The Irrelevance of Control Rights in Agency Models under Risk Neutrality

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Abstract: The allocation of control rights is a key issue in incomplete contracts. In this paper, we add an exit option, the right of early termination, to the standard agency model for employment contracts. We address two questions: (1) Who should have this right? (2) What is the effect of its inclusion in a contract? Under risk neutrality, we find that (1) it does not matter for investments and economic efficiency to whom this right belongs, although it may affect contractual terms and income sharing, and (2) although the allocation of this right does not matter, its inclusion can have either a positive or a negative effect on economic efficiency. Our analysis provides a benchmark for more general analyses on control rights under risk aversion.

Keywords: Information Revelation, Control Rights, Agency Model, Risk Neutrality

JEL Classification: D23, D2, D8
1. Introduction

As pointed out by Maskin and Tirole (1999), one of the three key assumptions in the incomplete contract literature is the risk-neutrality assumption. Agency models with risk neutrality are popular in applications because of the existence of an optimal linear contract. In a world where information is revealed ex post, control rights are an important part of the model. In this study, we address two questions: How does the inclusion of control rights affect investments and economic efficiency? How does the allocation of control rights affect investments and economic efficiency? We provide simple answers to these questions.

Following the pioneering works by Grossman and Hart (1986), Hart and Moore (1990) and Hart (1995), control rights have become a hot topic. In the world of incomplete contracts, control rights are generally defined as the ownership of residual rights. In our principal-agent model in particular, a control right is defined as the right of early exit. If the exit option is not exercised, the contractual agreement remains valid; if the exit option is exercised, the contractual agreement becomes invalid and a new contract may be negotiated ex post. In equilibrium, the optimal contract is renegotiation-proof and hence valid throughout the cooperation period.

Real world contracts, such as employment contracts, typically contain statements on early termination. In practice, there are large variations in exit options among employment contracts. In some contracts, employers have the control rights of exit. For example, in professional sports, employers impose transfer fees to prevent players from leaving without the employers’ consent. However, in most labor contracts, employees have the control rights of exit. According to Meccheri (2008, p.15-16), penalties on manager-grade staff for quitting jobs are rare in large Western corporations. In most cases, an employer’s early termination of employment is conditional, while employees have the control rights of exit, meaning that they can quit the job anytime without a penalty. For example, many large US firms commit to the so-called zero-firing policy that prevents them from laying off employees without employees’ consent. In Japan, it is a tradition for employers to provide lifetime employment. The observed variations in assignment of termination rights may be because termination rights are inconsequential so that they appear in many different forms, which is consistent with our irrelevance result; it may also be because the trading parties are risk averse so that termination rights are relevant and need to be specified conditional on the circumstances. In the latter case, our irrelevance result provides a benchmark for the understanding of such dependence.

In some cases, the right of early exit seems particularly crucial in a contractual relationship. For example, in the relationship between a venture capitalist and an entrepreneur in venture capital investment, the right to replace the entrepreneur ex post by a professional manager is considered by researchers to be an important incentive mechanism. It is believed that venture capitalists prefer to finance risky firms by convertibles because convertibles offer
them this right (Sahlman, 1990). In university hirings, tenure-track contracts are popular because such contracts give a department the right to grant lifetime job security ex post. The key here is the right to exercise the option ex post after the principal has obtained sufficient knowledge about the agent’s performance and ability. A contract between an employer and an employee typically includes a clause for early termination. Different countries and cultures may have very different practices regarding early termination of a contract. In some cases, an employee can quit and the employer can fire a worker anytime without a penalty, while in others, the unilateral early termination of a contract incurs a severe financial penalty. We wonder how much the right of early termination in particular and the allocation of control rights in general actually matter in a principal-agent relationship.

We have an agency model with double moral hazard. In many real-world principal-agent relationships, besides the agent, the principal applies effort and investment to affect outcomes. For example, the demand for a product is affected not only by a downstream firm’s (the agent) sales effort but also by an upstream firm’s (the principal) manufacturing inputs that determine the quality of the product. The relationship between a franchiser and a franchisee is another example. In her empirical analysis, Lafontaine (1992) concludes that a double moral-hazard argument on franchising best explains the data. In a university, university authorities try best to provide a good research environment by securing research funding sources, hiring star researchers, reducing teaching duty, and providing a sound research rewarding system. Early theoretical studies on agency models with double moral-hazard include Romano (1994), Bhattacharyya and Lafontaine (1995), and Kim and Wang (1998).

Our model environment is such that certain information and options become available over time. The issue of control rights is particularly relevant to such a setting. We focus on the right of early termination, by which an employer can dismiss an employee or an employee can quit without penalty. Such an option will ensure ex-post efficiency with Coasian bargaining and may sometimes lead to ex-ante efficiency. Ex-ante efficiency, which is typically referred to as economic efficiency, is the key in our investigation. As we know from Kim and Wang (1998), this agency model cannot achieve the first best (under double moral hazard) even if both parties are risk neutral. Our question is: Can control rights in this agency model improve efficiency if the control rights are allocated appropriately?

In our model, the two parties are asymmetric in the sense that one is the principal (with full bargaining power ex ante) and the other is the agent. We will derive the solutions for different allocations of control rights. With asymmetry and risk neutrality, we show that the allocation of control rights is irrelevant for investments and economic efficiency, although it does affect the terms in a contract. This result has not been known before. Also, we find that, although the allocation of the right does not affect efficiency, the inclusion of the right in a contract can enhance or reduce efficiency. In a parametric case, we derive conditions under which the inclusion of the right in a contract can enhance or reduce economic efficiency. In particu-
lar, although the control rights on job security do not affect investments and economic efficiency, they do affect the structure of the contract and the income sharing scheme in the contract as well as the ability threshold beyond which the agent is to leave the job voluntarily or forcefully.

The rest of the paper is organized as follows. Section 2 lays out the basic model. Section 3 presents the optimal solutions under different circumstances. Section 4 discusses the results. Section 5 concludes with a few remarks. All the derivations and proofs are in the appendix.

2. The Model

Consider an agency model in which a principal hires an agent to undertake a project. The project lasts two periods. At the beginning of the project, each party invests effort, \( a \) from the agent and \( b \) from the principal. The costs of effort are private and are respectively \( c(a) \) and \( C(b) \). The efforts \( a \) and \( b \) are not verifiable. The two effort investments generate a composite effort \( h(a,b) \), which affects the distribution of output.\(^3\) The output is also dependent on the agent’s type, which is represented by an index \( \theta \in \mathbb{R}_+ \). This index is unknown to both parties at time 0 (ex ante), and it becomes known to both parties at time 1 (ex post). We assume that the disinformation about \( \theta \) is symmetric, i.e., the two parties have a common distribution function \( \Phi(\theta) \) about \( \theta \) ex ante, with density function \( \phi(\theta) \). Given effort inputs \( (a,b) \) and the agent’s ability \( \theta \), the density function of output is \( f[x,\theta,h(a,b)] \).\(^4\)

As in the standard agency model, output is contractible. An income sharing rule is represented by a function \( s(x) \) from the output space \( \mathbb{X} \) to \( \mathbb{R} \), where \( s(x) \) is the payment to the agent when output is \( x \). The set of sharing rules is
\[
\mathcal{S} = \{s : \mathbb{X} \to \mathbb{R} | s \text{ is Lebesgue integrable}\}.
\]

In addition to an income-sharing rule, we allow for an ex-post option in a contract. Unlike the income rights defined by the income sharing rule, this option is a control right, which can be given either to the principal or the agent. The ability \( \theta \) of the inside agent is random ex ante. When \( \theta \) is revealed ex post, the principal considers to replace the inside agent by an outside agent from the labor market. Depending on who has the right, a monetary transfer between the principal and the inside agent from one direction or the other may be made. The

\(^3\)Here, efforts \( a \) and \( b \) may or may not be observable ex ante or ex post. Their observability is not an issue here since the equilibrium efforts \( a^* \) and \( b^* \) are known to both parties at any moment in time. Since the efforts are not verifiable, no matter whether they are observable or not, they are not enforceable by a court.

\(^4\)For example, in a special case when \( x = \theta h(a,b) + \xi \), where \( \xi \) is a random shock, the output distribution function is \( F_x(x) = F_{x-\theta h(a,b)} \), where \( F_x \) is the distribution of \( \xi \). This \( F_x[x-\theta h(a,b)] \) is a special case of our output distribution function \( F[x,\theta,h(a,b)] \).
outside agent with ability $\theta_0$ is available ex post for a fixed market wage rate $s_0$, where $\theta_0$ and $s_0$ are known constants. Here we assume that the firm is competitive in the labor market so that it takes the market wage rate as given. When the outside agent with ability $\theta_0$ is hired ex post, the output density function for $x$ is $\hat{f}[x,\theta_0, b(a,b)]$. Here, we assume a different density function $\hat{f}$ to allow for inputs by the outside agent. Hence, a contract in our model is of the form:

$$\text{contract} = \{\text{sharing rule } s(x), \text{ right to dismiss or quit}\}.$$  

The contractable option of exit is a unique feature in our model. The control right in our model refers to the control of this exit option. We are concerned with two issues: (1) Who should have this right? (2) What is the effect of the inclusion of this right in a contract?

Our model is different from a multi-agent model. An outside agent in our model gives a value to the exit option rather than competes directly with the inside agent. The outside agent serves as a second “instrument” in controlling incentives, in addition to the output sharing rule. The ability of a worker has been found to be a crucial factor in agency models in the literature. Baker and Holmström (1995) find that wage growth and promotion are not closely related. Instead, they find that ability is what drives both wage growth and promotion. They find strong evidence in support of the existence of internal labor markets, but they find that external labor markets have limited influence on internal labor markets. Gibbons and Waldman (1999) construct a selection model based on ability. Their model explains a few key empirical findings, such as a positive serial correlation in wage growth and a positive correlation between wage growth and promotions. Also, Lazear and Oyer (2004) find empirical evidence indicating an important impact of external labor markets on internal labor markets. These studies justify the inclusion of an employee’s ability and the existence of an outside agent as key factors in our model.

The timing of the events is illustrated in the following figure.

![Figure 1. The Timing of Events](image)

5 For example, $\hat{f}[x,\theta_0, b(a,b)]$ can be $f[x,\theta_0, h(a+a_0^*,b)]$, where $a_0^*$ is the outside agent’s optimal effort. Also, to represent the case where the inside agent’s investment $a$ does not have a large impact on output after he or she is dismissed, $\hat{f}[x,\theta_0, b(a,b)]$ can be $f[x,\theta_0, h_0(a_0^*,b)]$, where $h_0$ can be different from $h$. However, for simplicity, we do not explicitly model the outside agent’s optimal decision.
1. At $t = 0$, the principal offers a contract to the agent. If the contract is accepted, the agent and the principal simultaneously invest $a$ and $b$ and incur costs $c(a)$ and $C(b)$, respectively.

2. At $t = 1$, uncertainty about $\theta$ is resolved. The principal considers the option to replace the inside agent with an outside agent. Suppose that an outside agent with $\theta_0$ is available for a market wage range $s_0$. Depending on who has the control right, the principal may bargain with the inside agent for a new contract or simply fire him without having to pay any compensation.

3. At $t = 2$, the project is finished and payments are made based on the existing contract.

In practice, when one party proposes early termination of the relationship (either the employee expresses an intention to leave or the employer requests the employee to leave), the other party often offers to negotiate. Whoever has the control right of exit has an advantage in renegotiation. To make our model more realistic, we allow a contract to be renegotiated ex post when new information is available, i.e. when $\theta$ becomes known. By that time, the investments $(a, b)$ are sunk but the inside agent is replaceable. As new information arrives, the two parties may want to take advantage of the opportunity to improve ex-post efficiency and share the gain from renegotiation.

Given $(a, b, \theta)$, the expected revenue at $t = 1$ is

$$ r(a, b, \theta) \equiv \int x f[x, \theta, h(a, b)] dx. \quad (1) $$

The expected welfare gain at $t = 1$ from renegotiation is

$$ g(a, b, \theta) \equiv r(a, b, \theta_0) - r(a, b, \theta) - s_0. \quad (2) $$

We adopt Coasian bargaining, by which renegotiation yields ex-post efficiency. Hence, given the agent’s type $\theta$ with investments $(a, b)$, there will be renegotiation if and only if $g(a, b, \theta) > 0$. Define the threshold $\hat{\theta}(a, b)$ by equation:

$$ r(a, b, \theta_0) = r(a, b, \hat{\theta}) + s_0. \quad (3) $$

Then, if and only if $\theta < \hat{\theta}$ will the inside agent be replaced by an outside agent, regardless of who has the right of control. Without renegotiation, the expected revenue at $t = 0$ is

$$ R(a, b) \equiv \int r(a, b, \theta) d\Phi(\theta). $$

With renegotiation, the expected revenue at $t = 0$ is

$$ \hat{R}(a, b) \equiv \int_0^{\hat{\theta}(a, b)} [r(a, b, \theta_0) - s_0] d\Phi(\theta) + \int_{\hat{\theta}(a, b)}^{\infty} r(a, b, \theta) d\Phi(\theta). \quad (4) $$

Hence, the expected gain at $t = 0$ from renegotiation is

$$ G(a, b) \equiv \int_0^{\hat{\theta}(a, b)} g(a, b, \theta) d\Phi(\theta), \quad (5) $$
where
\[ \hat{R}(a,b) = R(a,b) + G(a,b). \] (6)

Standard assumptions are imposed on the functions:
\[ R_a(a,b) > 0, \quad R_\alpha(a,b) \leq 0, \quad R_b(a,b) > 0, \quad R_\alpha(b,a) \leq 0, \]
\[ c(a) > 0, \quad c'(a) > 0, \quad c''(a) > 0, \]
\[ C(b) > 0, \quad C'(b) > 0, \quad C''(b) > 0. \] (7)

### 3. The Solutions

#### 3.1. Employment without an Exit Option

As a benchmark, we first consider the standard agency model in which early termination is not an option. In this case, contracts are of the form:
\[ \text{contract} \equiv \{ \text{a sharing rule } s(x) \}. \]

In this case, the agent’s ex-ante payoff is
\[ U(a,b,s) = \int \int s(x)f[x,\theta, h(a,b)]\psi(\theta)d\theta, \]
and the principal’s ex-ante payoff is
\[ V(a,b,s) = \int \int [x-s(x)]f[x,\theta, h(a,b)]\psi(\theta)d\theta = R(a,b) - U(a,b,s). \]

Suppose that after the contract is signed, the two parties play a Nash game to determine efforts/investments \( a \) and \( b \). Then, the two first-order conditions (FOCs) are
\[ U_a(a,b,s) = c'(a), \quad V_b(a,b,s) = C'(b), \]
and the principal’s second-best problem is
\[
\Pi^* \equiv \max_{s \in S, a \in A, b \in B} R(a,b) - U(a,b,s) - C(b)
\]
\[ \text{s.t.} \quad IC_a : U_a(a,b,s) = c'(a), \]
\[ IC_b : V_b(a,b,s) = C'(b), \]
\[ IR : \quad U(a,b,s) \geq \bar{u} + c(a), \]
where \( \bar{u} \) is the agent’s reservation utility, \( IC \) stands for an incentive compatibility condition, and \( IR \) stands for an individual rationality condition. When both efforts are contractible, the principal’s first-best problem is
\[
\Pi^{**} \equiv \max_{s \in S, a \in A, b \in B} R(a,b) - U(a,b,s) - C(b)
\]
\[ \text{s.t.} \quad U(a,b,s) \geq \bar{u} + c(a). \]
Lemma 1. For the first-best problem, the first-best efforts \((a^{**}, b^{**})\) are determined by
\[
R_a(a, b) = c'(a), \quad R_b(a, b) = C'(b),
\]
and a fixed contract is optimal:
\[
s^{**}(x) = \bar{u} + c(a^{**}).
\]

Proposition 1. For the second-best problem, the second-best efforts \((a^*, b^*)\) are determined by
\[
\Pi^* = \max_{a \in A, b \in B} \left( R(a, b) - c(a) - C(b) - \bar{u} \right)
\text{s.t.} \quad R_a(a, b) = c'(a) + \frac{h_a(a, b)}{h_b(a, b)} C'(b),
\]
and a linear contract is optimal:
\[
s^*(x) = \alpha x + \beta,
\]
where \(\alpha = \frac{c'(a^*)}{R_a(a^*, b^*)}\) and \(\beta = \bar{u} + c(a^*) - \alpha R(a^*, b^*).\) We have \(0 < \alpha < 1\) and the first-best outcome is not achievable.

Bhattacharyya and Lafontaine (1995) were the first to provide this result for a special output process with a distribution function of the form \(F(x - h).\) Kim and Wang (1998) provided the same result with a general distribution function \(F(x, h).\) Here, the optimality of a linear contract is due to risk neutrality and the failure to achieve the first best is due to the principal’s inability to commit ex ante to the efficient level of effort.

Remark 1. One advantage of a linear contract is that the validity of the first-order approach (FOA), i.e., \(U_{aa}(a, b, s^*) < c''(a)\) and \(V_{bb}(a, b, s^*) < C''(b)\) for all \((a, b),\) can be easily guaranteed by the simple conditions in (7). Hence, we have no need to discuss the validity of the FOA.

Remark 2. We have implicitly assumed that an outside agent is not available ex ante. To justify this, we may think of the inside agent as the best available person for the job out of a pool of job applicants.

3.2. Employment with an Exit Option: Agent in Control

When early termination is allowed, the ownership of the control rights over this option becomes an issue. In this section, the inside agent has the control right, which means that he or she can quit anytime without penalty. In this case, the contract is of the form:

\[
\text{contract} = \{\text{sharing rule } s(x), \text{ right to quit by the agent}\}.
\]
At time 1, \( \theta \) is known. The principal considers the option of replacing the agent by an outside agent from the market. Given \( \theta \) and \( s(\cdot) \), if the principal wants to replace the inside agent, the principal has to bargain with the agent. Suppose that an outside agent with ability \( \theta_0 \) can be hired for a fixed wage \( s_0 \). By (3), the principal will replace the inside agent ex post if and only if \( \theta < \hat{\theta} \). With the distribution of bargaining powers \((\delta,1-\delta)\) and the net gain \( g(a,b,\theta) \) from renegotiation, where \( \delta \in (0,1) \), the Nash bargaining solution implies that the ex-post payoffs of the agent and principal are, respectively,

\[
u(\theta,\delta) = \int [x - s(x)]f[x,\theta,h(a,b)]dx + (1-\delta)g(a,b,\theta) = r(a,b,\theta_0) - s_0 - u(\theta,\delta),
\]

where \( r(a,b,\theta) \) and \( g(a,b,\theta) \) are defined in (1) and (2), respectively. Note that since both parties benefit ex post from renegotiation when \( \theta < \hat{\theta} \), the initial contract is abandoned voluntarily. Then, the ex-ante payoffs of the agent and the principal are, respectively,

\[
U(a,b,s,\delta) = \int_0^{\hat{\delta}(a,b)} u(\theta,\delta)\phi(\theta)d\theta + \int_{\hat{\delta}(a,b)}^{\infty} \int s(x)f[x,\theta,h(a,b)]\phi(\theta)dx d\theta
\]

\[
= \int s(x)f[x,\theta,h(a,b)]\phi(\theta)dx d\theta + \delta G(a,b),
\]

\[
V(a,b,s,\delta) = \int_0^{\hat{\delta}(a,b)} v(\theta,\delta)\phi(\theta)d\theta + \int_{\hat{\delta}(a,b)}^{\infty} \int [x - s(x)]f[x,\theta,h(a,b)]\phi(\theta)dx d\theta
\]

\[
= \hat{R}(a,b) - U(a,b,s,\delta),
\]

where \( G(a,b) \) and \( \hat{R}(a,b) \) are defined in (5) and (6), respectively. Then, the principal’s second-best problem is

\[
\hat{\Pi}^* = \max_{s \in S, a \in A, b \in B} \hat{R}(a,b) - U(a,b,s,\delta) - C(b)
\]

s.t. \( IC_a : U_a(a,b,s,\delta) = c'(a) \),

\( IC_b : V_b(a,b,s,\delta) = C'(b) \),

\( IR : U(a,b,s,\delta) \geq \bar{u} + c(a). \)

When both efforts are contractible, the principal’s first-best problem is

\[
\hat{\Pi}^{**} = \max_{s \in S, a \in A, b \in B} \hat{R}(a,b) - U(a,b,s,\delta) - C(b)
\]

s.t. \( U(a,b,s,\delta) \geq \bar{u} + c(a). \)

**Lemma 2.** For the first-best problem, the first-best efforts \((\hat{a}^{**}, \hat{b}^{**})\) are determined by

\[
\hat{R}_a(a,b) = c'(a), \quad \hat{R}_b(a,b) = C'(b),
\]

and a fixed contract is optimal:

\[
\hat{s}^{**}(x) = \bar{u} + c(\hat{a}^{**}) - \delta G(\hat{a}^{**},\hat{b}^{**}).
\]
Proposition 2. For the second-best problem, the second-best efforts \((\hat{a}^*, \hat{b}^*)\) are determined by

\[
\hat{\Pi}' = \max_{a \in A, b \in B} \hat{R}(a, b) - c(a) - C(b) - \tilde{u} \\
\text{s.t. } \hat{R}_a(a, b) = c'(a) + \frac{h_a(a, b)}{h_b(a, b)} C'(b),
\]

and a linear contract is optimal:

\[
\hat{s}^*(x) = \alpha x + \beta,
\]

where \(\alpha = \frac{c'(\hat{a}^*) - \delta G_a(\hat{a}^*, \hat{b}^*)}{R_a(\hat{a}^*, \hat{b}^*)}\) and \(\beta = \tilde{u} + c(\hat{a}^*) - \alpha R(\hat{a}^*, \hat{b}^*) - \delta G(\hat{a}^*, \hat{b}^*)\). The first-best outcome is not achievable.

3.3. Employment with an Exit Option: Principal in Control

In this section, the principal has the control right of exit, which means that he or she can dismiss the agent anytime without penalty. In this case, the contract is of the form:

contract = \{sharing rule \(s(x)\), right to terminate by the principal\}.

At time 1, \(\theta\) is known. The principal considers the option of replacing the inside agent by an outside agent. Given \(\theta\) and \(s(\cdot)\), the principal can simply dismiss the agent since the principal has the control right. Suppose that an outside agent with ability \(\theta_0\) can be hired for a fixed wage \(s_0\). The net gain from exercising the control right is \(g(a, b, \theta)\). By (3), the principal will replace the agent ex post if and only if \(\theta < \hat{\theta}\).

If the inside agent is not fired, the wage payment will still be based on the initial contract. But if the inside agent is fired, the ex-post payoffs of the principal and agent are, respectively,

\[
u(\theta) \equiv r(a, b, \theta_0) - s_0.
\]

Then, the ex-ante payoffs of the agent and the principal are, respectively,

\[
U(a, b, s) \equiv \int_{\hat{h}(a, b)}^{\infty} \int_{\hat{s}(a, b)}^{\infty} s(x) f[x, \theta, h(a, b)] \phi(\theta) dx d\theta,
\]

\[
V(a, b, s) \equiv \int_{0}^{\hat{v}(a, b)} v(\theta) \phi(\theta) d\theta + \int_{\hat{v}(a, b)}^{\infty} \left[ x - s(x) \right] f[x, \theta, h(a, b)] \phi(\theta) dx d\theta
\]

\[
= \hat{R}(a, b) - U(a, b, s).
\]
Hence, the principal’s second-best problem is

\[
\hat{\Pi}^* = \max_{s \in S, a \in A, b \in B} \hat{R}(a,b) - U(a,b,s) - C(b)
\]

s.t. \( IC_a : U_a(a,b,s) = c'(a) \),
\( IC_b : V_b(a,b,s) = C'(b) \),
\( IR : U(a,b,s) \geq \bar{u} + c(a) \).  

When both efforts are contractible, the principal’s first-best problem is

\[
\hat{\Pi}^{**} = \max_{s \in S, a \in A, b \in B} \hat{R}(a,b) - U(a,b,s) - C(b)
\]

s.t. \( U(a,b,s) \geq \bar{u} + c(a) \).

**Lemma 3.** For the first-best problem, the first-best efforts \((\hat{a}^{**}, \hat{b}^{**})\) are determined by

\[
\hat{R}_a(a,b) = c'(a), \quad \hat{R}_b(a,b) = C'(b),
\]

and a fixed contract is optimal:

\[
\hat{s}^{**}(x) = \frac{\bar{u} + c(\hat{a}^{**})}{1 - \Phi[\hat{\theta}(\hat{a}^{**}, \hat{b}^{**})]}.
\]

**Proposition 3.** For the second-best problem, the second-best efforts \((\hat{a}^*, \hat{b}^*)\) are determined by

\[
\hat{\Pi}^* \equiv \max_{a \in A, b \in B} \hat{R}(a,b) - c(a) - C(b) - \bar{u}
\]

s.t. \( \hat{R}_a(a,b) = c'(a) + \frac{h_a(a,b)}{h_b(a,b)} C'(b) \),

and a linear sharing rule is optimal:

\[
\hat{s}^*(x) = \alpha x + \beta,
\]

where \( \alpha = \frac{c'(\hat{a}^*)}{D_a(\hat{a}^*, \hat{b}^*)} \), \( \beta = \frac{\bar{u} + c(\hat{a}^*) - \alpha D(\hat{a}^*, \hat{b}^*)}{1 - \Phi[\hat{\theta}(\hat{a}^*, \hat{b}^*)]} \) and \( D(a,b) \equiv \int_{\theta(a,b)}^{\infty} r(a,b,\theta) d\Phi(\theta) \). The first-best outcome is not achievable.

**4. Discussion**

**4.1. Allocation of Control Rights**

One immediate question is: who should have the control right? By Propositions 2 and 3, the answer is surprisingly simple. We can see that problems (10) and (12) are exactly the same, which implies the following irrelevance result.
**Proposition 4** (Irrelevance). Under risk neutrality, control rights are irrelevant in agency problems. More specifically, investments (optimal efforts) and economic efficiency (social welfare) are independent of the allocation of control rights, although contractual terms and income sharing as well as the threshold for early termination are dependent on the allocation of control rights.

This irrelevance result requires the assumptions of risk neutrality and Coasian bargaining. Coasian bargaining ensures no loss of social welfare when the parties renegotiate. With risk neutrality, the Nash bargaining solution allocates the benefit of renegotiation proportionally to the two parties so that there is no distortion in aggregate payoff. This explains why the irrelevance result holds under risk neutrality and Coasian bargaining.

From Propositions 1–3, we further find that, under risk neutrality, the inclusion of control rights and the allocation of control rights will not affect the existence of an optimal linear contract.

**Proposition 5** (Linear Contract). Under risk neutrality, the inclusion of control rights and the allocation of control rights will not affect the existence of an optimal linear sharing rule.

The existence of an optimal linear income sharing rule under risk neutrality is well known. We extend this result to models with control rights. We show that the existence of an optimal linear contract as well as the independence of the existence on the allocation of control rights.

### 4.2. Inclusion of Control Rights

A further question is: how will the inclusion of control rights in a contract affect economic efficiency? That is, although the allocation of control rights does not matter, the inclusion of control rights may. The answer to this question is quite complicated. Under the first best, we know that the inclusion of control rights, or the inclusion of more choices in a contract generally, will always enhance economic efficiency. However, under the second best, the inclusion of control rights allows more choices but also causes more opportunistic behaviors. We find that, under the second best, the inclusion of control rights can sometimes reduce efficiency and at other times improve efficiency. To determine the roles of different factors affecting the answer to this question, we consider a parametric case in which a few key factors are represented by parameters.

For convenience of notation, let agent 1 be the inside agent and agent 2 be the principal. Consider the following parametric case:

\[ h(a,b) = \mu_1 a + \mu_2 b, \quad x(\theta, h) = \tilde{A}\theta h, \quad c(a) = \frac{\gamma_1}{2} a^2, \quad C(b) = \frac{\gamma_2}{2} b^2, \quad \bar{u} = 0, \quad (13) \]
where $\mu_i \geq 0$ represents the weight of agent $i$’s investment, $\tilde{A}$ is random ex post with $E(\tilde{A}) = 1$, and $\theta$ is random ex ante and takes two possible values $\theta_L \geq 0$ and $\theta_H > \theta_L$ with probability $\rho$ and $1 - \rho$, respectively. Suppose $\theta_L < \hat{\theta} < \theta_H$,\(^6\) and denote

\[
\overline{\theta} \equiv \rho \theta_L + (1 - \rho) \theta_H, \quad \bar{\theta} \equiv \rho \theta_0 + (1 - \rho) \theta_H.
\]

![Figure 2. The Distribution of Ability](image)

Then, the second-best solution for the agency model without control rights is

\[
a^* = \overline{\theta} \frac{\gamma_2}{\gamma_1 \gamma_1 \mu_1^2 + \gamma_2 \mu_1^2}, \quad b^* = \overline{\theta} \frac{\gamma_1}{\gamma_2 \gamma_2 \mu_2^2 + \gamma_1 \mu_2^2},
\]

\[
\Pi^* = \frac{\overline{\theta}^2}{2} \left( \frac{\mu_1^2}{\gamma_1} + \frac{\mu_2^2}{\gamma_2} - \frac{\mu_1 \mu_2^2}{\gamma_2 \mu_1^2 + \gamma_1 \mu_2^2} \right),
\]

and the second-best solution for the agency model with control rights is

\[
\hat{a}^* = \hat{\theta} \frac{\gamma_2}{\gamma_1 \gamma_1 \mu_1^2 + \gamma_2 \mu_1^2}, \quad \hat{b}^* = \hat{\theta} \frac{\gamma_1}{\gamma_2 \gamma_2 \mu_2^2 + \gamma_1 \mu_2^2},
\]

\[
\hat{\Pi}^* = \frac{\hat{\theta}^2}{2} \left( \frac{\mu_1^2}{\gamma_1} + \frac{\mu_2^2}{\gamma_2} - \frac{\mu_1 \mu_2^2}{\gamma_2 \mu_1^2 + \gamma_1 \mu_2^2} \right) - \rho s_0.
\]

Let

\[
p_1 = \frac{\mu_1^2}{\gamma_1}, \quad p_2 = \frac{\mu_2^2}{\gamma_2}.
\]

We call $p_i$ agent $i$’s productivity. Consider a condition on the productivity:

\[^6\text{This means } \frac{\hat{\theta} \gamma_1 \gamma_2 \mu_1^4 + \gamma_2 \mu_1^4}{\gamma_1 \gamma_2 \gamma_1 \mu_2^2 + \gamma_2 \mu_2^2} \geq \frac{s_0}{\theta_0 - \theta_L}, \text{ which is a minor restriction on the parameters.}\]
where
\[
p_1 + p_2 - \frac{p_1 p_2}{p_1 + p_2} = c, \tag{14}
\]

Equation (14) defines an elliptic curve on \( \mathbb{R}^2 \). The curve and the comparison of \( \Pi' \) against \( \hat{\Pi}' \) are shown in Figure 3.

![Figure 3. Effects of the Inclusion of Control Rights in a Contract](image)

The figure implies the following result.

**Proposition 6.** Given the parametric functions in (13), we have the following results:

(a) When either \( p_1 \) or \( p_2 \) is sufficiently large, the inclusion of control rights enhances efficiency. When both \( p_1 \) and \( p_2 \) are sufficiently small, the inclusion of control rights reduces efficiency.

(b) When \( \theta_0 \) is large or when \( s_0 \) is small, the inclusion of control rights tends to enhance efficiency.

(c) Although the two parties’ positions are asymmetric ex ante, their productivities \( p_1 \) and \( p_2 \) are equally relevant to the impact of the inclusion of control rights on efficiency.

Control rights allow the holder to renegotiate the terms in a contract. As indicated in (2), the expected welfare gain from renegotiation is \( g(a,b,\theta) \). Hence, the two parties’ incentives (the choices of \( a \) and \( b \)) are not only affected by the income-sharing rule but also by this welfare gain. Specifically, at \( t = 1 \), the income-sharing rule determines an allocation of \( r(a,b,\theta) \), while the Nash bargaining solution determines an allocation of \( g(a,b,\theta) \). The latter allocation causes an additional consideration (the so-called opportunistic behavior) by the parties when they make choices of \( a \) and \( b \) ex ante. That is, the possibility of renegotiation may cause the
parties to engage in opportunistic behaviors when making their initial investments. This opportunistic behavior, stemming from the right to renegotiate for better terms ex post, may enhance or reduce the incentive to invest. Proposition 6(a) indicates that, if at least one party is sufficiently productive, the benefit from this opportunistic behavior will outweigh its negative effect.

Control rights can also provide protection against risks. Both parties are uncertain about the inside agent’s ability (even the inside agent is uncertain about his or her own ability ex ante). The inclusion of the termination right in the contract allows one of the parties to renegotiate for a replacement by an outside agent when a more capable or lower cost outside agent is available. This possibility of replacement reduces the risk of having an inside agent whose ability is uncertain. This benefit is shared by both parties through Nash bargaining, which is built into our optimal renegotiation-proof contract. This explains Proposition 6(b).

Proposition 6(c) emphasizes the equal relevance of \( p_1 \) and \( p_2 \) even though the two parties’ positions are asymmetric. However, this equal relevance is for overall economic efficiency; income sharing and contractual terms are still in favor of the holder of the control right.

5. Concluding Remarks

As is well known, risk neutrality in income ensures the existence of an optimal linear contract. We show that risk neutrality in income also leads to the irrelevance of the allocation of control rights to investments and economic efficiency.

We have assumed Coasian bargaining. Without this assumption, based on Hart and Moore (2008) and Hart (2009), the expected welfare gain from renegotiation becomes \((1 - \lambda)g(a,b,\theta)\) for some constant \(\lambda \in (0, 1)\). With this change, we need to replace \(R(a,b)\) in Proposition 2 by \(\hat{R}(a,b) - \lambda G(a,b)\), but other results remains the same, where \(\hat{R}\) in Proposition 6 refers to that in Proposition 3. Since \(\hat{R}(a,b)\) in Proposition 2 is reduced to \(\hat{R}(a,b) - \lambda G(a,b)\) while no change in Proposition 3, the solution with the agent in control is less efficient than that with the principal in control. The explanation is simple: when the agent is in control, the principal has to negotiate with the agent when the principal wants to replace the agent, which results in some welfare loss; when the principal is in control, the principal does not need to renegotiate with the agent when the principal wants to replace the agent. This result suggests that, when renegotiation is likely to cause grievance of either or both sides, control rights should be given to the principal, otherwise they can be given to either side.

Although in this paper we discuss a specific right — the right of early termination — other kinds of control rights can be similarly analyzed. Also, although we call \(\theta\) the agent’s ability, it can certainly be used to represent other kinds of information that are only revealed over time.
Finally, our model serves as a benchmark for general models on control rights. It may help us understand how risk attitudes determine the optimal allocation of control rights in a more general model. For example, as we know, if the principal is less risk averse than the agent, the principal should share more risks; the question is: should the principal also be allocated more control rights? With risk neutrality, output risk does not matter. But with risk aversion, it does. If both the agent and the principal are risk averse, then on the one hand, the aversion against the risk of the external labor market will strengthen the principal’s preference for the right of exit; on the other hand, the aversion against the risk of losing the job will strengthen the agent’s preference for the right of exit. As in the current model, an optimal contract has to balance the incentives and the risks. Intuitively, giving the right to the agent improves the incentives for the agent, while giving the right to the principal improves the principal’s risk position. Hence, the answer to the question of who should have the right of exit will depend on a tradeoff between the agent’s incentives and the principal’s risk attitude.

**Appendix**

This appendix contains all the proofs and derivations.

**A.1. Proof of Lemma 2**

Since the IR condition must be binding, the first-best problem becomes

\[
\Pi^{**} \equiv \max_{s \in S, a \in A, b \in B} \hat{R}(a, b) - c(a) - C(b) - \bar{u}
\]

s.t. \( U(a, b, s, \delta) = \bar{u} + c(a) \).

This problem can be solved in two steps. We first find the optimal efforts \((a^{**}, b^{**})\) by maximizing the objective function without the constraint. Then, given the efforts, we consider a fixed contract \(s(x) = \beta\) that satisfies the constraint. We find

\[
\beta \int \int f(x, \theta, h(a^{**}, b^{**})) \phi(\theta) dx d\theta + \delta G(a^{**}, b^{**}) = \bar{u} + c(a^{**}),
\]

which immediately determines \(\beta\).

**A.2. Proof of Proposition 2**

From (3), we find

\[
\frac{\partial \hat{\theta}}{\partial a} = \frac{r_a(a, b, \theta_0) - r_a(a, b, \hat{\theta})}{r_a(a, b, \hat{\theta})}, \quad \frac{\partial \hat{\theta}}{\partial b} = \frac{r_b(a, b, \theta_0) - r_b(a, b, \hat{\theta})}{r_b(a, b, \hat{\theta})}.
\]

Since

\[
\begin{align*}
r_a(a, b, \theta) &= \int f_a \left( x, \theta, h(a, b) \right) dx, \\
r_b(a, b, \theta) &= \int f_b \left( x, \theta, h(a, b) \right) dx,
\end{align*}
\]
we have
\[
\frac{\hat{\theta}_a(a,b)}{h_a(a,b)} = \frac{\hat{\theta}_b(a,b)}{h_b(a,b)}.
\]
(15)

We also find
\[
G_a(a,b) = \left[r(a,b,\theta_0) - r(a,b,\hat{\theta}) - s_0\right] \phi(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial a} + \int_0^\theta \left[r_a(a,b,\theta_0) - r_a(a,b,\theta)\right] \phi(\theta)d\theta
\]
\[
= h_a \int_0^\theta \left\{ \int x f_a[x,\theta_0,h(a,b)] dx - \int x f_a[x,\theta,h(a,b)] dx \right\} \phi(\theta)d\theta,
\]
\[
G_b(a,b) = h_b \int_0^\theta \left\{ \int x f_b[x,\theta_0,h(a,b)] dx - \int x f_b[x,\theta,h(a,b)] dx \right\} \phi(\theta)d\theta,
\]
\[
\text{implying}
\]
\[
\frac{G_a(a,b)}{h_a(a,b)} = \frac{G_b(a,b)}{h_b(a,b)}.
\]
(16)

We also have
\[
\frac{R_a(a,b)}{h_a(a,b)} = \frac{R_b(a,b)}{h_b(a,b)}.
\]

Thus, (6) and (16) imply
\[
\frac{\hat{R}_a(a,b)}{h_a(a,b)} = \frac{\hat{R}_b(a,b)}{h_b(a,b)}.
\]

Therefore, the following two equations imply each other:
\[
\hat{R}_b(a,b) = C'(b) + \frac{h_b(a,b)}{h_a(a,b)} c'(a), \quad \hat{R}_a(a,b) = c'(a) + \frac{h_a(a,b)}{h_b(a,b)} C'(b).
\]

Since
\[
U_a(a,b,s,\delta) = h_a \int s(x) f_a[x,h(a,b)] \phi(\theta) dx d\theta + \delta G_a(a,b),
\]
\[
U_b(a,b,s,\delta) = h_b \int s(x) f_b[x,h(a,b)] \phi(\theta) dx d\theta + \delta G_b(a,b),
\]
by (16), we have
\[
U_b(a,b,s,\delta) = \frac{h_b}{h_a} U_a(a,b,s,\delta) - \frac{h_b}{h_a} \delta G_a(a,b) + \delta G_b(a,b) = \frac{h_b}{h_a} U_a(a,b,s,\delta).
\]

Thus,
\[
V_b(a,b,s,\delta) = \hat{R}_b(a,b) - U_b(a,b,s,\delta) = \frac{h_b}{h_a} \hat{R}_a(a,b) - \frac{h_b}{h_a} U_a(a,b,s,\delta) = \frac{h_b}{h_a} V_a(a,b,s,\delta).
\]

Therefore, the two IC conditions in problem (9) imply
\( c'(a) = U_a(a,b,s,\delta) = \hat{R}_a(a,b) - V_a(a,b,s,\delta) \)
\[= \hat{R}_a(a,b) - \frac{h_u}{h_b} V_b(a,b,s,\delta) = \hat{R}_a(a,b) - \frac{h_u}{h_b} C'(b). \] (17)

Also, since the IR condition in problem (9) must be binding, problem (9) becomes
\[
\Pi \equiv \max_{s \in S, a \in A, b \in B} \hat{R}(a,b) - c(a) - C(b) - \bar{u}
\]
s.t. \( \hat{R}_a(a,b) = c'(a) + \frac{h_u(a,b)}{h_b(a,b)} C'(b), \)
\[U_a(a,b,s,\delta) = c'(a), \]
\[U(a,b,s,\delta) = \bar{u} + c(a), \] (18)

This problem can be solved in two steps. We first find the optimal efforts \((\hat{a}^*, \hat{b}^*)\) from
\[
\Pi \equiv \max_{a \in A, b \in B} \hat{R}(a,b) - c(a) - C(b) - \bar{u}
\]
s.t. \( \hat{R}_a(a,b) = c'(a) + \frac{h_u(a,b)}{h_b(a,b)} C'(b). \) (19)

And then we find a contract that satisfies the following two conditions:
\[ U_a(a,b,s,\delta) = c'(a), \] (20)
\[ U(a,b,s,\delta) = \bar{u} + c(a), \] (21)

Conditions (20) and (21) can be written as
\[ c'(a) = h_u \int \int s(x) f(x, h(a,b)) \phi(\theta) dx d\theta + \delta G_a(a,b), \]
\[ \bar{u} + c(a) = \int \int s(x) f(x, h(a,b)) \phi(\theta) dx d\theta + \delta G(a,b). \]

Given efforts \((\hat{a}^*, \hat{b}^*)\) from problem (19), consider a linear contract of the form \( s(x) = \alpha x + \beta. \) Then, conditions (20) and (21) become
\[ c'(a) = \alpha R_a(a,b) + \delta G_a(a,b), \]
\[ \bar{u} + c(a) = \alpha R(a,b) + \beta + \delta G(a,b), \]
implying
\[ \alpha = \frac{c'(\hat{a}^*) - \delta G_a(\hat{a}^*, \hat{b}^*)}{R_a(\hat{a}^*, \hat{b}^*)}, \quad \beta = \bar{u} + c(\hat{a}^*) - \delta G(\hat{a}^*, \hat{b}^*) - \alpha R(\hat{a}^*, \hat{b}^*). \]
That is, there exists a linear contract satisfying (20) and (21).

Finally, by comparing the IC condition in problem (19) with condition \( \hat{R}_a(a,b) = c'(a) \) for the first best in Lemma 2, we immediately know that \((\hat{a}^*, \hat{b}^*)\) of (19) cannot achieve the first best.
A.3. Proof of Lemma 3

Since the IR condition must be binding, the first-best problem becomes

\[
\Pi^{**} = \max_{s \in S, a \in A, b \in B} \hat{R}(a, b) - c(a) - C(b) - \bar{u}
\]

s.t. \( U(a, b, s) = \bar{u} + c(a) \).

This problem can be solved in two steps. We first find the optimal efforts \((\hat{a}^{**}, \hat{b}^{**})\) by maximizing the objective function without the constraint. Then, given the efforts, we consider a fixed contract \(s(x) = \beta\), the simplest contract, that satisfies the constraint. We find

\[
\beta \int_{\hat{\theta}^{**}}^{\infty} \int f(x, \theta, h(\hat{a}^{**}, \hat{b}^{**})) \phi(\theta)d\theta dx = \bar{u} + c(\hat{a}^{**}),
\]

or

\[
\beta \int_{\hat{\theta}^{**}}^{\infty} \phi(\theta)d\theta = \bar{u} + c(\hat{a}^{**}),
\]

where \(\hat{\theta}^{**} \equiv \hat{\theta}(\hat{a}^{**}, \hat{b}^{**})\). This equation determines \(\beta\).

A.4. Proof of Proposition 3

We have

\[
U_a(a, b, s) = h_a \int_{\hat{\theta}}^{\infty} \int s(x)f_a[x, \theta, h(a, b)] \phi(\theta)d\theta dx - \phi(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial a} \int s(x)f[x, \hat{\theta}, h(a, b)]dx,
\]

\[
U_b(a, b, s) = h_b \int_{\hat{\theta}}^{\infty} \int s(x)f_b[x, \theta, h(a, b)] \phi(\theta)d\theta dx - \phi(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial b} \int s(x)f[x, \hat{\theta}, h(a, b)]dx.
\]

By (15),

\[
U_a(a, b, s) = \frac{h_b}{h_a} U_a(a, b, s).
\]

Thus,

\[
V_b(a, b, s) = \hat{R}_b(a, b) - U_b(a, b, s) = \frac{h_b}{h_a} \hat{R}_a(a, b) - \frac{h_b}{h_a} U_a(a, b, s) = \frac{h_b}{h_a} V_a(a, b, s).
\]

Therefore, the two IC conditions in problem (11) imply

\[
c'(a) = U_a(a, b, s) = \hat{R}_a(a, b) - V_a(a, b, s) = \hat{R}_a(a, b) - \frac{h_b}{h_a} V_b(a, b, s) = \hat{R}_a(a, b) - \frac{h_b}{h_b} C'(b).
\]

Also, since the IR condition in problem (11) must be binding, problem (11) becomes
\[
\Pi^* \equiv \max_{s \in S, a \in A, b \in B} \hat{R}(a, b) - c(a) - C(b) - \bar{u}
\]
\[
s.t. \quad \hat{R}_a(a, b) = c'(a) + \frac{h_a(a, b)}{h_b(a, b)} C'(b), \quad (24)
\]
\[
U_a(a, b, s) = c'(a), \quad U(a, b, s) = \bar{u} + c(a),
\]

This problem can be solved in two steps. We first find the optimal efforts \((\hat{a}^*, \hat{b}^*)\) from
\[
\Pi^* \equiv \max_{a \in A, b \in B} \hat{R}(a, b) - c(a) - C(b) - \bar{u}
\]
\[
s.t. \quad \hat{R}_a(a, b) = c'(a) + \frac{h_a(a, b)}{h_b(a, b)} C'(b). \quad (25)
\]

And then we find a contract that satisfies the following four conditions:
\[
U_a(a, b, s) = c'(a), \quad (26)
\]
\[
U(a, b, s) = \bar{u} + c(a), \quad (27)
\]

Conditions (26) and (27) are
\[
c'(a) = \frac{\partial}{\partial a} \int_{\theta}^{\infty} \int s(x) f(x, \theta, h(a, b)] \phi(d\theta) dxd\theta
\]
\[
\bar{u} + c(a) = \int_{\theta}^{\infty} \int s(x) f(x, \theta, h(a, b)] \phi(d\theta) dxd\theta.
\]

Given efforts \((\hat{a}^*, \hat{b}^*)\) from problem (25), consider a linear contract of the form \(s(x) = \alpha x + \beta\).

Then, conditions (26) and (27) become
\[
c'(\hat{a}^*) = \alpha D_a(\hat{a}^*, \hat{b}^*), \quad \bar{u} + c(\hat{a}^*) = \alpha D(\hat{a}^*, \hat{b}^*) + \beta[1 - \Phi(\hat{b}^*)],
\]
where \(\hat{b}^* = \hat{b}(\hat{a}^*, \hat{b}^*)\) and
\[
D(a, b) \equiv \int_{\theta(a, b)}^{\infty} r(a, b, \theta) d\Phi(\theta).
\]

That is, there exists a linear contract that satisfies (26) and (27) with
\[
\alpha = \frac{c'(\hat{a}^*)}{D_a(\hat{a}^*, \hat{b}^*)}, \quad \beta = \frac{\bar{u} + c(\hat{a}^*) - \alpha D(\hat{a}^*, \hat{b}^*)}{1 - \Phi(\hat{b}^*)}.
\]

Finally, by comparing the IC condition in problem (25) with condition \(R_a(a, b) = c'(a)\) for the first best in Lemma 3, we immediately know that \((\hat{a}^*, \hat{b}^*)\) of (25) cannot achieve the first best.
A.5. The Example

We find

\[ \hat{\theta} = \theta_0 - \frac{s_0}{h(a,b)}, \]
\[ r(a,b,\theta) = \theta h(a,b), \]
\[ R(a,b) = \bar{\theta} h(a,b), \]
\[ G(a,b) = h(a,b) \int_{\hat{\theta}}^{\bar{\theta}} \Phi(\theta) d\theta = \rho \left( \hat{\theta} - \theta_L \right) h(a,b) = \rho \left[ (\theta_0 - \theta_L) h(a,b) - s_o \right], \]
\[ \hat{R}(a,b) = \left[ \rho \theta_L + (1 - \rho) \theta_H + \rho (\theta_0 - \theta_L) \right] h(a,b) - \rho s_o = \bar{\theta} h(a,b) - \rho s_o, \]
\[ D(a,b) = h(a,b) \int_{\hat{\theta}(a,b)}^{\bar{\theta}(a,b)} \theta \phi(\theta) d\theta = (1 - \rho) \theta_H h(a,b). \]

The concavity of \( R(a,b) \) implies the concavity of \( \hat{R}(a,b), G(a,b) \) and \( D(a,b) \). Consider a general form

\[ R(a,b) = A + Bh(a,b), \]

with some constants \( A \) and \( B \). For the second-best problem, the IC condition is

\[ B \mu_1 = \gamma_1 a + \frac{\mu_1}{\mu_2} \gamma_2 b. \]

Then, the second-best problem is

\[ \Pi^* = \max_{a \in A, b \in B} A + Bh(a,b) - \gamma_1 a^2 - \frac{\gamma_2}{2} b^2 \]
\[ \text{s.t. } B = \frac{\gamma_1 a + \gamma_2 b}{\mu_1}. \]

The Lagrange function is

\[ L = A + Bh(a,b) - \gamma_1 a^2 - \frac{\gamma_2}{2} b^2 + \lambda \left( B \mu_1 - \gamma_1 a + \gamma_2 b - B \right) \]
\[ = \left( B \mu_1 + \frac{\gamma_1}{\mu_1} \right) a + \left( B \mu_2 + \frac{\gamma_2}{\mu_2} \right) b - \gamma_1 a^2 - \frac{\gamma_2}{2} b^2 + A. \]

The FOCs immediately imply

\[ a = B \frac{\mu_1}{\gamma_1} + \frac{\lambda}{\mu_1}, \quad b = B \frac{\mu_2}{\gamma_2} + \frac{\lambda}{\mu_2}. \]

Substituting these into the IC condition yields

\[ \lambda = -\frac{B}{\frac{\gamma_1}{\mu_1} + \frac{\gamma_2}{\mu_2}}. \]
Then,

\[ a^* = B \frac{\gamma_2 \mu_3}{\gamma_1 \gamma_2 \mu_2 + \gamma_3 \mu_1}, \quad b^* = B \frac{\gamma_1 \mu_3}{\gamma_2 \gamma_3 \mu_1 + \gamma_1 \mu_2}, \]

and

\[ \Pi^* = A + B(\mu_1 a^* + \mu_2 b^*) - \frac{\gamma_1}{2} (a^*)^2 - \frac{\gamma_2}{2} (b^*)^2 = A + \frac{B^2}{2} \left( \frac{\mu_2^2 + \mu_2^2}{\gamma_1 \gamma_2 \mu_2 + \gamma_1 \mu_2} - \frac{\mu_2^2 \mu_2}{\gamma_2 \mu_1^2 + \gamma_1 \mu_2^2} \right), \]

and

\[ h(a^*, b^*) = \mu_1 a^* + \mu_2 b^* = \frac{B}{\gamma_1 \gamma_2 \mu_2 + \gamma_1 \mu_2} \gamma_2^2 \mu_1^4 + \gamma_1^2 \mu_2^4. \]

Therefore, for the agency model without renegotiation, with \( A = 0 \) and \( B = \bar{\theta} \), we have

\[ a^* = \bar{\theta} \frac{\gamma_2 \mu_3}{\gamma_1 \gamma_2 \mu_2 + \gamma_3 \mu_1}, \]

\[ b^* = \bar{\theta} \frac{\gamma_1 \mu_3}{\gamma_2 \gamma_3 \mu_1 + \gamma_1 \mu_2}, \]

\[ \Pi^* = \frac{\bar{\theta}^2}{2} \left( \frac{\mu_2^2 + \mu_2^2}{\gamma_1 \gamma_2 \mu_2 + \gamma_1 \mu_2} - \frac{\mu_2^2 \mu_2}{\gamma_2 \mu_1^2 + \gamma_1 \mu_2^2} \right). \]

For the agency models with renegotiation, with \( A = -\rho s_o \) and \( B = \bar{\theta} \), we have

\[ \hat{a}^* = \bar{\theta} \frac{\gamma_2 \mu_3}{\gamma_1 \gamma_2 \mu_2 + \gamma_3 \mu_1}, \]

\[ \hat{b}^* = \bar{\theta} \frac{\gamma_1 \mu_3}{\gamma_2 \gamma_3 \mu_1 + \gamma_1 \mu_2}, \]

\[ \hat{\Pi}^* = \frac{\bar{\theta}^2}{2} \left( \frac{\mu_2^2 + \mu_2^2}{\gamma_1 \gamma_2 \mu_2 + \gamma_1 \mu_2} - \frac{\mu_2^2 \mu_2}{\gamma_2 \mu_1^2 + \gamma_1 \mu_2^2} \right) - \rho s_o. \]

References


