effect of storage demand, which is absent in alternative competitive contexts (e.g., Salop and Stiglitz, 1982). A firm's storage sales may decrease with its rival's price reduction. Intuitively, for example, if P_{B1} is reduced, x_B would increase due to the *own-price* effect of storage demand. As a result, P_{A2}^* is anticipated to be lower, as suggested by Proposition 1, which in turn drives down the expected payoff of stockpiling product A. Interestingly, this *cross-price* effect is not due to brand switching as in a static setting, that is, the decreased storage sales are not transferred to the rival firm but carried over to the second period. Another interesting result is on the impact of consumer patience on storage demand. On one hand, the discount factor influences the threshold for positive storage to arise: the more patient the consumers are, the smaller the price cut

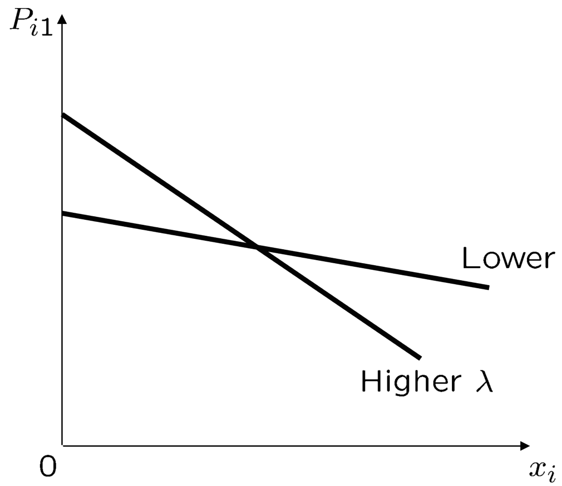

FIGURE 3. CONSUMER DISCOUNTING AND STORAGE DEMAND

needed to induce consumer stockpiling. On the other hand, a higher discount factor increases the present value of the nonstockpiling option, making storage demand less responsive to current prices. Therefore, the overall impact of the consumer discount factor is to rotate the storage demand function, as illustrated in Figure 3.

If consumer preferences are uniform, the storage demand functions can be simplified:

$$(x_A, x_B) = \begin{cases} (0, 0), & \text{if } P_{A1} \geq \lambda t \text{ and } P_{B1} \geq \lambda t; \\ \left(\dfrac{3\lambda t - 3P_{A1}}{4\lambda t}, 0\right), & \text{if } P_{A1} < \lambda t \text{ and } P_{B1} \geq \dfrac{\lambda t + P_{A1}}{2}; \\ \left(0, \dfrac{3\lambda t - 3P_{B1}}{4\lambda t}\right), & \text{if } P_{A1} \geq \dfrac{\lambda t + P_{B1}}{2} \text{ and } P_{B1} < \lambda t; \\ \left(\dfrac{\lambda t - 2P_{A1} + P_{B1}}{2\lambda t}, \right. & \\ \left. \quad \dfrac{\lambda t + P_{A1} - 2P_{B1}}{2\lambda t}\right), & \text{otherwise.} \end{cases}$$

Lemma 3: *Under uniform consumer preferences, the storage demand is more price sensitive than the static-consumption demand.*

This lemma pertains to comparing the impact of price on consumer inventory and on brand switching under uniform consumer preferences. It suggests that storage demand expansion can be more responsive to price than secondary demand shifts. This paper identifies a rationale for greater storage demand sensitivity, which relies on the differential propensity for consumers to stockpile, and consumers being forward-looking. To see this, note that the static-consumption demand is drawn from switching the marginal consumer away from the competitor. The price has to be cut sufficiently to compensate the consumer's efficiency loss resulting from consumption switching. In contrast, the stockpiling consumers in the current model tend to be "loyals" who have higher preferences than the "switchers." They rationally expect that in equilibrium they are going to purchase and consume the same product no matter whether or not they stockpile (recall that Lemma 2 implies interior equilibria in the second period). Therefore, by stockpiling the preferred product, the "loyals" do not lose efficiency in consumption and hence respond more to price changes.

4. Pricing Concerns and the No-Storage Equilibrium

Let us now study the firms' first-period incentives to promote consumer inventories. To facilitate the exposition, the analysis focuses on the uniform preference distribution. We first lay out the potential concerns considered by the competing firms in price setting, given the differential consumer propensity to stockpile and the dynamic effects of consumer inventory on the subsequent price competition. The first-period equilibrium pricing decisions are then investigated.

A forward-looking firm considers how its first-period price influences its present profits, as well as the expected future payoffs. The present sales may consist of both static-consumption and storage demand. A firm caring about present profits makes the trade-off between increasing both static and storage sales at the margin and losing inframarginal rents. In the current context, the marginal consumer for static consumption is at $x_1 = \frac{t - P_{A1} + P_{B1}}{2t}$, and at x_A and $1 - x_B$ for storage sales.

The firms may also take into account the dynamic impacts of consumer inventory on expected future payoffs. First of all, consumer stockpiling generates a *demand-shifting* effect, which decreases a firm's inframarginal consumers in the future period. Note, for example, that the size of firm A's second-period inframarginal consumers (i.e., $x_2^* - x_A$) shrinks with increasing x_A. This is an outcome of consumer stockpiling recognized in Anton and Das Varma (2005). However, in the current study, the *demand-shifting* effect of increasing a firm's storage sales is completely shouldered in the future by the firm itself (in contrast to being shared by all firms, if the products are not differentiated and the firms compete in quantity). Recall that this difference results from the differential consumer propensity to stockpile in differentiated markets.

Moreover, increasing a firm's storage sales may create a *strategic* impact on the rival's second-period price (Proposition 1). For example, as x_A increases, firm B would cut its second-period price which in turn hurts firm A's future profits. Therefore, both the *demand-shifting* and the *strategic* impact of consumer inventory mitigate a firm's incentive to stimulate storage. There might also exist an *indirect strategic* impact on the second-period prices, if the rival has positive storage sales. Recall from Proposition 2 that, if firm A cuts its price, firm B's storage demand decreases in response, which would drive up firm B's future price. As a result, this *indirect strategic* effect encourages the firms to cut prices.

Depending upon the prices charged, some of the pricing concerns identified above may not arise. For example, if a firm's price is sufficiently high, its storage sales are zero and hence all the dynamic effects of consumer inventory are absent. If, on the other hand, the rival's price is sufficiently high, the *indirect strategic* impact would be missing since the rival is not having positive storage sales. Recall also that whether one or both of the firms have positive storage sales is dependent upon how dispersed the charged prices are. Therefore, there might *ex ante* exist three pricing scenarios where none, one, or both of the firms induce positive inventories. It turns out that, however, only the no-storage scenario emerges as an equilibrium.

Proposition 3: *Under uniform consumer preferences and with a common discount factor, there exists a unique equilibrium, in which there is no storage:* $P_{A1}^* = P_{B1}^* = t$, *and* $\pi_A^* = \pi_B^* = \frac{(1+\lambda)t}{2}$.

This proposition establishes an important result of the paper, that product storability may not necessarily lead to equilibrium consumer storage in differentiated markets. In other words, the possibility of shifting demand intertemporally *per se* may not be a sufficient condition for the competing firms to actually do so. Despite the possibility of the products being stockpiled, in equilibrium the firms charge the same prices as in the benchmark case and no consumer stockpiles any product. This no-storage result highlights the endogenous impact of consumer stockpiling on market differentiation and price competition in the following period. This proposition demonstrates that the *demand-shifting* and the direct *strategic* effects of consumer inventory can be so strong as to dominate the *indirect strategic* effect. In this particular case, the net dynamic inventory effect hence completely undermines the firms' incentives to cut prices.

In particular, when the common discount factor is large, the firms are patient and the consideration weight for the dynamic effects of consumer inventory is high among the pricing concerns. When the discount factor is small, the firms care more about current than about future payoffs. But then the first-period price competition becomes more intense, since the price sensitivities of storage demand decrease with the consumer discount factor (Proposition 2). Decreasing the consumer discount factor has an additional *threshold* effect: the less patient the consumers are, the deeper the price cut is needed to induce any positive storage sales (see Figure 3). This deeper price cut lowers the firms' inframarginal rents on the static-consumption sales, which is hence a force against lowering prices. Overall, if the consumers and the firms are equally patient, the firms in equilibrium never offer low enough prices to induce consumer stockpiling.

This no-storage equilibrium is obtained for any $t > 0$. This implies that as long as there exists market differentiation, albeit small in magnitude, the strategic effects of consumer inventory necessarily arise and may completely shut down the firms' incentives to induce consumer stockpiling. As investigated below, if the firms are not too forward-looking in comparison to the consumers, there may be a possibility of equilibrium stockpiling, as the firms care less about the future competition-intensifying effect of stockpiling.

One may wonder whether this result is due to not allowing for the possibility of firms offering quantity discounts. The next result shows that the result above is robust to this generalization.

Lemma 4: *Suppose that the firms can offer quantity discounts: P'_{i1} for the first unit and P_{i1} for the second unit, $i = A, B$. Under uniform consumer preferences and with a common discount factor, there exists a unique equilibrium, in which there is no storage: $P'^{*}_{A1} = P^{*}_{A1} = P'^{*}_{B1} = P^{*}_{B1} = t$, and $\pi^{*}_{A} = \pi^{*}_{B} = \frac{(1+\lambda)t}{2}$.*

This lemma further reinforces the importance of considering the dynamic effects of consumer inventory, by showing that the no-storage result still holds even when we allow the firms to price differently the units consumed in the current period and the units that are stored. This result reinforces the intuition presented above. Firms do not want to offer price cuts that generate consumer stockpiling, because such stockpiling will induce more intense price competition in the future.

5. Extensions and Storage Equilibria

The previous section shows an example of a no-storage equilibrium. The driving force for this result is the endogenous impact of consumer inventory on subsequent market differentiation and price competition. The difference between this result and that obtained from previous research can be attributed to the different market structure investigated (differentiated versus homogeneous markets), the different form of competition (price versus quantity competition), and the resulting different perspectives with respect to consumer heterogeneity in stockpiling. Nevertheless, given the existence in some markets of consumer stockpiling, it would be insightful to examine within the current framework what factors may lead competing firms to charge inventory-inducing prices. By doing this, we may be able to better understand the varying significance of the "primary demand" effects of price.[4]

Two alternative extensions to the base model are therefore made, which can potentially mitigate the firms' concerns about the dynamic impact of consumer inventory. First, the dynamic impact might be less significant if the extent to which the subsequent market differentiation is influenced by consumer inventory is weakened. We investigate whether changing consumer preferences can undermine the dynamic concerns. Alternatively, the weight assigned to the dynamic concerns might be lower if firms care less about future payoffs. In particular, we also pursue whether relaxing the assumption of a common discount factor may allow the firms to harvest the present value of demand expansion while caring less about its negative impact on subsequent profits. Throughout this section, the focus is on symmetric storage equilibria.

5.1 Changing Consumer Preferences

In some differentiated markets, consumer preferences might not be constant over time. In the base model, consumer preferences in the second period are assumed to remain the same as in the previous period. At the opposite extreme, a consumer's preference can be independently distributed over time.[5] To capture the impact of changing consumer preferences, let us consider a simple modification of the base model. Specifically, suppose that each of these two extreme scenarios takes place with a probability of one half.

Following the independent-preference scenario, the second-period market is composed of a continuum of consumers uniformly located at [0, 1]. Given consumer inventories in the first period, the

4. The term "primary demand" has commonly been used to capture both pure stockpiling and category expansion.

5. Consumers may have limited memory (Dow, 1991; Chen et al., 2005), or forget about what they learned about their preferences from previous experiences. Equilibrium storage could also possibly result if firms have different information about the consumer preferences (Kuksov, 2006).

mass of the second-period market is $1 - x_A - x_B$. Therefore, under independent preferences consumer inventories just scale down the subsequent market size without changing market differentiation, a scenario similar to previous studies (e.g., Salop and Stiglitz, 1982). As a result, the (symmetric) second-period equilibrium prices are equal to t. In contrast, in the fixed-preference scenario, recall from Section 3.2, that the second-period equilibrium prices are given by $P^*_{A2} = (1 - \frac{4x_A + 2x_B}{3})t$ and $P^*_{B2} = (1 - \frac{2x_A + 4x_B}{3})t$, respectively.

In anticipation of the potential preference change, the expected net utilities of the consumer at x_A who is indifferent between stockpiling A and not are: $E[\mu^S_A(x_A)] = \frac{1}{2}\lambda(\nu - x_A t) + \frac{1}{2}\lambda(\nu - \frac{1}{2}t) - P_{A1}$, and $E[\mu^{NS}(x_A)] = \frac{1}{2}\lambda(\nu - x_A t - (1 - \frac{4x_A + 2x_B}{3})t) + \frac{1}{2}\lambda(\nu - \frac{1}{4}t - t)$. Similarly, the expected net utilities of the consumer at $1 - x_B$ who is indifferent between stockpiling B and not are given by: $E[\mu^S_B(1 - x_B)] = \frac{1}{2}\lambda(\nu - x_B t) + \frac{1}{2}\lambda(\nu - \frac{1}{2}t) - P_{B1}$, and $E[\mu^{NS}(1 - x_B)] = \frac{1}{2}\lambda(\nu - x_B t - (1 - \frac{2x_A + 4x_B}{3})t) + \frac{1}{2}\lambda(\nu - \frac{1}{4}t - t)$. Conditional on positive storage sales for both firms, we can then write the storage demand functions as: $x_A = \frac{7\lambda t - 16P_{A1} + 8P_{B1}}{8\lambda t}$ and $x_B = \frac{7\lambda t + 8P_{A1} - 16P_{B1}}{8\lambda t}$.[6] Notice that now the storage demand is more price sensitive than when preferences are fixed, because consumers anticipate that with probability 1/2 they will pay the high price t if remaining in the second-period market.

The firms also take into account the impact of potential consumer preference change on their pricing decisions. For example, firm A solves:

$$\underset{0<P_{A1}<\lambda t}{\text{Max}}\ \pi_A = P_{A1} \cdot (x_1 + x_A) + \frac{1}{2}\lambda P^*_{A2}\left(x^*_2 - x_A\right) + \frac{1}{2}\lambda t \cdot \frac{(1 - x_A - x_B)}{2}. \tag{4}$$

Compared to the base model, allowing for changing preferences introduces four changes to the firms' pricing concerns identified in Section 4: (1) As noted above, consumer inventories are more sensitive to price than when preferences are fixed; (2) The *demand-shifting* effect is now with probability 1/2 shared with the rival; (3) The direct *strategic* effect of consumer inventory on the rival's second-period price is discounted by one half; and (4) The second-period market size can be enhanced through the *indirect strategic* effect by decreasing the rival's storage sales.

Proposition 4: *Suppose that consumer preferences are either fixed or independent over time with equal probability. With a common discount factor: (i) There exists a (symmetric) equilibrium with positive consumer storage if the*

6. It is easy to check that $x_A = \frac{3(7\lambda t - 8P_{A1})}{16\lambda t}$ and $x_B = 0$, if $P_{A1} < \frac{7\lambda t}{8}$ and $P_{B1} \geq \frac{7\lambda t + 8P_{A1}}{16}$. Similarly, $x_A = 0$ and $x_B = \frac{3(7\lambda t - 8P_{B1})}{16\lambda t}$, if $P_{A1} \geq \frac{7\lambda t + 8P_{B1}}{16}$ and $P_{B1} < \frac{7\lambda t}{8}$.

discount factor is sufficiently large: $P_{A1}^* = P_{B1}^* = \frac{7\lambda t}{4(2+\lambda)}$, $x_A^* = x_B^* = \frac{7\lambda}{8(2+\lambda)}$, *and* $\pi_A^* = \pi_B^* = \frac{3(40-\lambda)\lambda t}{64(2+\lambda)}$; *(ii) The equilibrium profits with consumer storage and changing preferences are lower than those in the no-storage equilibrium with fixed preferences.*

This result demonstrates that changing preferences can permit greater storage demand expansion while counteracting the competition-intensifying impact exerted by consumer stockpiling. When the discount factor is sufficiently high, the price sensitivity of storage demand decreases and hence the rival's strategic reaction in response to price cuts becomes less intense. As a result, low prices are offered (in both periods) and consumer inventories emerge in equilibrium. However, the profits under the storage equilibrium are lower than in the no-storage case with fixed preferences. Changing preferences intensify first-period price competition because of the softer future competition.

5.2 Different Discount Factors

So far, we have assumed that the consumers and the firms share the same discount factor λ. Here we consider how different discount factors might influence the various forces in affecting the firms' pricing decisions. Let us assume that the consumers and the firms value future payoffs differently, that is, $0 \leq \lambda_C \leq 1$ and $0 \leq \lambda_F \leq 1$, respectively. The consumer discount factor λ_C determines the responsiveness of storage demand to price. It has been shown in Proposition 2 that the price sensitivity of storage demand decreases with the consumer discount factor. Therefore one can expect that the higher the consumer discount factor, all else being equal, the more likely inventories are induced because the rival's retaliation in response to price cuts is less intense. The firm discount factor λ_F affects the firms' incentive to allocate sales across time periods. The less patient the firms are, all else being equal, the more they are inclined to sell products immediately. A lower firm discount factor hence can enhance the firms' motivation to shift future demand into the current period, by discounting the expected negative impacts of consumer stockpiling on future payoffs.

Proposition 5: *Under uniform consumer preferences, there exists a (symmetric) equilibrium with positive storage if the consumer discount factor is sufficiently large relative to the firm discount factor, that is, if* $\lambda_C \geq \lambda_C^+$ *where* λ_C^+ *is a function of* λ_F *as characterized in the Appendix. In the storage equilibrium,* $P_{A1}^* = P_{B1}^* = \frac{2\lambda_C^2 t}{\lambda_C^2 + 3\lambda_C - 2\lambda_F}$, *and* $\pi_A^* = \pi_B^* = \frac{2\lambda_C^2(\lambda_C^2 + 2\lambda_C - \lambda_F)t}{(\lambda_C^2 + 3\lambda_C - 2\lambda_F)^2}$.

The set of parameters supporting the positive-storage equilibrium are indicated by CDE in Figure 4. Note that even when the firms

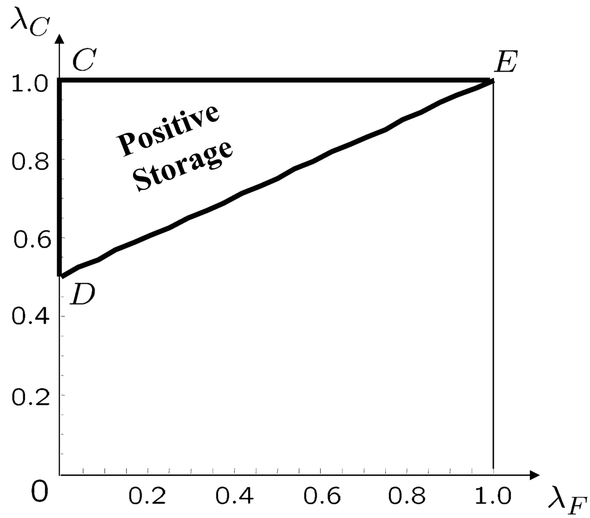

FIGURE 4. DIFFERENTIAL DISCOUNT FACTORS AND THE POSITIVE-STORAGE EQUILIBRIUM

care only about present payoffs (λ_F converges to zero), the consumer discount factor still needs to be large enough to sustain the storage equilibrium. This is because lower consumer patience exacerbates the *threshold* effect (see Figure 3), making deviations to higher prices more attractive, because a larger price cut is needed to induce *any* positive storage. As the firms become more concerned about future profits, the dynamic impact of consumer inventory receives higher consideration weight. As a result, to support the storage equilibrium, consumers need to be increasingly patient to soften the first-period price competition.

Let us then investigate the comparative statics of the different discount factors.

Proposition 6: *Under uniform consumer preferences and the (symmetric) storage equilibrium: (i) The first-period equilibrium prices increase with λ_F, and increase with λ_C if λ_F is small and decrease otherwise. (ii) The expected profits increase with λ_F, and increase with λ_C if λ_F is small and decrease otherwise.*

The first-period equilibrium prices reflect the firms' strategic concerns in stimulating consumer inventories. As the firms become more patient, the negative dynamic effects of consumer stockpiling are assigned higher consideration weight. The firms hence strategically limit the amount of intertemporal demand shifts by raising first-period prices. As a result, the first-period equilibrium prices increase with firm patience. The impact of consumer patience on the first-period equilibrium prices is influenced by the relative importance of the firms' immediate versus future payoffs. When the firms are myopic, their strategic interaction hinges mainly on the responsiveness of storage demand. Then, the first-period equilibrium prices increase with consumer patience, as storage demand becomes less price sensitive. When the firms become sufficiently forward-looking, they desire to allocate sales to the more profitable period. Note that the current profitability, relative to that of the future, increases with consumer patience, because the equilibrium profit margins satisfy $P_{i1}^* = \lambda_C P_{i2}^*$ (equations (2) and (3)). Therefore, the first-period equilibrium prices may go down in order to drive up more storage sales, as consumers become sufficiently patient.

Proposition 6 also suggests that being more forward-looking can improve the competing firms' equilibrium profits. This is because the more firms care about future strategic competition, the less intensely they compete in the first period for consumer inventories. Actually, it turns out that higher firm patience increases not only the first-period equilibrium profits, but also the expected second-period profits. As the first-period equilibrium prices go up with increasing firm patience, less consumer inventories are induced, which in turn softens subsequent competition and enhances future profitability.

Interestingly, it can be shown that the first-period equilibrium profits are higher, whereas the second-period profits are lower, as consumer patience increases. This is again caused by consumer patience increasing the equilibrium current profit margin relative to that of the future period. Overall, the impact of consumer patience on total expected profits depends on the relative importance of current versus future profitability. As a result, expected overall profits increase with consumer patience when firms are myopic, but decrease when the firms' future payoffs become more important.

6. Concluding Remarks

6.1 Discussion

We have established that no consumer storage is the unique equilibrium in a standard model of horizontal differentiation. The driving force is the strategic impact of consumer inventory on subsequent price competition, which is derived from differential consumer stockpiling propensity. We highlight this strategic effect in accounting for the difference between this no-storage result and those obtained from previous studies. This strategic effect is necessarily absent in models of monopolistic competition (Salop and Stiglitz, 1982). If we impose the assumption that consumers with stronger preferences are less likely to stockpile (Hong et al., 2002), we can obtain the reverse result that consumer inventory softens future price competition. Similarly, if we look at quantity competition with homogeneous goods (Anton and Das Varma, 2005), the level of market differentiation remains unchanged across periods and we can expect consumer stockpiling to arise in equilibrium.

Note that this strategic effect is different from the storage-cost explanation for the absence of equilibrium low prices to induce storage (Salop and Stiglitz, 1982). It is also important to stress the difference between product differentiation and monopoly power, which may both sustain the no-storage equilibrium.[7] First, in our model the no-storage result holds for any positive level of product differentiation (i.e., any $t > 0$). We also extend the model to allow for asymmetric/arbitrary product locations under uniform consumer preferences, that is, product A at a and B at $1 - b$, where $0 \leq a < 1 - b \leq 1$. In this case the no-storage outcome also emerges as the unique equilibrium as long as the product locations are sufficiently differentiated.[8] Similarly, if the distribution of consumer preferences is asymmetric (but not too asymmetric) the no-storage outcome would remain as the market equilibrium. Second, total consumption is a function of price in the monopoly case in Anton and Das Varma (2005), although not so in this competition model. As a result, in inducing consumer inventories, a monopoly firm cares about the total market demand, whereas the competing firms in our model care about the composition of the storage demand as well, that is, the differential consumer stockpiling propensity.

Nevertheless, this does not imply that consumer stockpiling is necessarily absent in any differentiated markets. In the light of the equilibrium relationship between no storage and the strategic impact of consumer inventory that we establish in this paper, one may expect that consumer stockpiling could be recovered in equilibrium by economic forces that remove the positive link between consumer preference and stockpiling propensity and/or sufficiently weaken this strategic impact. One of such forces is changing consumer preferences. If consumers change preferences in the second period of the model (e.g., change of residence, new job, getting married, resolution of uncertainty), then the different preference between the no-stockpiling and stockpiling consumers, existing in the first period, may not necessarily remain in the second period. Alternatively, if firms care less about future payoffs than consumers do, we might obtain both lower firm decision

7. It seems that intertemporally growing demand is a necessary condition under monopoly for consumer storage to arise in equilibrium. Equilibrium consumer stockpiling is absent under monopoly when demand does not grow over time (Anton and Das Varma, 2005), whereas could arise when demand and prices increase intertemporally (Dudine et al., 2006). This is related to the result in this paper in the sense that intertemporally increasing prices, which are driven either by demand growth or by softened future differentiation, are critical for equilibrium consumer storage.

8. The analysis is available from the authors upon request.

weight on the strategic effect and smaller price sensitivity of storage demand.[9] Therefore, even within the current framework, consumer stockpiling could be sustained in equilibrium if consumer preferences are sufficiently unstable and/or the firm discount factor is sufficiently lower than that of the consumers.

6.2 Summary and Future Research

This paper examines competitive pricing strategies in differentiated markets when consumers may stockpile and purchase in advance for future consumption. We investigate the strategic incentives for intertemporal demand shifts and revisit the necessity of equilibrium consumer storage for storable products. We conclude that product storability *per se* is not sufficient for consumer stockpiling to be induced in equilibrium. The analysis establishes the role of the *competition-intensifying* effect of consumer inventory in inducing forward-looking firms not to offer price promotions. The rationale behind this result highlights the impact of differential consumer stockpiling propensity, which is not considered in previous studies focusing on homogeneous goods.

In the current model, consumer commitments to consume the stockpiled product do not result in efficiency loss, since there is no learning involved. If instead consumers could learn about product preferences through usage (e.g., Villas-Boas, 2004; Bergemann and Valimaki, 2006), stockpiling may imply not using the information gained from their product experience. Consumers' incentives to stockpile might then be impeded. On the other hand, the learned preferences may mitigate future price competition and facilitate inventory-inducing low prices. It is therefore interesting to investigate the impact of usage learning on consumer storage and equilibrium pricing strategies. Another interesting issue to look at is that stockpiling decisions may be intertwined with preferences for consumption flexibility (Guo, 2006), where consumers may have an incentive to stockpile a less preferred product. Finally, it would be interesting to understand the interaction between consumer stockpiling and the different actions of players in a distribution channel.

Appendix

Proof of Lemma 1. Let us first establish that a firm $i = A, B$ in equilibrium has positive second-period sales if $x_i < x_j$, $j = B, A$. Without

9. Note that the different influences exerted by the firm versus consumer discounting are on the relative importance of the strategic concerns, which is different from the inventory-cost shifting explanation in a monopoly model.

loss of generality, let us look at firm A. Note that $x_A < x_B$ implies $x_A < 1/2$. For any given $P_{B2} \geq 0$, if firm A charges $P_{A2} = P_{B2} + \epsilon$ with $0 < \epsilon < (1 - 2x_A)t$, its profit is $\hat{\pi}_{A2} = P_{A2} \cdot [F(\hat{x}) - F(x_A)]$, where $\hat{x} = \min\{\frac{t-\epsilon}{2t}, 1 - x_B\}$. Note that $\hat{\pi}_{A2} > 0$ since $\hat{x} > x_A$ by construction.

To solve for the equilibrium, consider two alternative situations. Let us first look at the case $x_A < x_B$. From the above, we know firm A's profit function is reduced to $\pi_{A2} = P_{A2} \cdot [F(\bar{x}(P_{A2}, P_{B2})) - F(x_A)]$, where $\bar{x}(P_{A2}, P_{B2}) = \min\{x_2(P_{A2}, P_{B2}), 1 - x_B\}$. Equivalently, firm A's problem can be rewritten as

$$\operatorname*{Max}_{P_{A2}} P_{A2} \cdot [F(x_2(P_{A2}, P_{B2})) - F(x_A)], \text{ subject to } P_{A2} \geq P_{B2} + 2x_B t - t.$$

The derivative of the objective function with respect to P_{A2}, $[F(x_2(P_{A2}, P_{B2})) - F(x_A)] - \frac{1}{2t} P_{A2} \cdot f(x_2(P_{A2}, P_{B2}))$, while evaluated at $P_{A2} = P_{B2} + 2x_B t - t$, is positive if and only if $2t[F(1 - x_B) - F(x_A)] - (P_{B2} + 2x_B t - t) f(1 - x_B) > 0$. This yields firm A's best response function:

$$P_{A2} = \begin{cases} \dfrac{2t \cdot [F(x_2(P_{A2}, P_{B2})) - F(x_A)]}{f(x_2(P_{A2}, P_{B2}))}, & 2t[F(1 - x_B) - F(x_A)] \\ & \quad - (P_{B2} + 2x_B t - t) f(1 - x_B) > 0; \\ P_{B2} + 2x_B t - t, & \text{otherwise.} \end{cases} \tag{A1}$$

Similarly, firm B's problem can be reduced to $\operatorname{Max}_{P_{B2}} P_{B2} \cdot [F(1 - x_B) - F(x_2(P_{A2}, P_{B2}))]$, subject to $P_{B2} \leq P_{A2} - 2x_B t + t$. The best response function is then given by

$$P_{B2} = \begin{cases} \dfrac{2t \cdot [F(1 - x_B) - F(x_2(P_{A2}, P_{B2}))]}{f(x_2(P_{A2}, P_{B2}))}, & P_{A2} - 2x_B t + t > 0; \\ 0, & \text{otherwise.} \end{cases} \tag{A2}$$

Combining (A1) and (A2), if $[F(1 - x_B) - F(x_A)] + f(1 - x_B)(1/2 - x_B) > 0$, then an interior solution is obtained where both firms have positive sales. Otherwise, the equilibrium is at the boundary $1 - x_B$, where $P_{A2}^* = 2x_B t - t$ and $P_{B2}^* = 0$.

In the alternative case $x_B < x_A$, the cut-off consumer type can be reduced: $\bar{x}(P_{A2}, P_{B2}) = \max\{x_A, x_2(P_{A2}, P_{B2})\}$. Similarly, an interior solution can be obtained if $[F(1 - x_B) - F(x_A)] + f(x_A)(1/2 - x_A) > 0$. If otherwise, the boundary solution is at $P_{A2}^* = 0$, and $P_{B2}^* = 2x_A t - t$.

Note that $F(1 - x_B) = 1 - F(x_B)$, and $f(1 - x_B) = f(x_B)$. So if $1 - F(x_A) - F(x_B) + f(\dot{x})(1/2 - \dot{x}) > 0$, where $\dot{x} = \max\{x_A, x_B\}$, an interior equilibrium can be obtained by solving simultaneously

$$P_{A2} = \frac{2t \cdot [F(x_2(P_{A2}, P_{B2})) - F(x_A)]}{f(x_2(P_{A2}, P_{B2}))},$$

$$P_{B2} = \frac{2t \cdot [F(1 - x_B) - F(x_2(P_{A2}, P_{B2}))]}{f(x_2(P_{A2}, P_{B2}))}. \tag{A3}$$

Denoting $x_2 = \frac{t - P_{A2} + P_{B2}}{2t}$, from (A3) one can obtain $G(x_2) \equiv \frac{[1 + F(x_A) - F(x_B) - 2F(x_2)]}{f(x_2)} + \frac{1}{2} - x_2 = 0$. It can be shown that $G'(x_2) = -3 - \frac{[1 + F(x_A) - F(x_B) - 2F(x_2)] f'(x_2)}{f(x_2)^2} < 0$ using the MHR. Moreover, evaluating $G(\cdot)$ at x_A and $1 - x_B$, we have $G(x_A) = \frac{1 - F(x_A) - F(x_B) + f(x_A)(1/2 - x_A)}{f(x_A)} > 0$ and $G(1 - x_B) = -\frac{1 - F(x_A) - F(x_B) + f(x_B)(1/2 - x_B)}{f(1 - x_B)} < 0$, respectively, as indicated by the interior-solution conditions. There must then exist a unique x_2^* solving $G(x_2) = 0$, where $x_A < x_2^* < 1 - x_B$. □

Proof of Proposition 1. Recall from the proof of Lemma 1 that in an interior equilibrium $G'(\cdot) < 0$, $G(x_A) > 0$, and $G(1 - x_B) < 0$. Moreover, we have $G(1/2) = \frac{F(x_A) - F(x_B)}{f(1/2)}$. Therefore, $x_2^* \leq 1/2$ if $x_A \leq x_B$. Note also that $x_2^* = \frac{t - P_{A2}^* + P_{B2}^*}{2t} = 1/2 + \frac{P_{B2}^* - P_{A2}^*}{2t}$. Hence, we have $P_{A2}^* \geq P_{B2}^*$ if and only if $x_A \leq x_B$. From (A3), the ratio of firm A's market share to that of firm B is given by $\frac{F(x_2^*) - F(x_A)}{F(1-x_B) - F(x_2^*)} = \frac{P_{A2}^*}{P_{B2}^*}$. This shows that a firm has a larger equilibrium demand if only its equilibrium price is higher than its rival firm's. The first part of Proposition 1 follows.

To prove the second part, let us totally differentiate the first-order conditions given in (A3) with respect to x_A: $I_{1A}\frac{dP_{A2}^*}{dx_A} + I_{1B}\frac{dP_{B2}^*}{dx_A} - \frac{2t \cdot f(x_A)}{f(x_2^*)} = 0$, $I_{2A}\frac{dP_{A2}^*}{dx_A} + I_{2B}\frac{dP_{B2}^*}{dx_A} = 0$, where $I_{1A} = \{-2 + \frac{[F(x_2^*) - F(x_A)] f'(x_2^*)}{f(x_2^*)^2}\}$, $I_{1B} = \{1 - \frac{[F(x_2^*) - F(x_A)] f'(x_2^*)}{f(x_2^*)^2}\}$, $I_{2A} = \{1 - \frac{[F(1-x_B) - F(x_2^*)] f'(x_2^*)}{f(x_2^*)^2}\}$, and $I_{2B} = \{-2 + \frac{[F(1-x_B) - F(x_2^*)] f'(x_2^*)}{f(x_2^*)^2}\}$. Note that by MHR, we have $I_{1A} < 0, I_{1B} > 0, I_{2A} > 0, I_{2B} < 0$, $|I_{1A}| > |I_{1B}|$, and $|I_{2A}| < |I_{2B}|$. We therefore have

$$\frac{dP_{A2}^*}{dx_A} = \frac{\frac{2t \cdot f(x_A)}{f(x_2^*)} I_{2B}}{I_{1A} \cdot I_{2B} - I_{1B} \cdot I_{2A}} < 0,$$

$$\frac{dP_{B2}^*}{dx_A} = \frac{\frac{2t \cdot f(x_A)}{f(x_2^*)} I_{2A}}{I_{1B} \cdot I_{2A} - I_{1A} \cdot I_{2B}} < 0, \quad \text{and} \quad \left|\frac{dP_{A2}^*}{dx_A}\right| > \left|\frac{dP_{B2}^*}{dx_A}\right|. \qquad \square$$

Proof of Lemma 2. Let us proceed by contradiction. Without loss of generality, suppose that firm A has zero sales in the second period: $x_2^* = x_A$. From Lemma 1, we must have $P_{A2}^* = 0$ and $P_{B2}^* = 2x_A t - t$. Consequently, consumers in the first period rationally believe that $P_{A2}^E = 0$, $P_{B2}^E = 2x_A t - t$, and all consumers at $[x_A, 1 - x_B]$ will buy product B in the second period, if consumers at $[0, x_A]$ and $[1 - x_B, 1]$ stockpile product A and B, respectively. Now consider the marginal consumer at x_A. By construction, this consumer is indifferent between stockpiling product A and nothing: $E[\mu_A^S(x_A)] = \lambda(\nu - x_A t) - P_{A1} = E[\mu^{NS}(x_A)] = \max\{\lambda(\nu - x_A t - P_{A2}^E), \lambda(\nu - (1 - x_A)t - P_{B2}^E)\} = \lambda(\nu - x_A t)$. This leads to $P_{A1} = 0$, implying that firm A makes no positive profits in either period. However, firm A could always make profitable deviations, for example, charging a higher P_{A1} with zero storage and having positive sales in the second period. This proves that in equilibrium it can never be the case that firm A has zero sales in the second period. The case for firm B is similar. □

Proof of Proposition 2. To prove that the storage demand $x_A(P_{A1}, P_{B1})$ and $x_B(P_{A1}, P_{B1})$ are continuous functions, note first that the four demand regions are mutually exclusive. The continuity in the first three regions can be obtained from the monotonicity of $P_{i2}^*(x_A, x_B)$ in both x_A and x_B. For the fourth region, let us proceed by contradiction. Suppose that there exist $(\underline{x}'_A, \underline{x}'_B) \neq (\underline{x}_A, \underline{x}_B)$ that also solve equations (2) and (3). Given the monotonicity of $P_{i2}^*(x_A, x_B)$, without loss of generality we can consider only the case that $\underline{x}'_A > \underline{x}_A$ and $\underline{x}'_B < \underline{x}_B$. By construction, we have $P_{A2}^*(\underline{x}_A, \underline{x}_B) = P_{A2}^*(\underline{x}'_A, \underline{x}'_B)$ and $P_{B2}^*(\underline{x}_A, \underline{x}_B) = P_{B2}^*(\underline{x}'_A, \underline{x}'_B)$. From Proposition 1, $|\frac{dP_{A2}^*}{dx_A}| > |\frac{dP_{B2}^*}{dx_A}|$ and $|\frac{dP_{B2}^*}{dx_B}| > |\frac{dP_{A2}^*}{dx_B}|$. It follows that: $0 = P_{A2}^*(\underline{x}'_A, \underline{x}'_B) - P_{A2}^*(\underline{x}_A, \underline{x}_B) = [P_{A2}^*(\underline{x}'_A, \underline{x}'_B) - P_{A2}^*(\underline{x}'_A, \underline{x}_B)] + [P_{A2}^*(\underline{x}'_A, \underline{x}_B) - P_{A2}^*(\underline{x}_A, \underline{x}_B)] < [P_{B2}^*(\underline{x}'_A, \underline{x}'_B) - P_{B2}^*(\underline{x}'_A, \underline{x}_B)] + [P_{B2}^*(\underline{x}'_A, \underline{x}_B) - P_{B2}^*(\underline{x}_A, \underline{x}_B)] = 0$, which cannot be true.

The proposed features of price sensitivities of storage demand in the first three regions are obvious from the monotonicity of $P_{i2}^*(x_A, x_B)$. In the fourth region, without loss of generality let us totally differentiate equations (2) and (3) with respect to P_{A1}:

$$\lambda \frac{dP_{A2}^*}{dx_A} \frac{dx_A}{dP_{A1}} + \lambda \frac{dP_{A2}^*}{dx_B} \frac{dx_B}{dP_{A1}} - 1 = 0, \qquad \lambda \frac{dP_{B2}^*}{dx_A} \frac{dx_A}{dP_{A1}} + \lambda \frac{dP_{B2}^*}{dx_B} \frac{dx_B}{dP_{A1}} = 0.$$

Note that Proposition 1 implies $\frac{dP_{A2}^*}{dx_A} < \frac{dP_{A2}^*}{dx_B} < 0$, and $\frac{dP_{B2}^*}{dx_B} < \frac{dP_{B2}^*}{dx_A} < 0$. Solving the above equations simultaneously, we therefore prove the proposition by noting that

$$\frac{dx_A}{dP_{A1}} = \frac{\dfrac{1}{\lambda}\cdot\dfrac{dP^*_{B2}}{dx_B}}{\dfrac{dP^*_{A2}}{dx_A}\cdot\dfrac{dP^*_{B2}}{dx_B} - \dfrac{dP^*_{A2}}{dx_B}\cdot\dfrac{dP^*_{B2}}{dx_A}} < 0,$$

$$\frac{dx_B}{dP_{A1}} = \frac{\dfrac{1}{\lambda}\cdot\dfrac{dP^*_{B2}}{dx_A}}{\dfrac{dP^*_{A2}}{dx_B}\cdot\dfrac{dP^*_{B2}}{dx_A} - \dfrac{dP^*_{A2}}{dx_A}\cdot\dfrac{dP^*_{B2}}{dx_B}} > 0, \quad \left|\frac{dx_A}{dP_{A1}}\right| > \left|\frac{dx_B}{dP_{A1}}\right|, \quad \text{and}$$

$$\frac{d^2x_A}{dP_{A1}d\lambda} = \frac{-\dfrac{1}{\lambda^2}\cdot\dfrac{dP^*_{B2}}{dx_B}}{\dfrac{dP^*_{A2}}{dx_A}\cdot\dfrac{dP^*_{B2}}{dx_B} - \dfrac{dP^*_{A2}}{dx_B}\cdot\dfrac{dP^*_{B2}}{dx_A}} > 0,$$

$$\frac{d^2x_B}{dP_{A1}d\lambda} = \frac{-\dfrac{1}{\lambda^2}\cdot\dfrac{dP^*_{B2}}{dx_A}}{\dfrac{dP^*_{A2}}{dx_B}\cdot\dfrac{dP^*_{B2}}{dx_A} - \dfrac{dP^*_{A2}}{dx_A}\cdot\dfrac{dP^*_{B2}}{dx_B}} < 0. \qquad \square$$

Proof of Proposition 3. To prove the existence, note first that no storage is induced for both firms if and only if $P_{A1} \geq \lambda t$ and $P_{B1} \geq \lambda t$. In this case, the pricing competition is reduced to the benchmark case and the unique (local) equilibrium is $P_{A1} = P_{B1} = t$. To show that this is indeed a global equilibrium, without loss of generality let us check whether firm A wants to deviate given $P_{B1} = t$. Conditional on $P_{A1} < \lambda t$, firm A can deviate and solve

$$\underset{0<P_{A1}<\lambda t}{\text{Max}}\ \pi_A = P_{A1}\cdot(x_1 + x_A) + \lambda P^*_{A2}(x_A, x_B)\left[x^*_2(x_A, x_B) - x_A\right], \qquad \text{(A4)}$$

where $x_1 = \frac{t-P_{A1}-P_{B1}}{2t}$, $x_A = \frac{3\lambda t - 3P_{A1}}{4\lambda t}$, $x_B = 0$, $P^*_{A2}(x_A, x_B) = (1 - \frac{4x_A+2x_B}{3})t$, and $x^*_2(x_A, x_B) = \frac{1}{2} + \frac{x_A - x_B}{3}$. The first-order condition, evaluated at $P_{A1} = \lambda t$ and $P_{B1} = t$, is $\frac{5}{4} - \lambda > 0$. Because the second-order condition is negative, this establishes the existence of the no-storage equilibrium. The equilibrium profits are then obvious.

Let us then prove the uniqueness by showing that there exists no equilibrium where one or both firms have positive storage. Suppose first that only firm A has positive storage. Note that firm A's profit maximization problem is given by (A4), which yields the best response function: $P_{A1}(P_{B1}) = \frac{5\lambda t + 2\lambda P_{B1}}{2+4\lambda}$. Because firm B is inducing no storage, its best response function is given by $P_{B1}(P_{A1}) = \frac{t+P_{A1}}{2}$. However, the resulting solution, $P_{A1} = \frac{6\lambda t}{2+3\lambda}$, is above λt and therefore does not qualify as an equilibrium. Suppose then that both firms have positive storage. Firm A's problem is still given by (A4), except that now $x_A = \frac{\lambda t - 2P_{A1} + P_{B1}}{2\lambda t}$, $x_B = \frac{\lambda t + P_{A1} - 2P_{B1}}{2\lambda t}$. The best response function is then $P_{A1}(P_{B1}) = \frac{2\lambda t + (1+\lambda)P_{B1}}{2+2\lambda}$. Given the symmetry of firm B's problem, the only solution is $P_{A1} = P_{B1} = \frac{2\lambda t}{1+\lambda}$, which is again greater than λt. This completes the proof. □

Proof of Lemma 4. Note first that the firms' pricing decisions for the first unit, P'_{i1}, are the same as the benchmark case. Therefore, we have $P'^*_{A1} = P'^*_{B1} = t$. The firms' pricing decisions for the second unit, P_{i1}, are similar to the case without quantity discounts, except that the static-consumption demand x_1 (and $1 - x_1$) is independent of the storage-inducing prices. The proof for the lemma is then similar to that for Proposition 3, by plugging $x_1 = 0$ into (A4). □

Proof of Proposition 4. Conditional on $x_A > 0$ and $x_B > 0$, firm A's profit maximization is given by equation (4). By plugging $x_1 = \frac{t - P_{A1} + P_{B1}}{2t}$, $x_A = \frac{7\lambda t - 16P_{A1} + 8P_{B1}}{8\lambda t}$, $x_B = \frac{7\lambda t + 8P_{A1} - 16P_{B1}}{8\lambda t}$, $P^*_{A2} = (1 - \frac{4x_A + 2x_B}{3})t$, and $x^*_2 = \frac{1}{2} + \frac{x_A - x_B}{3}$, the best response function can be obtained from the first-order condition: $P_{A1}(P_{B1}) = \frac{7\lambda t + 4(2+\lambda)P_{B1}}{8(2+\lambda)}$. Imposing symmetry, one can obtain $P_{A1} = P_{B1} = \frac{7\lambda t}{4(2+\lambda)}$, and $\pi_A = \pi_B = \frac{3(40-\lambda)\lambda t}{64(2+\lambda)}$.

For this to be an equilibrium, there must be no profitable deviations for both firms. Without loss of generality, suppose that firm A deviate such that its storage sales are driven down to zero. Given $P_{B1} = \frac{7\lambda t}{4(2+\lambda)}$ and now $x_A = 0$, $x_B = \frac{3(7\lambda t - 8P_{B1})}{16\lambda t}$, by re-solving equation (4) we can show that the best deviating profit is $\pi'_A = \frac{(64 + 400\lambda - 19\lambda^2)t}{256(2+\lambda)}$. It is easy to check that $\pi_A - \pi'_A$ is increasing in λ, $\pi_A > \pi'_A$ if $\lambda \to 1$, and $\pi_A < \pi'_A$ if $\lambda \to 0$. This proves that the proposed solution is indeed an equilibrium if λ is sufficiently large. It is also easy to check that $\pi^*_A = \frac{3(40-\lambda)\lambda t}{64(2+\lambda)} > \frac{(1+\lambda)t}{2}$. □

Proof of Proposition 5. To prove the proposition, we first solve the firms' local profit maximization problem in the positive-storage scenario, and then identify the conditions under which no firm wants to deviate. Conditional on $x_A > 0$ and $x_B > 0$, firm A solves:

$$\underset{0 < P_{A1} < \lambda_C t}{\text{Max}} \ \pi_A = P_{A1} \cdot (x_1 + x_A) + \lambda_F P^*_{A2}(x_A, x_B)\big[x^*_2(x_A, x_B) - x_A\big], \qquad \text{(A5)}$$

where $x_1 = \frac{t - P_{A1} + P_{B1}}{2t}$, $x_A = \frac{\lambda_C t - 2P_{A1} + P_{B1}}{2\lambda_C t}$, $x_B = \frac{\lambda_C t + P_{A1} - 2P_{B1}}{2\lambda_C t}$, $P^*_{A2}(x_A, x_B) = (1 - \frac{4x_A + 2x_B}{3})t$, and $x^*_2(x_A, x_B) = \frac{1}{2} + \frac{x_A - x_B}{3}$. The first-order condition is $2\lambda_C^2 t + (\lambda_C + \lambda_C^2)P_{B1} - (2\lambda_C^2 + 4\lambda_C - 2\lambda_F)P_{A1} = 0$. Solving the first-order condition yields the best-response function, $P_{A1}(P_{B1}) = \frac{2\lambda_C^2 t + (\lambda_C + \lambda_C^2)P_{B1}}{2\lambda_C^2 + 4\lambda_C - 2\lambda_F}$. Similarly, the best-response function for firm B is

$P_{B1}(P_{A1}) = \frac{2\lambda_C^2 t + (\lambda_C + \lambda_C^2)P_{A1}}{2\lambda_C^2 + 4\lambda_C - 2\lambda_F}$. These best-response functions have slopes less than one if and only if $\lambda_C \geq \frac{\sqrt{9+8\lambda_F}-3}{2}$. Solving the best-response functions simultaneously therefore yields: $P_{A1} = P_{B1} = \frac{2\lambda_C^2 t}{\lambda_C^2 + 3\lambda_C - 2\lambda_F}$.

We also want to make sure that $P_{i1} < \lambda_C t$, $i = A, B$, which is reduced to $\lambda_C > \frac{\sqrt{1+8\lambda_F}-1}{2}$. Note also that $\frac{\sqrt{1+8\lambda_F}-1}{2} \geq \frac{\sqrt{9+8\lambda_F}-3}{2}$ for all $0 \leq \lambda_F \leq 1$. So, if $\lambda_C > \frac{\sqrt{1+8\lambda_F}-1}{2}$, a symmetric local maximum exists in the positive-storage region with profits $\pi_A = \pi_B = \frac{2\lambda_C^2(\lambda_C^2 + 2\lambda_C - \lambda_F)t}{(\lambda_C^2 + 3\lambda_C - 2\lambda_F)^2}$.[10]

For this to be a global equilibrium, one has to ensure that no firm prefers to deviate. The best deviating profit is obtained by solving (A4), except that now $x_A = 0$ and $x_B = \frac{3\lambda_C t - 3P_{B1}}{4\lambda_C t}$. It can be verified that the difference between the best deviating and the above proposed equilibrium profit is strictly decreasing in λ_C for all $\lambda_C \in [\frac{\sqrt{1+8\lambda_F}-1}{2}, 1]$. In addition, when λ_C takes value 1 the best deviating profit is strictly below the proposed equilibrium profit, and strictly above when λ_C goes to $\frac{\sqrt{1+8\lambda_F}-1}{2}$. There must therefore exist a unique $\lambda_C^+ \in [\frac{\sqrt{1+8\lambda_F}-1}{2}, 1]$ such that deviations are strictly dominated and the above local equilibrium is indeed a global one if and only if $\lambda_C \geq \lambda_C^+$. □

Proof of Proposition 6. The proof for the impacts of the firm and the consumer discount factors on the first-period equilibrium prices is straightforward by taking the derivative of $P_{i1}^* = \frac{2\lambda_C^2 t}{\lambda_C^2 + 3\lambda_C - 2\lambda_F}$ with respect to λ_F and λ_C, respectively.

To prove the second part of the proposition, note that the equilibrium profits are given by $\pi_{i1}^* = \frac{2\lambda_C^2(\lambda_C^2 + 2\lambda_C - 2\lambda_F)t}{(\lambda_C^2 + 3\lambda_C - 2\lambda_F)^2}$, $\pi_{i2}^* = \frac{2\lambda_C^2 t}{(\lambda_C^2 + 3\lambda_C - 2\lambda_F)^2}$, and $\pi_i^* = \frac{2\lambda_C^2(\lambda_C^2 + 2\lambda_C - \lambda_F)t}{(\lambda_C^2 + 3\lambda_C - 2\lambda_F)^2}$. It is easy to check that $\frac{d\pi_{i1}^*}{d\lambda_F} > 0$ and $\frac{d\pi_{i2}^*}{d\lambda_F} > 0$. By taking the derivative of π_{i1}^* with respect to λ_C, we can obtain that $\frac{d\pi_{i1}^*}{d\lambda_C} > 0$ if and only if $4\lambda_F^2 - (2\lambda_C^2 + 6\lambda_C)\lambda_F + 3\lambda_C^2 + 2\lambda_C^3 > 0$. This is a quadratic function of λ_F. Because $(2\lambda_C^2 + 6\lambda_C)^2 - 4 \times 4 \times (3\lambda_C^2 + 2\lambda_C^3) = 4\lambda_C^2(\lambda_C + 1)(\lambda_C - 3) < 0$ for all $\lambda_F \in [0, 1]$, the quadratic function is always positive. One could also readily verify that $\frac{d\pi_{i2}^*}{d\lambda_C} < 0$.

Taking the derivative of π_i^* with respect to λ_C, one can see that $\frac{d\pi_i^*}{d\lambda_C} > 0$ if and only if $2\lambda_F^2 - (3\lambda_C^2 + 6\lambda_C)\lambda_F + 3\lambda_C^2 + 2\lambda_C^3 > 0$. This is a quadratic function of λ_F. Solving $2\lambda_F^2 - (3\lambda_C^2 + 6\lambda_C)\lambda_F + 3\lambda_C^2 + 2\lambda_C^3 = 0$ gives rise to $\lambda_{F1} = \frac{\lambda_C}{4}(3\lambda_C - \sqrt{9\lambda_C^2 + 20\lambda_C + 12} + 6)$ and

10. The second-order conditions require that $(1 + \lambda_C)^2 \geq 1 + \lambda_F$, which is satisfied if $\lambda_C > \frac{\sqrt{1+8\lambda_F}-1}{2}$.

$\lambda_{F2} = \frac{\lambda_C}{4}(3\lambda_C + \sqrt{9\lambda_C^2 + 20\lambda_C + 12} + 6)$. It can be verified that $0 \leq \lambda_{F1} \leq 1$, and $\lambda_{F2} > 1$ if $\lambda_C \geq \lambda_C^+$.[11] As a result, $\frac{d\pi_i^*}{d\lambda_C} > 0$ if and only if $\lambda_F \leq \lambda_{F1}$. This completes the proof. □

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11. A quick check is to note that $\lambda_C^+ > 1/2$, and that $\lambda_{F2} > 1$ when $\lambda_C = 1/2$.

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