Staged Financing in Venture Capital:  
Moral Hazard and Risks

Susheng Wang and Hailan Zhou*

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Abstract

This paper investigates staged financing in an environment where an entrepreneur faces an imperfect capital market and an investor faces moral hazard and uncertainty. Staged financing plays two roles in this model: to control risk and to mitigate moral hazard. Using parametric functions and comparing staged financing with upfront financing, we discover a few interesting properties of staged financing. In particular, we show that when used together with a sharing contract, staged financing acts as an effective complementary mechanism to contracting in controlling agency problems.

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Our addresses are, respectively, Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, and Johnson School of Management, Cornell University, Ithaca, NY 14850, U.S.A.
1. Introduction

Staged financing has been widely used in venture capital, especially in the United States. Can staged financing be used by venture capitalists to reduce risks and to control moral hazard? Can staged financing improve efficiency, especially for highly promising firms?

One prominent characteristic of many new startup companies in high-tech industries is the high risk due to the great uncertainty about returns, the lack of substantial tangible assets and the lack of a track record in operations. Many high-tech startups may face many years of negative earnings before they start to see profits. According to Bergemann and Hege (1998), the fraction of such projects for which investors can successfully cash out, mostly through IPO’s, is twenty percent or less. Given this situation, banks and other intermediaries are reluctant to or even prohibited from lending money to such firms. Furthermore, these financial institutions usually lack expertise in investing in young and high-risk companies. Consequently, these startups often seek venture capitalists to be involved in their activities by offering revenue sharing in the form of equity joint ventures in order to obtain necessary funding and to benefit from the venture capitalists’ experience in management and finance.

Key characteristics in venture capital financing are staging the commitment of capital and preserving the option to abandon the project. Instead of providing all the necessary capital upfront, venture capitalists invest in stages to keep the project under control. Staged investment allows venture capitalists to monitor the firm before they make refinancing decisions. The information about the viability of a project acquired through such monitoring helps venture capitalists to avoid throwing money at bad projects. It reduces losses from inefficient continuation and creates an exit option for venture capitalists. The higher the risk in the project, the higher the value this option has to venture capitalists. This option to quit is similar to debt in that it limits potential financial losses.

Further, by monitoring and credibly threatening termination, venture capitalists also have better control over potential moral hazards. There may be several agency costs in a joint venture. If cash flows are not completely verifiable, entrepreneurs may appropriate investments. If effort is not verifiable, entrepreneurs may shirk job responsibilities. Also, if there are private benefits from continuing a project, entrepreneurs may keep the project going even if it has negative expected profits. Gompers (1995) provides an empirical study on the factors affecting the structure of staged financing when moral hazard exists. He shows that in financing high-risk companies with per-
asive moral hazards, staged financing allows venture capitalists to gather information and to monitor the progress of projects while maintaining the option to quit.

Our paper is among a few recent papers to offer a formal model for staged financing in controlling risks and moral hazard in venture capital. We find a closed-form solution, by which analysis of the complementary roles of staged financing and contracting can be made. We focus on the problems of moral hazard and uncertainty involved in financing new startups. Specifically, we consider a financially constrained entrepreneur (EN) and a venture capitalist (VC) who is interested in investing in the EN’s project. The project is risky and the EN’s effort is unverifiable. The EN faces an imperfect capital market and the VC is the only potential investor who understands the project (i.e., the VC knows the distribution function of the output). The VC offers a sharing contract and finances the project strategically in stages. Using parametric functions, we are able to derive some interesting properties of staged financing. Our results clearly show that, in addition to contracting, staged financing is an effective mechanism for venture capitalists to reduce agency costs and to control risks.

With the flexibility of staged financing, many projects, which may otherwise be abandoned under upfront financing, become profitable. We show that the efficiency of staged financing approaches the first best for highly promising firms. However, staged financing is not always dominant over upfront financing in terms of social welfare. When the project does not look very promising, staged financing is inferior to upfront financing. The reason is that VCs may underinvest in a project in the early stages when the project does not look very promising, which may cause a viable project to fail and result in a loss of social welfare.

There is a relatively small and recent literature on venture capital, which emphasizes the role of entrepreneurial incentives and studies how venture capital financing can mitigate incentive distortions. Sahlman (1990) discusses various aspects of venture capital, in which the agency problem is emphasized. Sahlman focuses on legal definitions, organizational forms and relationships among venture capitalists, investors and entrepreneurs. Our theoretical finding is consistent with Sahlman’s observation that staged financing is the most potent control mechanism a venture capitalist can employ to deal with the agency problem. Admati and Perry (1991) consider a situation in which enforceable contracts are not possible and investments are sunk as soon as they are committed. In such a case, staged financing is used as a way to reduce the cost of commitment. We, however, allow an enforceable revenue sharing contract. Since a firm’s revenue can be audited in reality, simple revenue sharing contracts are usually enforceable by the law. Admati and Pfleiderer (1994) consider robust contracting in
inducing an optimal continuation decision on investment. Their contract has the VC hold a fixed fraction of payoffs. In their model, the first investment is given, while the second investment is endogenous. In our model, the total amount of investment is given, while the distribution of investments across periods is endogenous. The second crucial difference is that while the VC provides financial investments, the EN in our model also provides an indispensable (non-financial) effort; such a role for the EN is not considered by Admati and Pfleiderer. Using a unique data set from Venture Economics, Hellmann (1994) argues that venture capitalists provide staged financing in order to avoid risk, but at the same time, staged financing triggers the EN’s short-term behavior. To deal with this short-termism, the venture capitalist is allocated a larger share, which induces an incentive to monitor the EN. We show that staged financing under equity financing can achieve approximately the first best for high-potential ventures, suggesting that the short-termism caused by staged financing is of minor consequence. Gompers (1995) provides an empirical study of venture capital when agency and monitoring costs exist. His empirical findings are consistent with our theoretical predictions of the roles of agency costs and the ex-post refinancing decision. Repullo and Suarez’s (1998) optimal contract resembles a convertible preferred stock in a model with double moral hazard and two-stage financing. In their model, the amount of investment in each period is exogenously given, while we endogenously solve the optimal allocation of funds across periods. It is the endogenity of the investment allocation in our model that plays a crucial role in mitigating the moral hazard. Bergemann and Hege (1998) emphasize the learning of the truth potential of a venture through staged financing. The effort input from the EN is not considered in their model, while an incentive condition for no stealing is considered. In our model, the EN’s unverifiable effort plays a crucial role. Neher (1999) studies staged financing in mitigating the EN’s commitment problem in a model without risks and the agency problem. By staging the investment, early rounds of investment create collateral that protects the VC’s claim from being renegotiated downward in the future. Complementary to Neher (1999), we avoid the commitment problem by not allowing renegotiation; instead, we focus on the agency problem in a risky project.

This paper proceeds as follows. In the next section, we define the model of venture-capital financing in an environment with uncertainty and moral hazard. In Section 3, we consider several variations of the model, including both the second-best and the first-best solutions under both staged financing and upfront financing with and without ex-post renegotiation. In Section 4, we discuss the properties and features of staged financing in our model. Finally, Section 5 concludes the paper.
2. The Model

2.1. The Project

Consider an innovative entrepreneur (EN) who relies on a venture capitalist (VC) for investment. The project lasts two periods. The EN’s effort, $x$, is applied throughout the two periods, and the VC provides a total investment, $k$, in the two periods. The cost of effort for the EN is $c(x)$. With effort input, $x$, from the EN throughout the two periods and with total capital input, $k$, from the VC in the two periods, the expected output is $F(x,k)$. The output faces a random shock, $\mu$, such that the realized output is $y = \mu F(x,k)$. The VC knows the distribution function of $\mu$ at the beginning of the first period (ex ante), and the shock is realized and publicly revealed at the beginning of the second period (ex post).

The VC offers to provide a total of $k_1$ in funds at the beginning of the first period. After the uncertainty is realized at the beginning of the second period, the VC considers providing a total of $k_2$ in funds. To realize the output by the end of the second period, the necessary amount, $k$, of investment must be made, i.e., $k_1 + k_2 \geq k$.

If the project is abandoned in the middle by either the EN or the VC, the project fails without any output and the initial investment $k_1$ is lost. The EN is indispensable to the project; without the EN, the project cannot succeed. The EN and VC share the revenue at the end of the project based on the existing contract.

2.2. Contracting

As in a typical agency model, we assume that only the output is contractible ex ante.\textsuperscript{1} In particular, the EN’s effort, $x$, is unverifiable and the VC’s investment strategy is non-contractible. The latter is consistent with the reality that the VC typically has the option of abandoning a project in the future. Thus, we have a model with double moral hazard.\textsuperscript{2}

Venture capital typically takes two forms of financing: equity financing or debt financing.\textsuperscript{3} Since the EN has no money to pay off a debt if the project fails, we will

\textsuperscript{1}The known justification for this is that contracts in reality are typically very simple and are generally based on some well-known aggregate measures.

\textsuperscript{2}Refer to Kim and Wang (1998) for a theoretical foundation of an agency model with double moral hazard.

\textsuperscript{3}Many financial intermediaries, such as banks, use debt financing simply because they are prohibited from using equity financing. Many researchers, such as Sahlman (1990), specifically define
consider equity financing only. In other words, the contract proposed by the VC is a sharing contract \((s, 1-s)\) on revenue/output, with revenue share \(s\) for the EN and \(1-s\) for the VC. Facing the EN’s effort, \(x\), and a random shock, \(\mu\), the VC offers a sharing contract and finances the project strategically in stages. The variables \(s\), \(x\), \(k_1\) and \(k_2\) are determined endogenously in equilibrium.

As we will show later, the optimal equity contract, assisted by staged financing, can approximately achieve the first best for high-potential firms. Thus, restricting to a sharing contract means little loss of efficiency, which may be the reason that a sharing contract is widely adopted in venture-capital financing.

As some information becomes available when \(\mu\) is realized ex post, the two parties may want to renegotiate the contract. We will consider both the cases of allowing and not allowing ex-post renegotiation. Renegotiation can ensure ex-post efficiency, but it causes ex-post opportunistic behaviors, which may result in ex-ante inefficiency. In other words, there is a tradeoff: no renegotiation may result in ex-post inefficiency, but renegotiation may result in ex-ante inefficiency. Consequently, economic agents in reality sometimes find ways to commit themselves to no-renegotiation. For example, governments in many circumstances enact laws to prevent renegotiation and economic agents may invite a third party to enforce the rule of no-renegotiation, especially in a repeated environment where reputation is important. In our theoretical analysis, we view both the cases to be equally important and thus consider both.

### 2.3. Timing

The timing of the events is illustrated in the following figure.

<table>
<thead>
<tr>
<th>Contract: (s)</th>
<th>Effort: (x)</th>
<th>Realization: (\mu)</th>
<th>Investment: (k_1)</th>
<th>Investment: (k_2)</th>
<th>Output: (\mu F(x, k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ex ante</td>
<td>Ex post</td>
<td>End</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. At \(t = 0\), the VC offers a sharing contract \((s, 1-s)\) to the EN and decides on an investment plan \((k_1, k_2)\); if the contract is accepted, the VC invests \(k_1\) and the EN applies effort \(x\) and incurs cost \(c(x)\).
2. At \( t = 1 \), the uncertainty is resolved. The VC considers the options to quit and to renegotiate. If the project is bad, she abandons the project without investing \( k_2 \). If the project is mediocre, she demands to renegotiate a new contract. If the project is good, she continues to invest in the project and provides the necessary investment \( k_2 \). The EN, on the other hand, would never want to renegotiate in our model, since once the project starts, his effort is sunk.

3. At \( t = 2 \), the project is finished, and the VC and EN divide the output based on the existing contract.

### 2.4. Assumptions

A few assumptions are needed for a tractable model. We first assume that the capital investment \( k \) is nondivisible. This means that the project requires a minimum amount, \( k \), of capital investment. In other words, \( k \) is a given constant, and the project is expected to generate output \( F(x, k) \) with effort \( x \) if and only if the capital investment is no less than \( k \). Venture capital financing focuses on the survival of a new product. To develop this new product, a certain amount of capital is required, which is our constant \( k \).

However, although \( k \) is fixed, the VC can divide the required amount into two installments, \( k_1 \) and \( k_2 \), for the two periods, and the VC also has the option to abandon the project ex post by not providing the planned second installment, \( k_2 \). Although the distribution of investments across the two periods does not affect output directly, it may affect the EN’s incentive to work and hence affect the output indirectly. The VC may benefit from the resolution of uncertainty by investing ex post. However, ex-post investment will impose a risk on the EN (from the VC’s options to quit and to renegotiate), which may lead to a lower effort. Therefore, the VC needs to choose the distribution of investments properly in order to balance the EN’s incentive to work and the benefit of late investment.

Assume there is a linear production function \( f(x) = \alpha x \), and the output is \( y = \mu f(x) \) with \( \mu \) being random ex ante and observable ex post.\(^4\) In order to have a

\(^4\)Since \( k \) is fixed, our production technology of the form \( ax - c(x) \) is actually equivalent to a production technology of the form \( h_1(x)h_2(k) - C(x) \) with increasing and concave \( h_i(\cdot) \) and convex \( C(\cdot) \). In particular, the separability among inputs \( x \) and \( k \) is a typical assumption for a production technology — the Cobb-Douglas technology is such an example.
closed-form solution, we will also use the following specific parametric functions:

\[
c(x) = \frac{1}{\beta} x^{\beta}, \quad \text{with} \quad \beta \geq 1.
\]

\[
\mu \sim U[0, 2],
\]

where \( U[0, 2] \) denotes the uniform distribution on interval \([0, 2]\). These two functional forms are not essential, but they lead to a unique closed-form solution, which allows us to arrive at a few clear-cut conclusions quickly. Here, a value of \( \mu < 1 \) means a contraction of output and a value of \( \mu > 1 \) means an expansion of output, and the costs in our equilibrium solutions converge to zero quickly as \( \beta \to \infty \).

Finally, assume that both the EN and the VC are risk neutral in income. For simplicity, also assume no discount of time preferences and no interest rate across periods.

3. Venture-Capital Financing

We examine several variations of the model in this section. In the first two subsections, we do not allow ex-post renegotiation. In the third subsection, we do not allow staged financing; in this case, renegotiation does not matter. In the last subsection, we allow ex-post renegotiation.

3.1. Staged Financing

We examine staged financing without renegotiation in this subsection. At the beginning of the second period, given the investment, \( k_1 \), the EN’s effort level, \( x \), and the resolved uncertainty, \( \mu \), the VC decides whether to continue investing in the project or to abandon it. Since the EN would always like to continue, only the VC’s refinancing decision matters. Let \( \bar{\mu} \equiv \frac{k - k_1}{\alpha (1 - \bar{s})} \). Implied by the VC’s ex-post individual rationality (IR) condition, the VC will provide the refinancing if \( \mu \geq \bar{\mu} \), otherwise the VC will stop investing in the project.

Thus, \( \bar{\mu} \) is the threshold of termination. A larger effort, \( x \), a smaller share, \( s \), and a larger initial investment, \( k_1 \), will all lower the threshold, implying a higher chance of refinancing.

Expecting the possibility of termination ex post, the EN’s expected profit from the
project is
\[ \Pi_{EN} = \int_{\mu}^{\infty} s\alpha \mu x g(\mu) d\mu - c(x). \]

Let \( X \) be the effort space and \( \mathbb{R}_+ \) be the capital space. Assuming that the EN’s reservation profit is zero, the VC’s ex-ante profit maximization problem is

\[
\Pi_{VC}^* = \max_{s \in [0, 1], k_1, k_2 \geq 0, x \in X} \int_{\frac{k_2}{\alpha x(1-s)}}^{\infty} [(1-s)\alpha \mu x - k_2] g(\mu) d\mu - k_1 \\
\text{s.t.} \quad \int_{\frac{k_2}{\alpha x(1-s)}}^{\infty} s\alpha \mu x g(\mu) d\mu \geq c(x), \\
\frac{\partial}{\partial x} \int_{\frac{k_2}{\alpha x(1-s)}}^{\infty} s\alpha \mu x g(\mu) d\mu = c'(x), \\
k_1 + k_2 \geq k.
\]

That is, the VC maximizes her ex-ante profit subject to her ex-post IR condition, \( \mu \geq \bar{\mu} \), the EN’s ex-ante IR condition (1a), the EN’s incentive compatibility (IC) condition (1b), and the capital requirement condition (1c). We call the solution in this case the second best.

Note that without renegotiation, the VC’s ex-post profit may not be maximized. This restriction eliminates ex-post opportunistic behavior from the VC and avoids an ex-ante incentive reaction from the EN. This restriction will be dropped in Subsection 3.4.

Problem (1) is found to have a unique closed-form solution in the following. The derivations for this solution and all other solutions are presented in the appendix.

**Solution 1.** In the case of staged financing without renegotiation, the second-best solution is

\[
s^* = \frac{1}{\beta}, \\
x^* = \left( \frac{2\alpha}{1+\beta} \right)^{\frac{1}{\beta-1}}, \\
k_2^* = \frac{\beta-1}{\beta} \left( \frac{2\alpha}{1+\beta} \right)^{\frac{\beta}{\beta-1}} \sqrt{\beta^2 - 1}, \\
\Pi_{VC} = (\beta - 1) \left( \frac{2\alpha}{1+\beta} \right)^{\frac{\beta}{\beta-1}} - k.
\]
3.2. The First Best

We now suppose that the EN’s effort is verifiable. Without moral hazard, the VC can demand an effort level in the contract without providing sufficient inducement and staged financing is used only to control risk. We call the solution in this case the first best.

By dropping the IC condition in (1), the VC’s problem becomes

\[
\Pi_{VC}^{**} = \max_{s \in [0, 1], k_1, k_2 \geq 0, x \in \mathbb{X}} \int_{\frac{k_1}{\mu x(1-x)}}^{\infty} \left[(1-s)\alpha \mu x - k_2\right]g(\mu)d\mu - k_1
\]

s.t. \[\int_{\frac{k_2}{\mu x(1-x)}}^{\infty} s\alpha \mu x g(\mu)d\mu \geq c(x), \quad (2a)\]

\[k_1 + k_2 \geq k. \quad (2b)\]

This problem (2) is found to have a unique closed-form solution also, which is

**Solution 2.** In the case of staged financing without renegotiation, the first-best solution is

\[s^{**} = \frac{1}{\beta},\]

\[x^{**} = \left(\frac{2\alpha}{\beta}\right)^{\frac{1}{\alpha - 1}},\]

\[k_2^{**} = \left(\frac{2\alpha}{\beta}\right)^{\frac{1}{\alpha - 1}} (\beta - 1) \sqrt{1 - \frac{2}{\beta}},\]

\[\Pi_{VC}^{**} = \frac{(\beta - 1)^2}{\beta} \left(\frac{2\alpha}{\beta}\right)^{\frac{2}{\alpha - 1}} - k.\]

From the IR condition (2a), we can see that the VC can either make a higher contract payment \(s\) or a higher initial investment \(k_1\) to keep the EN interested in the project. Thus, the first-best solution does not imply that the VC should withhold all investment until the second period.

**Remark 1.** We have not imposed a minimum initial capital requirement condition such as \(k_1 \geq k_1^*\) for a given \(k_1^* > 0\) in our model. This will, however, not affect our main conclusions. For example, for Solution 1, with the minimum initial capital requirement condition, we need \(\frac{\beta - 1}{\beta} \left(\frac{2\alpha}{1+\beta}\right)^{\frac{2}{\alpha - 1}} \sqrt{\beta^2 - 1} \leq k_1 + k\), which amounts to a minor restriction on the parameters.
Remark 2. We have implicitly assumed that all the investments in the firm are turned into firm-specific (physical) assets. That is, we have assumed a zero liquidation value for the firm when it is abandoned ex post. The solutions will not change much if we assume a non-zero liquidation value for the firm. Specifically, given the initial investment $k_1$, suppose that the firm can be liquidated ex post for the value $\lambda k_1$, where $0 \leq \lambda < 1$. To be consistent with reality, we assume that the VC gets the liquidation value. Then, given the ex-ante contract, the firm will be liquidated ex post if and only if \( \mu < \bar{\mu} \equiv \frac{k - (1-\lambda)k_1}{\alpha x(1-s)} \). Hence, the ex-ante expected utility values are

\[
\Pi_{EN} = \int_{\bar{\mu}}^{\infty} s \alpha x \mu g(\mu) d\mu,
\]

\[
\Pi_{VC} = \int_{0}^{\bar{\mu}} \lambda k_1 g(\mu) d\mu + \int_{\bar{\mu}}^{\infty} [(1-s) \alpha x \mu - k_2] g(\mu) d\mu - k_1.
\]

We find that the second-best and first-best solutions in this case are, respectively, the same as the second-best and first-best solutions listed in Solutions 1 and 2, except that we should replace the optimal second-period capital investment by $\frac{k_2 - \lambda k}{1-\lambda}$ and $\frac{k_2^* - \lambda k}{1-\lambda}$, respectively, where $k_2^*$ and $k_2^{**}$ are defined in Solutions 1 and 2 (see Appendix).

3.3. Upfront Financing

In this subsection, we consider the case in which the VC is required to invest all the required capital, $k$, upfront. In this case, the VC loses the option to quit. She only has the binary choice of whether or not to finance at the beginning of the project. The effort is unverifiable and the sharing contract is now the only instrument that can be used to control moral hazard.

By dropping the capital requirement condition and the VC’s ex-post IR condition, $\mu \geq \bar{\mu}$, in (1), the VC’s problem becomes

\[
\bar{\Pi}_{VC} = \max_{s \in [0,1], x \in \mathbb{X}} \int_{0}^{\infty} (1-s) \alpha x \mu g(\mu) d\mu - k
\]

s.t. \( \int_{0}^{\infty} s \alpha x \mu g(\mu) d\mu \geq c(x), \) \( \frac{\partial}{\partial x} \int_{0}^{\infty} s \alpha x \mu g(\mu) d\mu = c'(x). \)

This problem (3) is found to have a unique closed-form solution, which is
Solution 3. In the case of upfront financing, the second-best solution is

\[ s^* = \frac{1}{\beta}, \]
\[ x^* = \left( \frac{\alpha}{\beta} \right)^{\frac{1}{\xi}}, \]
\[ \Pi_{VC}^* = (\beta - 1) \left( \frac{\alpha}{\beta} \right)^{\frac{\beta}{\sigma-\xi}} - k. \]

Unlike in staged financing, the EN's IR condition in upfront financing is not binding. The EN is able to earn part of the surplus ex ante. The reason that the VC cannot expropriate all the surplus is that, in the case of upfront financing, she only has one instrument, contracting on revenue shares, to control moral hazard and to reduce risks. With staged financing, the VC has two instruments, contracting at \( t = 0 \) and refinancing at \( t = 1 \). Staged financing gives the VC additional power, which allows her to expropriate all the ex-ante surplus.

3.4. Renegotiation

In this subsection, we allow the possibility of renegotiation ex post. When the uncertainty is resolved ex post, the project may turn out to be bad or mediocre so that the VC is unwilling to continue investing given the existing contract. However, the project may still be socially viable. In this case, the EN has no alternative but to renegotiate with the VC for a new contract. We assume that the VC has the control rights so that she gets all the efficiency gain from ex-post renegotiation.

In the real world, when the project turns out to be bad, it will be abandoned; when it turns out to be mediocre, the VC will take over the control of the company and she will usually fire the EN and replace him by a professional manager from the market. Thus, our assumption of VC control is realistic, although, as indicated later by Remark 3, it is not necessary.

Specifically, there are three possible cases:

1. When \( \mu \geq \frac{k_2}{\alpha \varepsilon (1-s)} \), the project continues without renegotiation. Renegotiation cannot increase ex-post efficiency in this case and thus one of the parties will refuse to renegotiate.
2. When \( \frac{k_2}{ax} \leq \mu < \frac{k_2}{ax(1-s)} \), the project is still socially viable and the VC needs a new contract to continue financing. In this case, the VC must be provided with extra compensation and the EN has no alternative but to yield to the VC’s demands. Having the control rights, the VC takes over the firm and captures all the efficiency gain \( \alpha x \mu - k_2 \).

3. When \( \mu < \frac{k_2}{ax} \), the project is no longer socially viable and it will be terminated.

Although renegotiation will not have an impact under upfront financing, it will generally have an impact under staged financing. As shown later, in some of our main conclusions, the possibility of ex-post renegotiation can make a crucial difference, while in others it does not do so.

With renegotiation, the VC’s second-best problem under staged financing becomes

\[
\hat{\Pi}_{VC}^* \equiv \max_{s \in [0, 1], k_2 \geq 0, x \in X} \int_{\frac{k_2}{ax(1-s)}}^{\infty} [(1-s)\alpha x \mu - k_2] g(\mu) d\mu + \int_{\frac{k_2}{ax}}^{\infty} (\alpha x \mu - k_2) g(\mu) d\mu - k_1 \\
\text{s.t.} \int_{\frac{k_2}{ax(1-s)}}^{\infty} s\alpha x g(\mu) d\mu \geq c(x), \\
\frac{\partial}{\partial x} \int_{\frac{k_2}{ax(1-s)}}^{\infty} s\alpha x g(\mu) d\mu = c'(x).
\]

The second-best and first-best problems are found to have unique closed-form solutions also, which are:

**Solution 4.** In the case of staged financing with renegotiation, the second-best solution is

\[
\hat{s}^* = \frac{2}{\beta} \frac{1}{1 + \sqrt{1 - 2 \frac{(\beta-1)(2\beta-1)}{\beta^2}}}, \\
\hat{x}^* = \left[ \frac{4\alpha}{1 + \beta} \frac{1}{1 + \sqrt{1 - 2 \frac{(\beta-1)(2\beta-1)}{\beta^2}}} \right]^{\frac{1}{\beta-1}}, \\
\hat{k}_2^* = 2\alpha \hat{x}^*(1 - \hat{s}^*) \sqrt{\frac{\beta - 1}{\beta + 1}}, \\
\hat{\Pi}_{VC}^* = \frac{2\alpha \beta(\beta - 1)}{(1 + \beta)(2\beta - 1)} \hat{x}^*(2 - \hat{s}^*) - k.
\]
Solution 5. In the case of staged financing with renegotiation, the first-best solution is

\[
\hat{s}^{**} = \frac{1}{3} + \sqrt{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt{\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}},
\]

\[
\hat{x}^{**} = \left(\frac{2\alpha\beta(\hat{s}^{**})^2}{1 + \hat{s}^{**}}\right)^{\frac{1}{\beta - 1}},
\]

\[
\hat{k}_2^{**} = 2\alpha\hat{x}^{**}(1 - \hat{s}^{**}) \sqrt{\frac{1 - \hat{s}^{**}}{1 + \hat{s}^{**}}},
\]

\[
\bar{\Pi}_{VC}^{**} = \alpha\hat{x}^{**}\left[2 + \left(\hat{s}^{**}\right)^2\right] \frac{(1 - \hat{s}^{**})}{1 + \hat{s}^{**}} - k,
\]

where \( p \equiv \frac{4\beta + 1}{3(2\beta - 1)} \) and \( q \equiv \frac{14\beta - 52}{27(2\beta - 1)} \).

Remark 3. Once renegotiation is allowed in a model, things can become very complicated. With ex-post renegotiation, the equilibrium solution may depend on the allocation of (ex-post) control rights and the distribution of (ex-post) bargaining power. What is an appropriate allocation of control rights or should we consider the optimal allocation of control rights? Fortunately, as shown by Wang (2002), in agency models with risk neutrality in income, control rights and bargaining power are irrelevant in the sense that the allocation of control rights and the distribution of bargaining power will not affect ex-ante efficiency and optimal investments/efforts. Thus, we have the freedom to arbitrarily assign the control rights to the VC in our model, which happens to be consistent with reality.
## 4. Analysis

To understand the role of staged financing in alleviating moral hazard and in sharing risks, we consider five different cases. Their solutions are summarized in the following table, where social welfare is defined by $SW = \Pi_{VC} + \Pi_{EN}$.

<table>
<thead>
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We will use SF, UF, SB and FB to stand for staged financing, upfront financing, the second best and the first best, respectively.

### Proposition 1. (Efficiency). With or without renegotiation, the solution under staged financing approaches the first best when $\beta$ goes to infinity.

Proposition 1 suggests that staged financing, coupled with a sharing contract, can approximately achieve the first best. This may explain why many joint ventures use the simple form of equity sharing, since an equity joint venture may approximately achieve the first best with the aid of staged financing. This is also the reason that we restrict ourselves to a sharing contract since the first best can be approximately obtained even if the contract itself may not be optimal. An optimal contract may take a much more complicated form, but, with the aid of staged financing, we have little loss of efficiency.

The comparison of the three social welfare curves is shown in Figure 1. The top line is the social welfare for the first best under staged financing; the middle line is the social welfare for the second best under staged financing; and the bottom line is the
social welfare for the second best under upfront financing. The effort level and welfare under staged financing approach the first-best solution quickly as \( \beta \) gets larger.

![Figure 1. Efficiency](image)

The intuition is clear. The \( \beta \) coefficient measures the strength of the moral hazard problem, with higher values of \( \beta \) signifying a weaker entrepreneurial moral hazard. This explains why as \( \beta \to \infty \), the difference in efficiency between the second best and the first best diminishes.

**Proposition 2. (Social Welfare).** Without renegotiation, there exists a \( \beta > 1 \) such that, if and only if \( \beta > \beta \), we have \( SW_{SB-SF} > SW_{SB-UF} \). With renegotiation, we have \( SW_{SB-SF} > SW_{SB-UF} \) for any \( \beta > 1 \).\(^5\)

Figure 2 presents a graphic illustration of Proposition 2 when renegotiation is not allowed. It suggests that when the moral hazard is serious (with a small \( \beta \)), upfront financing is better. The reason is that while the VC prefers staged financing, the EN prefers upfront financing. The EN has zero welfare under staged financing, but he has positive welfare and substantially more effort under upfront financing. Thus, there may be cases in which the balance tips towards the EN and upfront financing becomes better. More specifically, given effort \( x \), we have \( \Pi_{VC} = (\beta - 1)x^\beta - k \) for both the cases of staged financing without renegotiation and upfront financing. The VC prefers staged financing to upfront financing since \( x^* \) is larger than \( \bar{x}^* \). However, as \( \beta \) gets smaller, \( x^*/\bar{x}^* \to 1 \) and this preference becomes weaker until the EN’s preference for upfront financing makes a difference.

\(^5\)Here, the other parameters \( \alpha \) and \( k \) are irrelevant in the comparisons.
Figure 2. Welfare Comparison: SF vs. UF

On the other hand, ex-post renegotiation gives the VC an extra mechanism to control moral hazard. With an improvement in ex-post efficiency from saving a mediocre project, the VC can offer a higher share to the EN to induce more effort. This mechanism becomes particularly important when the moral hazard problem is serious (with a small $\beta$), and, as a consequence, it results in a sufficient improvement of ex-ante efficiency in this case so that staged financing continues to be better than upfront financing.

Proposition 3. (Incentives). With or without renegotiation, the effort level under staged financing is higher than that under upfront financing.

A comparison of the three effort levels is shown in Figure 3. A higher effort level in staged financing can be easily understood. Upfront financing involves single moral hazard, while staged financing involves double moral hazard. Staged financing introduces a threat that the VC may possibly abandon the project, which induces the EN to work extra hard in an attempt to provide the VC with incentives for ex-post refinancing. In other words, the moral hazard on the part of the VC under staged financing will have an extra impact on the EN’s incentives, which induces a higher effort level.
Proposition 4. (Contract). Without renegotiation, the optimal shares in all the cases are the same. With renegotiation, the second-best share for the EN is higher than the first-best share.

Proposition 4 suggests that without the option of ex-post renegotiation, the contract plays only a marginal role in controlling moral hazard because the contract is not adjusted in very different situations. Thus, as a complementary mechanism to contracting, staged financing must play a crucial role in controlling moral hazard. This is further verified by the fact that the VC commits less initial investment under the second best than that under the first best. More late investment imposes a pressure on the EN to input more effort. On the other hand, with the option of ex-post renegotiation, since the VC is in control, she is willing to offer a higher share to the EN, which induces more effort from the EN.

Proposition 4 can be understood from a cost-benefit analysis of the two different mechanisms. There are costs to the VC for using staged financing and a sharing contract. The cost of staged financing is the risk on the early investment; the cost of a sharing contract is a lower share for the VC. When the marginal cost of using staged financing is always lower than the marginal cost of using a sharing contract, the VC will rely heavily on her financing strategy and leave her contracting option unchanged. Only when the VC has the option to renegotiate will she use the contracting option as well.

Interestingly, Sahlman’s (1990) and Admati and Pfleiderer’s (1994) findings are consistent with our Proposition 4. Admati and Pfleiderer (1994) derive robust finan-
cial contracts when lead venture capitalists are better informed than others. They demonstrate that a contract in which the lead VC maintains a constant fraction of the firm’s equity is the only form of financing that is robust to small changes in possible outcomes. Sahlman (1990) finds that “the most important mechanism to controlling the venture is staging the infusion of capital”.

In addition to the above properties, there are also a few other more intuitive properties of staged financing. First, with the flexibility of staged financing, a project will generally generate a higher expected value to the VC. Thus, projects that are not profitable under upfront financing may become profitable under staged financing.

Second, the threshold $\bar{\mu}$ is higher under staged financing than under the first best. In other words, moral hazard creates a greater possibility for the project to be abandoned in the middle of its development.

Third, when $\beta$ is larger, since the incentive problem becomes less important, the revenue share for the EN is smaller.

Finally, the VC is inclined to provide less early investment under moral hazard. Since late investment imposes a pressure on the EN, the VC uses her investment patterns to control moral hazard.

5. Concluding Remarks

Staged financing is a widely adopted form of investment in venture capital. In an environment where an entrepreneur faces an imperfect capital market and an investor faces uncertainty and moral hazard, we study how staged financing is used to mitigate moral hazard and to reduce risks. Under certain assumptions, we obtain some unique results on the performance and role of staged financing, which may enhance our understanding of venture capital.

We find that staged financing can achieve high efficiency, especially for highly promising ventures. Besides the benefit of reducing risks through late investment, as a complementary mechanism to contracting, staged financing plays a crucial role in controlling moral hazard. In particular, staged financing induces a higher effort from the entrepreneur. However, for less promising ventures, upfront financing may be a socially better choice, since initial underinvestment by the venture capitalist may overkill a viable venture.
While our paper refers to venture capital, staged financing has been used in many different circumstances. For example, the sequential investment in R&D and the staged installment of payments in procurement all involve staged investment over multiple periods. Our model should be readily applicable to those cases involving investment over time.

In reality, a venture capitalist’s financing plan may involve many options in a complex environment. Control variables may include the number of periods, the duration of each period, and the amount to be invested in each period. In addition, a venture capitalist can also consider how intensively to monitor a firm, when to go public with the firm, and how to arrange control rights properly. A more comprehensive study of staged financing is needed for a better understanding of these issues. For discussion of some of these issues, see some recent theoretical papers on venture capital such as Hellmann (1998), Marx (1998), Cornelli and Yosha (2001), Dessi (2001), and Schmidt (2002).
Appendix

A.1. Derivation of Solution 1

With $g(\mu) = \frac{1}{2}$ for $\mu \in [0, 2]$, we have

$$\Pi_{EN} \equiv \int_{\frac{k_2}{\alpha x(1-s)}}^{\infty} s \alpha x g(\mu) d\mu = \alpha s x - \frac{s k_2^2}{4 \alpha x^2 (1-s)^2},$$

$$\frac{\partial \Pi_{EN}}{\partial x} = \alpha s + \frac{s k_2^2}{4 \alpha x^2 (1-s)^2},$$

$$\Pi_{VC} \equiv \int_{\frac{k_2}{\alpha x(1-s)}}^{\infty} [(1-s) \alpha x - k_2] g(\mu) d\mu = \alpha x (1-s) + \frac{k_2^2}{4 \alpha x (1-s)} - k.$$

The IR condition must be binding. Then, problem (1) becomes

$$\Pi_{VC} = \max_{s \in [0, 1], k_2 \geq 0, x \in \mathbb{R}} \alpha x (1-s) + \frac{k_2^2}{4 \alpha x (1-s)} - k \quad (A1)$$

s.t. $\alpha s - \frac{s k_2^2}{4 \alpha x^2 (1-s)^2} = \frac{1}{\beta} x^{\beta-1},$

$$\alpha s + \frac{s k_2^2}{4 \alpha x^2 (1-s)^2} = x^{\beta-1}.$$

From the two constraints, we find

$$s = \frac{1 + \beta}{2 \alpha \beta} x^{\beta-1}. \quad (A2)$$

Then, the IR condition implies

$$\left( \frac{k_2}{1-s} \right)^2 = \frac{4 \alpha x^2}{s} (x^{\beta-1} - \alpha s) = 4 \alpha x^2 \frac{\beta - 1}{\beta + 1}, \quad (A3)$$

implying

$$k_2 = 2 \alpha x (1-s) \sqrt{\frac{\beta - 1}{\beta + 1}} = \left( 2 \alpha x - \frac{1 + \beta}{\beta} x^{\beta} \right) \sqrt{\frac{\beta - 1}{\beta + 1}}. \quad (A4)$$

Then, by (A2) and (A3),

$$\Pi_{VC} = \alpha x (1-s) \left[ 1 + \frac{1}{4 \alpha^2 x^2} \left( \frac{k_2}{1-s} \right)^2 \right] - k = \frac{2 \alpha \beta}{\beta + 1} x - x^{\beta} - k. \quad (A5)$$

The first-order condition (FOC) for $x$ immediately implies

$$x^* = \left( \frac{2 \alpha}{1 + \beta} \right)^{\frac{1}{\beta-1}}.$$

Then, (A2), (A4) and (A5) immediately give us $s^*, \ k_2^*$ and $\Pi_{VC}$ in Solution 1.
A.2. Derivation of Solution 2

We now drop the IC condition from problem (A1) and it becomes

\[
\Pi^{**}_{VC} = \max_{s \in [0, 1], k_2 \geq 0, x \in \mathbb{X}} \alpha x (1 - s) + \frac{k_2^2}{4\alpha x (1 - s)} - k \tag{A6}
\]

s.t. \( \alpha x - \frac{sk_2^2}{4\alpha x (1 - s)^2} = \frac{1}{\beta} x^\beta \). \tag{A7}

By the IR condition (A7),

\[
\Pi_{VC} = 2\alpha x (1 - s) - \frac{1 - s}{\beta s} x^\beta - k.
\]

Thus, we can solve problem (A6) in two steps: we first solve the following problem without the IR constraint:

\[
\Pi^{***}_{VC} = \max_{s \in [0, 1], x \in \mathbb{X}} 2\alpha x (1 - s) + \left(1 - \frac{1}{s}\right) \frac{1}{\beta} x^\beta - k. \tag{A8}
\]

Afterwards, we solve for \( k_2 \) from the IR constraint. The FOCs for (A8) immediately give us \( s^{**} \) and \( x^{**} \) in Solution 2. Substituting them into (A8) gives us \( \Pi^{**}_{VC} \). Then, the IR condition implies \( k_2^{**} \).

A.3. Derivation of the Solutions in Remark 2

The Second Best

We have

\[
\Pi_{EN} = \alpha x s - \frac{s}{4\alpha x} \left[ \frac{\lambda k + (1 - \lambda) k_2}{1 - s} \right]^2,
\]

\[
\frac{\partial \Pi_{EN}}{\partial x} = \alpha s + \frac{s}{4\alpha x^2} \left[ \frac{\lambda k + (1 - \lambda) k_2}{1 - s} \right]^2,
\]

\[
\Pi_{VC} = (1 - s) \alpha x + \frac{1 - s}{4\alpha x} \left[ \frac{\lambda k + (1 - \lambda) k_2}{1 - s} \right]^2 - k.
\]

The IR condition must be binding. Then, the problem becomes

\[
\Pi^{**}_{VC} = \max_{s \in [0, 1], k_2 \geq 0, x \in \mathbb{X}} (1 - s) \alpha x + \frac{1 - s}{4\alpha x} \left[ \frac{\lambda k + (1 - \lambda) k_2}{1 - s} \right]^2 - k \tag{A9}
\]

s.t. \( \alpha s - \frac{s}{4\alpha x^2} \left[ \frac{\lambda k + (1 - \lambda) k_2}{1 - s} \right]^2 = \frac{1}{\beta} x^{\beta - 1} \),

\( \alpha s + \frac{s}{4\alpha x^2} \left[ \frac{\lambda k + (1 - \lambda) k_2}{1 - s} \right]^2 = x^{\beta - 1} \).
Let $\tilde{k}_2 \equiv \lambda k + (1 - \lambda) k_2$. Then, problem (A9) becomes

$$\Pi_{VC}^* = \max_{s \in [0, 1], k_2 \geq 0, x \in \mathbb{X}} (1 - s) \alpha x + \frac{\tilde{k}_2^2}{4\alpha x (1 - s)} - k \quad \text{subject to} \quad \alpha s - \frac{s \tilde{k}_2^2}{4\alpha x^2 (1 - s)^2} = \frac{1}{\beta} x^{\beta - 1},$$

$$\alpha s + \frac{s \tilde{k}_2^2}{4\alpha x^2 (1 - s)^2} = x^{\beta - 1}.$$

We can see that (A10) is the same as problem (A1) and thus the solution for $(s, x, \tilde{k}_2, \Pi_{VC})$ is the same as Solution 1, which implies

$$k_s^* = \frac{\tilde{k}_2^* - \lambda k}{1 - \lambda} = \frac{1}{1 - \lambda} \left[ \frac{\beta - 1}{\beta} \left( \frac{2\alpha}{1 + \beta} \right)^{\frac{\beta}{1 - \beta}} \sqrt{\beta^2 - 1 - \lambda k} \right].$$

**The First Best**

Problem (A10) becomes

$$\Pi_{VC}^* = \max_{s \in [0, 1], k_2 \geq 0, x \in \mathbb{X}} (1 - s) \alpha x + \frac{\tilde{k}_2^2}{4\alpha x (1 - s)} - k \quad \text{subject to} \quad \alpha s - \frac{s \tilde{k}_2^2}{4\alpha x^2 (1 - s)^2} = \frac{1}{\beta} x^{\beta - 1}.$$

We can see that (A10) is the same as problem (A6) and thus the solution for $(s, x, \tilde{k}_2, \Pi_{VC})$ is the same as Solution 2, which implies

$$k_s^{**} = \frac{\tilde{k}_2^{**} - \lambda k}{1 - \lambda} = \frac{1}{1 - \lambda} \left[ \left( \frac{2\alpha}{\beta} \right)^{\frac{\beta}{1 - \beta}} (\beta - 1) \sqrt{1 - \frac{2}{\beta} - \lambda k} \right].$$

**A.4. Derivation of Solution 3**

Problem (3) is

$$\bar{\Pi}_{VC}^* = \max_{s \in [0, 1], x \in \mathbb{X}} (1 - s) \alpha x - k \quad \text{subject to} \quad \alpha s x \geq \frac{1}{\beta} x^{\beta},$$

$$\alpha s x = x^{\beta - 1}.$$
Since the IC condition implies the IR condition, the problem can be reduced to

$$\max_{s \in [0, 1], x \in X} \quad (1-s)\alpha x - k$$
$$\text{s.t.} \quad \alpha s = x^{\beta-1},$$

which can be reduced further to

$$\bar{\Pi}_C^* = \max_{x \in X} (\alpha - x^{\beta-1})x - k,$$

which immediately implies $\bar{x}^*$ and then $\bar{\Pi}_C^*$ and $\bar{s}^*$ in Solution 3.

**A.5. Derivation of Solution 4**

The IR condition must be binding, which implies

$$\frac{1}{\beta} x^\beta = \int_{\frac{k_2}{\alpha x (1-s)}}^{\infty} s\alpha \mu x g(\mu) d\mu = s\alpha x - \frac{sk_2^2}{4\alpha x (1-s)^2}. \quad (A11)$$

Hence, the IC condition implies

$$x^\beta = \alpha x s + \frac{sk_2^2}{4\alpha x (1-s)^2}. \quad (A12)$$

By (A11) and (A12),

$$s = \frac{1}{2\alpha\beta} x^{\beta-1}. \quad (A13)$$

By (A12) and (A13),

$$\bar{\Pi}_C = \int_{\frac{k_2}{\alpha x (1-s)}}^{\infty} [(1-s)\alpha x - k_2] g(\mu) d\mu + \int_{\frac{k_2}{\alpha x (1-s)}}^{\frac{k_2}{\alpha x (1-s)}} (\alpha x - k_2) g(\mu) d\mu - k_1$$

$$= \alpha x (1 - s) + \frac{(1-s + s^2)k_2^2}{4\alpha x (1-s)^2} - k$$

$$= \frac{2\alpha\beta}{1+\beta} x - x^\beta + \frac{\beta^2 - 1}{4\alpha\beta^2} x^{2\beta-1} - k.$$

The FOC on $x$ is then

$$\frac{2\alpha\beta}{1+\beta} - \beta x^{\beta-1} + \frac{(\beta^2 - 1)(2\beta - 1)}{4\alpha\beta^2} x^{2\beta-2} = 0,$$

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implying
\[ x^{\beta-1} = \frac{4\alpha}{1 + \beta} \frac{1}{1 + \sqrt{1 - 2 (\beta - 1) (2\beta - 1)/\beta^3}}. \]

The second-order condition (SOC) on \( x \) indicates that one is the minimum solution and the other is the maximum solution. The maximum solution is
\[ \hat{x}^* = \left[ \frac{4\alpha}{1 + \beta} \frac{1}{1 + \sqrt{1 - 2 (\beta - 1) (2\beta - 1)/\beta^3}} \right] \frac{1}{\beta}. \]

Then, by (A13), we find \( \hat{s}^* \). By (A12) and (A13), we further find \( \hat{k}^* \) and \( \Pi_{VC}^* \).

### A.6. Derivation of Solution 5

Again, since the IR condition must be binding, we have (A11). Then, by (A11),
\[ \Pi_{VC} = \alpha x (1 - s) + \frac{(1 - s + s^2)k_2^2}{4\alpha x (1 - s)^2} - k = \alpha x (2 - 2s + s^2) - \frac{1 - s + s^2}{s} \frac{1}{\beta} x^{\beta} - k. \]

We can thus solve the problem in two steps. We first solve the following problem for \( (s, x) \) without the IR constraint:
\[ \Pi_{VC}^* \equiv \max_{s \in [0, 1], x \in \mathbb{R}} \alpha x (2 - 2s + s^2) - \frac{1 - s + s^2}{s} \frac{1}{\beta} x^{\beta} - k. \quad (A14) \]

Afterwards, we solve for \( k_2 \) from (A11).

The FOCs for (A14) imply
\[ \frac{2\alpha \beta s^2}{1 + s} = x^{\beta-1}, \quad (A15) \]
\[ s^3 - s^2 + \frac{2\beta}{2\beta - 1} s - \frac{2}{2\beta - 1} = 0. \quad (A16) \]

Substituting \( s = y + \frac{1}{3} \) into (A16) yields
\[ y^3 + \frac{4\beta + 1}{3(2\beta - 1)} y + \frac{14\beta - 52}{27(2\beta - 1)} = 0. \quad (A17) \]

Let \( p \equiv \frac{4\beta + 1}{3(2\beta - 1)} \) and \( q \equiv \frac{14\beta - 52}{27(2\beta - 1)} \). Since \( p > 0 \), there is a unique real root from (A17), which is
\[ \hat{y} = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}. \]

Thus, \( \hat{s}^{**} = \frac{1}{3} + \hat{y} \). Then, by (A11), (A14) and (A15), we find \( \hat{x}^{**}, \Pi_{VC}^{**} \) and \( \hat{k}_2^{**} \).
A.7. Derivation for Proposition 2

In the second-best cases, we have

\[ SW_{SB-SF} = (\beta - 1) \left( \frac{2\alpha}{1 + \beta} \right)^{\frac{\beta}{2}} - k, \]

\[ SW_{SB-UF} = \frac{\beta^2 - 1}{\beta} \left( \frac{\alpha}{\beta} \right)^{\frac{\beta}{2} - 1} - k. \]

Then, \( SW_{SB-SF} > SW_{SB-UF} \) if and only if \( \left( \frac{1 + \beta}{\beta} \right)^{\frac{2\beta - 1}{\beta}} < 2 \). We find that the equation \( \left( \frac{1 + \beta}{\beta} \right)^{\frac{2\beta - 1}{\beta}} = 2 \) has a unique solution \( \bar{\beta} \in (1, \infty) \). Thus, \( SW_{SB-SF} > SW_{SB-UF} \) if and only if \( \beta > \bar{\beta} \). In fact, \( \bar{\beta} \approx 1.4 \).
References


