Definition of Incomplete Contracts

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This note defines incomplete contracts and explains simple contracts. Although widely used in practice, incomplete contracts have not been well defined in the literature.

1. Verifiability and Admissibility

We say that a variable, such as profit or cost, is verifiable to a court if each of its possible values is verifiable to the court. A verifiable variable must first and foremost be observable to the court and the trading parties. That is, it must be observable to everyone. Besides observability, any evidence concerning the value of the variable must be admissible to the court. When this condition is met, we say that the variable is an admissible variable. For example, illegally obtained information is not admissible to a court even though it may offer truth. Hence, information necessary for verifiability must be admissible to a court. The purpose of verifiability is so that a contractual term can be enforced by a court. A variable is said to be contractible if it is verifiable and admissible, meaning that the court can enforce the contractual terms relating to any value of this variable. As in the literature, we assume any verifiable variable is admissible to a court.

2. Two Types of Variables and Rights

Any economic model involves two types of variables: value variables and action variables. Values of action variables describe actions taken by economic agents. For example, investment

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² This note will be updated often. Comments are welcome. This edition is a substantial improvement over the first edition in my book Wang (2012, Chapter 2).
and effort are often action variables in a model. If investment is denoted by \( I \), then \( I \) is an action variable and its value indicates the amount of investment. Action variables are independent variables, meaning that the economic agents can decide on their values directly. Value variables describe the value of activities in a project. For example, profit, revenue, income and cost are often value variables in a model. If the output of a project is denoted by \( x \), then \( x \) is a value variable and the value of \( x \) indicates the amount of output. Value variables are dependent variables that depend on action variables, other value variables and random events. A random variable is a value variable which may be influenced by action variables, other value variables and random variables. Economic agents may not be able to decide on the value of a value variable directly; however, they can influence its value through their control and influence over action variables and other value variables. When the dependence of a value variable is explicitly specified, this dependence is called a value function. The value of a variable is often, but not always, a real number.

For example, suppose we have a production function \( x = f(a_1, a_2, \bar{e}) \), where \( a_1 \) and \( a_2 \) are actions and \( \bar{e} \) is a random shock. Then \( x \) is a value variable, \( a_1 \) and \( a_2 \) are action variables, and \( f(a_1, a_2, \bar{e}) \) is a value function. Similarly, suppose we have a payoff function \( U = u[s(x), a] \), where \( s(x) \) is a value function and \( a \) is an action. Then \( U \) is a value variable and \( a \) is an action variable.

For value variables, there is an issue of how value such as income or cost is shared. A contract may indicate how the parties involved will share the values of value variables. For action variables, there are issues of contractual value and control rights. A contract may indicate what value an action variable must take, or alternatively, who has the right to decide on the value. When the value of an action variable has been specified by a contract (and is enforceable), the control right over the action no longer serves any function, since control rights cannot override the contract.

Hence, there are two kinds of contractibility over an action variable: contractibility of value, and contractibility of rights. Contractibility of value means that the value of a variable is contractible. For example, a labor contract typically states income rights, indicating how much income an employee will receive when certain conditions are met. This means that the value of income is contractible. Similarly, contractual investment refers to how much investment an investor has to make. Contractibility of rights means that the decision-making rights over an action variable are contractible. For example, if the action variable is investment and one is granted the decision-making rights over this variable, then one can decide how much to invest when the time comes, or alternatively one can sell the rights for money. The fact that you have the right to decide does not necessarily mean that you will actually decide; the right to decide gives you a better bargaining position and you may sell this right for money if this serves you better.
Between contractibility of value and contractibility of rights, if the value of an action variable is contractible, then it does not matter whether or not the rights over this variable are contractible. Once an action is contracted (meaning that the contract has specified a value for this action variable), it cannot be overridden since contractual terms come before personal rights. Only if an action cannot be contracted is the contractibility of rights a contractual option. That is, when an action cannot be contracted, we can consider granting someone the right to decide on the action.

Contractibility in the standard agency model refers to the contractibility of value. For incomplete contracts, both value and rights are contractible. A key contribution of Grossman & Hart (1986) is the idea that rights are contractible.

There are two types of rights: income rights and control rights. Income rights are the rights to claim income. These rights are defined by the value-sharing scheme in a contract. Control rights are the decision-making rights over actions. These rights are defined by the decision-making rights over action variables in a contract. For each action variable, a contract either specifies a value or who has control rights over the variable. Each contract specifies income rights defining a sharing scheme for value variables and the values or control rights over action variables.

In sum, contracts involve two types of variables—value and action variables, two types of contractibility—contractibility of value and contractibility of rights, and two types of rights—income and control rights. The definition of incomplete contracts depends on whether or not these rights are completely specified in a contract.

3. Definition of Incomplete Contracts

All contracts have the following form:

\[
\text{contract} = \begin{cases} 
\text{a specification of a value-sharing scheme for value variables,} \\
\text{a specification of the values or control rights over action variables}
\end{cases}
\]

where a blank contract or the lack of one is considered as a “special contract”. A value-sharing scheme defines how value is shared among partners. It is an allocation of the realized values of the value variables among partners. A value-sharing scheme is a complete value-sharing scheme if it is well defined for every possible value of the value variables. Otherwise it is an incomplete value-sharing scheme. Besides a value-sharing scheme, a contract may also specify the values or control rights over action variables. When the value of an action variable is specified, we say that the action is specified; when the control rights of an action variable are specified, we say that the action is under control. In an extreme case, a contract may specify a value for all action variables involved. A contract is a complete contract if it has complete specifications of both value sharing and actions. That is, a complete contract has a complete value-sharing scheme and a complete specification of all actions. Otherwise it is an incomplete con-
tract. That is, a contract is incomplete if either its value-sharing scheme or its actions are not completely specified.

When the value of an action variable is specified in a contract, control rights over this variable is meaningless, since control rights cannot override a contract. Hence, when a contract is complete, control rights have no role to play. Only when a contract is incomplete do control rights play a role. Hence, any contract involving control rights is an incomplete contract. The so-called organizational approach to economic problems relies on the use of control rights. This explains why incomplete contracting is the paradigm of organization theory.

A contract in a model must first and foremost be admissible. That is, we need to define a set of admissible contracts in the model. An admissible contract is well defined in the model and is one of the contract options available to the economic agents. In an economic problem, the economic agents’ contract options are limited to admissible contracts in the model.

An enforceable contract is a contract in which all items are enforceable by a court. However, a court has no right to enforce a contract if it has not been requested to do so by any of the participating parties. That is, enforceability is subject to a request from one of the participating parties. An admissible contract is not necessarily an enforceable contract; conversely, an enforceable contract is not necessarily an admissible contract, although it is generally admissible. For convenience, we assume that all enforceable contracts are admissible in this note.

When an action variable in a contract is not verifiable, for enforceability, the contract must offer incentives to ensure that the economic agents will voluntarily take the contractual value of the action variable. The condition that guarantees they do so is called an incentive compatibility (IC) condition. An enforceable contract must satisfy necessary IC conditions. For some action variables, neither their values nor their control rights are contractible. For such action variables, an IC condition is needed to ensure that the decision-maker will take the expected action in the equilibrium solution.

**Example 1.** In the classical contract theory (Holmström, 1979), a principal hires an agent to work on a project. Let $x$ be ex post output. Output is random ex ante and is denoted by $\bar{x}$. Given the agent’s effort $a$, the distribution function of $\bar{x}$ is $F(x; a)$, which is conditional on $a$. The agent’s effort is not verifiable and is thus not enforceable. But output $x$ is contractible and its value is enforceable. Hence, the principal can offer a value-sharing scheme $s(x)$ based on the contractible output $x$. This scheme pays amount $s(x)$ to the agent when output is $x$. The set of admissible contracts is

$$C = \{(s(x), \hat{a})| s(x) \geq 0 \text{ is the agent's income, } \hat{a} \text{ is the value of effort}\}.$$ 

These admissible contracts are complete since each contract $C = \{s(x), \hat{a}\}$ has a complete specification of the output-sharing scheme $s(x)$ and a complete specification of the action
variable \( a \). When the value of the action variable is enforceable, the optimal contract (called a first-best contract) \( C_1 = \{s^*(x), a^*\} \) is enforceable. When the value of the action variable is not enforceable, the optimal contract (called a second-best contract) \( C_2 = \{s(x), a^*\} \) is also enforceable since the value-sharing scheme satisfies the IC condition by which the agent will voluntarily accept the proposed action \( a^* \) (i.e., the agent voluntarily takes the proposed value of the action variable). The IC condition ensures that the second-best contract is enforceable. The first-best contract does not require an IC condition for the proposed effort \( a^* \) since the value of the action variable is enforceable.

The principal may alternatively offer a contract of the form \( C_3 = \{s(x)\} \). This is an incomplete contract since it does not have a specification of value for the action variable. If sufficient incentives are provided for \( a^* \), this contract is equally as efficient as \( C_2 \).

The principal may also offer a contract of the form \( C_4 = \{s(x), \text{control rights of } a\} \). Such a contract is incomplete since it does not have a specification of value over the action variable. The optimal contract in this case may be \( C_5 = \{s^{**}(x), \text{the agent controls } a\} \) under certain conditions. \( C_5 \) is equally as efficient as \( C_1 \) if \( a^{**} \) is the unique optimal effort for the agent given contract \( s^{**}(x) \).

**Example 2.** Consider the Coase Theorem. A chemical company wants to dump waste into a stream, while a group of fishermen wants to fish in the stream. The value of the stream is \( v_e(a_c, a_f) \) to the chemical company and \( v_f(a_c, a_f) \) to the fishermen, where \( a_c \) is the chemical company’s action and \( a_f \) is the fishermen’s action. Ownership offers a full set of rights. That is, ownership (or full ownership) offers the owner full income rights and control rights, i.e., a claim on all income and control over all actions. Coase proposes a simple contract of the form: contract = \{who has ownership\}. The set of admissible contracts is

\[
\mathcal{C} = \{C_1, C_2, C_3\},
\]

where

\[
C_1 = \{\text{the company has ownership}\},
C_2 = \{\text{the fishermen have ownership}\},
C_3 = \{\text{an arbitrary third party has ownership}\}.
\]

Each of the admissible contract is an incomplete contract, since it does not specify what action the owner is to take. Instead, each contract specifies income rights and control rights over the actions, as implied by ownership. Coase shows that any one of the admissible contracts will lead to efficiency and they are equally efficient if no transaction costs are incurred for an ex post ownership transfer.
**Example 3.** Consider a project in which two partners 1 and 2 produce output \( x \in \mathbb{R} \), which is random ex ante and verifiable ex post. The project requires inputs \( a_1 \) and \( a_2 \) from the two partners. Then the following contract is incomplete:

\[
C_1 = \{ s(x) = ax \text{ for } x \in [0, 10] \}.
\]

Its value-sharing scheme does not mention what will happen if \( x \) is negative or exceeds 10. Related to this contract, the following two contracts are also incomplete:

\[
C_2 = \{ s(x) \text{ for all } x \in \mathbb{R} \},
\]

\[
C_3 = \{ s(x) \text{ for all } x \in \mathbb{R}, \text{ other rights} \}.
\]

Contract \( C_2 \) has a complete specification of income rights. Contract \( C_2 \) is more complete than contract \( C_1 \), and contract \( C_3 \) is more complete than contract \( C_2 \). When an event not mentioned in a contract occurs, the two partners may negotiate to divide the value ex post and \( C_1 \) is equivalent to the following contract:

\[
C_4 = \{ s(x) \text{ if } x \in [0, 10]; \text{ negotiate ex post to divide } x \text{ if } x < 0 \text{ or } x > 10 \}.
\]

Although the value-sharing scheme in \( C_4 \) appears complete, it is not since it does not specify income sharing completely ex ante; it only specifies the two partners’ right to negotiate for income ex post. However, \( C_1 \), or more explicitly \( C_4 \), can perform better than \( C_2 \) in a risky environment (see Wang (2010) for an explanation).

**Example 4.** A debt contract is a complete contract since it specifies a return under every possible circumstance. No action variable is required in a debt deal.

A common stock contract is an incomplete contract since it does not offer a value-sharing scheme. It offers a voting right, which allows the holder to influence the decision on value sharing.

A convertible security is an incomplete contract. It has an action variable, by which the security holder has the right to decide ex post whether or not to convert his investment into equity. The holder has the decision-making right on this conversion option. It is an incomplete contract since the conversion decision is not predetermined in the contract.

**Example 5.** Suppose that income is contractible and no action is required in a deal. There are only two possible states, good or bad. Then, a contract that specifies payments in both states is a complete contract, while one that specifies a payment in the good state only is an incomplete contract. That is,

- Complete contract \( C_1 \): \( s_1(x) = \begin{cases} w_g \text{ if good state}, \\ w_b \text{ if bad state}, \end{cases} \)
- Incomplete contract \( C_2 \): \( s_2(x) = \begin{cases} w_g \text{ if good state}, \\ ? \text{ if bad state}. \end{cases} \)
In contract $C_2$, there is no mention of a payment in the bad state. However, under rationality, as long as the principal is aware that the world could evolve into a bad state, she will have made plans for it. For example, her plan may be that, she will decide what to do if and when the world evolves into a bad state. She may not explicitly mention her plan of action in a contract. If as is the common practice the principal has the default right to decide what to do if a state not described in a contract is realized, then she has no need to mention her plan in the contract. The contract in the latter case is:

$$C_3: s_3(x) = \begin{cases} w_g & \text{if good state}, \\ \text{the principal decides ex post if bad state}. \end{cases}$$

Although contract $C_3$ looks complete, it is not according to our definition. The principal is given control rights over the pay in the bad state, but how much the agent will actually be paid is not specified.

The definition of incomplete contracts in the literature is very different from ours. In the literature, a contract is considered incomplete if it appears to be incomplete. Researchers have been trying to explain the concepts of “apparent incompleteness” or “incompleteness in appearance.” An apparently incomplete contract is often a simple contract, which may or may not be an incomplete contract. We give a clear definition of incomplete contracts based on whether or not value sharing and actions are completely specified.

### 4. Why Simple Contracts?

Economists have been perplexed by just how complex contracts are in theory when they are often very simple in practice. In theory, a contract tends to be complicated since we must clearly specify all items in a contract. In practice, however, contracts are often very simple. Most of them have a linear sharing scheme plus some boundary conditions. A contract is said to be a simple contract if its value-sharing scheme is simple.

Economic activities are governed by rights. Those rights defined by a contract are called contractual rights, while all other rights are called residual rights. In the presence of residual rights, many mechanisms besides contracts may be at work. Consequently, contracts define contractual rights, while other mechanisms define residual rights.

Why are contracts in practice often so simple? If, besides a contract, many other mechanisms are at work to deal with incentive and information problems, then a contract can be incomplete and an incomplete contract can be simple. That is, our explanation for why contracts are so simple in practice is that multiple other mechanisms are simultaneously at work.

For example, corporate culture, tradition and common practice can play a role in regulating behavior in addition to contracts. In particular, ownership can also play a role. If a con-
tract does not specify how a particular piece of property may be used in a certain event, the owner has the default right to make that decision should said event occur.

An incomplete contract can be simple. For example, consider the following contract:

\[ C = \{ax + \beta, \text{rights}\}, \]

where \(ax + \beta\) is a linear sharing scheme over revenue \(x\). However, in practice, many rights are not explicitly written in a contract. Constitutional rights and rights defined by corporate culture, tradition and common practice are examples. In particular, labor contracts rarely mention what rights workers have. Workers know they have certain rights even though they are not explicitly written in the contract. Hence, the above contract may simply be written as \(C_0 = \{ax + \beta\}\). For an employee, the purpose of a labor contract may mainly be to indicate the difference between his rights and those of others, particularly income rights, while common rights applicable to all employees need not be mentioned in the contract. Indeed, labor contracts in practice often focus on value sharing and other rights simply follow the common practice. For example, a company may have the practice of promoting an employee to a management position after he/she has worked in the company for a number of years. This promotion, however, is not guaranteed and typically not written into a labor contract. That is, the written labor contract may be \(C_0\), but it implicitly includes the possibility of a promotion and many other employee rights. The written labor contract can be simple since it is an incomplete contract, and this incomplete contract can serve efficiently since other mechanisms are at work to help deal with information and incentive problems. Wang & Zhou (2004), Qiu & Wang (2009) and Wang (2009) find a simple income-sharing scheme to be optimal with the assistance of other mechanisms.

According to property rights theory (Grossman & Hart, 1986; Hart & Moore, 1990), control rights, ownership and property rights in particular, are important since they form part of an incomplete contract. Hart argues that, given a simple income-sharing scheme, control rights can play a role in regulating behavior. We emphasize the opposite argument that, since control rights can form part of a contract, the income-sharing scheme can be very simple indeed. In Wang & Zhou (2004) and Wang (2009), a simple income-sharing scheme is found to be optimal when the agent has certain control rights. Wang (2010) further shows that in an uncertain market, the principal will choose a less complete, simpler contract.

Many contracts in practice only appear simple or incomplete. For example, consider the law, which is a contract between law enforcement authorities and potential violators. The argument is that the law itself must be complete, but the written law is incomplete. The law must be complete so that a court is able to make a judgement on any case. However, the written law, or the law in writing, is obviously incomplete. The written law is incomplete since the court can also rely on the common practice, tradition and culture when making decisions.
That is, the law is complete only with the help of other mechanisms. In general, a contract can be very simple and incomplete as long as other mechanisms are at work.

5. Why Incomplete Contracts?

There are two major issues concerning incomplete contracts: why should economic agents choose an incomplete contract over a complete contract, and what is the optimal degree of contractual incompleteness?

5.1. Reasons for Contractual Incompleteness

Transaction Costs

Tirole (1999) classifies the justifications for contractual incompleteness in the literature into four types: enforcement costs, writing costs, unforeseen contingencies, and renegotiation.

On enforcement cost, it is argued that contracts are incomplete since enforcing a contract is costly and requires an elaborate system consisting of courts, laws and police forces to be set up. However, such a system is already in existence in every country. Since all countries have sufficient power to enforce contracts, why do we still have incomplete contracts?

On writing cost, some researchers argue that, if specifying contractual contingencies is costly, the optimal contract should trade the efficiency loss from contractual incompleteness off against the cost of adding more contractual clauses. A model of contractual incompleteness based on this idea is offered by Dye (1985). However, many contracts with large stakes are very much incomplete; in fact, contracts with larger stakes are not more complete than contracts with smaller stakes. Writing cost cannot explain this.

When someone hires a babysitter, they usually forgo a written contract. However, while the babysitter is on duty an accident could happen resulting in the death of the child. Both parties are obviously aware of this possibility, yet both agree to enter a contractual relationship without a written contract. Why? This cannot be explained by the writing cost of a contract.

Observability

When the participating parties can observe each other’s valuations ex post, it may be more beneficial to negotiate the trade ex post after they have done so, rather than rigidly contract it ex ante. However, this approach is quite unrealistic in practice. In theory, the equilibrium solution of a trade based on ex post negotiation is actually equivalent to an ex ante renegotiation-proof contract. Since this contract is renegotiation-proof, verifiability of value is unneces-
sary. Such a contract is generally very simple and easily implemented in practice. Hence, observability can ensure the existence of an ex ante renegotiation-proof contract, which is equally as efficient as the outcome of ex post negotiation. However, this contract is generally less efficient than the optimal contract under verifiability.

To improve efficiency, under the same condition that the contracting parties can observe each other’s valuations ex post, a contract can be made indirectly contingent on these valuations by allowing the parties to send messages and conditioning the trade on these messages (see Moore 1992 for a survey of the relevant implementation literature). Maskin & Tirole (1999) show that, if the contracting parties can perfectly foresee their own payoffs (not necessarily valuations) in each future contingency (a common assumption in the literature), the parties can design a message game that effectively describes all the trades ex post that have not been described ex ante. They show that if the parties can play such a game, inability to foresee future contingencies by itself does not constrain contracting. Further, in the context of a holdup problem, Rogerson (1992) shows that by using multi-stage message games and applying the concept of subgame-perfect implementation suggested by Moore & Repullo (1988), the first-best investment can be implemented. However, although such message games are shown to greatly improve efficiency, they are not seen in practice.

**Unforeseen Contingencies**

Hart & Moore (1999) justify contractual incompleteness with the following story. A buyer requires a good from a seller. Suppose that the quality of the good is dependent on the state of the world. If the state of the world is uncertain at the time of contracting, the two parties might write a contingent contract specifying exactly what quality of good to be delivered in each possible state. However, many contingencies are unforeseen at the time of contracting. Hence, to start with the two parties settle for an incomplete contract specifying trading deals for a few highly possible states only. That is, some aspects of future trades may not be foreseeable on the contracting date and must therefore be left to future negotiation. For this reason, contracts are often incomplete.

However, it is not clear what constitutes unforeseen events. Perhaps the contracting parties are aware that the events could occur but do not know the probability of occurrence; perhaps they are not even aware that the events could occur in the first place. If the parties know the occurrence probability, it is a problem of uncertainty; if the parties are aware that the events could occur but do not know the probability, they can apply subjective beliefs to the events based on the Bayesian approach so that these events can be considered in the contract; and finally if the trading parties are unaware that the events could occur in the first place, the problem is beyond all existing theories.
Renegotiation

Any contract can be renegotiated consensually. This possibility reduces the benefits of complete contracts. Because contracts are renegotiable, economic agents may refrain from making an early investment. This is the so-called holdup problem. Hence, although renegotiation has the advantage of ensuring ex-post efficiency (by the assumption of Coasian bargaining), it may lead to ex-ante inefficiency. Consequently, incomplete contracts can be more efficient than complete contracts under certain circumstances.

In a contractual setting that allows renegotiation, the equilibrium contract is renegotiation-proof, since the economic agents have taken into account the possibility of renegotiation in equilibrium. Hence, the real question is: why is an optimal contract incomplete? The explanation is that the condition of “renegotiation-proof” restricts the set of admissible contracts. If renegotiation-proofness rules out all complete contracts, then the optimal contract will be an incomplete contract.

Complexity

Segal (1999) argues that when the environment is complex, many IC conditions are needed to provide incentives. Each IC condition reduces efficiency further and represents a cost of complexity. When the environment is complex, the total cost of complexity is high. When this cost is high enough, no contract (which is a special incomplete contract) is superior.

A Contract as a Commitment

We offer two additional explanations for why optimal contracts are incomplete. First, a contract serves as an early commitment. During renegotiation, the original agreement is an alternative to any negotiating party. Without the early contract, there would be no constraints — anything would be up for negotiation. Since a less complete contract places fewer constraints on future negotiations, an incomplete contract can be more beneficial than a complete contract.

Wang (2010) considers two contracts where one is more complete than the other. He shows that in an uncertain market, the principal will choose a less complete contract. The reason is that a less complete contract offers more flexibility for an uncertain future.

Qiu & Wang (2009) explain a popular contract in private-public joint ventures with contract theory according to which contractual incompleteness is used to control product quality. The main gist is that contractual incompleteness allows options and flexibility in a contract so that the contract can handle contingencies optimally.
Multiple Mechanisms

Second, often several mechanisms are at work in an economic problem, where a contract is one of them. An optimal contract can be very simple and incomplete since there are other mechanisms in the model that can also deal with incentive and information problems.

Wang & Zhou (2004) consider an investment problem, in which the principal offers a debt contract and staged financing. The debt contract defines value sharing between the investor and the firm, and staged financing allows the investor to optimally allocate his investment across multiple periods. Here staged financing serves as a second mechanism in regulating behavior in addition to the debt contract. They show that the optimal debt contract can be very simple and incomplete with the presence of staged financing.

5.2. The Optimal Degree of Incompleteness

If incompleteness of a contract can play a role in regulating behavior, then an optimal degree of incompleteness must exist. In a model with internal and external risks together with incentive problems, Wang (2010) investigates the role of a risky environment in contractual incompleteness. With an incomplete contract, even if many items are missing from the contract, the contracting parties know what will happen ex post and they make choices ex ante based on this knowledge. In a complete contract, even if all necessary items are in the contract, they may still need to negotiate a new contract ex post. Wang’s main findings are that (1) external risks determine contractual incompleteness, and (2) a complete contract can better handle incentives, while an incomplete contract can better handle external risks.

References


